Design of Machines and Structures, Vol. 13, No. 1 (2023), pp. 114–121.
https://doi.org/10.32972/dms.2023.011

DETERMINATION OF TOOTH SURFACE POINTS ON BEVEL GEARS FOR CHECKING ON A COORDINATE MEASURING MACHINE

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Abstract: The use of coordinate measuring machines is an integral part of gear design. In order to this tool to be truly effective, it is necessary to know the surfaces to be measured and to provide comparative values. In the study, the mathematical modelling of the bevel gear tooth surfaces and the exact description of the production are used to produce the theoretical surfaces on which points can be added in any desired arrangement and number. These points form the basis for the evaluation of the measurement by comparison with the measured values.

Keywords: *coordinate measuring machine, bevel gear, mathematical modelling, grid points*

1. INTRODUCTION

Measurements on coordinate measuring machines (CMM), which can be used to check the accuracy of tooth surfaces, are essential tools for the development of gear drive pairs.

In order to evaluate the measurement results, reference values are needed, as an ideal solution, knowledge of the theoretically accurate tooth surfaces. We can produce these by mathematical modelling of production processes. As a result of the mathematical modelling, we have the position vector and the normal vector of the tooth surface in a coordinate system defined by us. During CMM measurements, the tooth surface is checked as a set of points. For this, the theoretical tooth surface is specified with a finite number of point sets. The points form a surface mesh (grid). The question may arise, how many points do we need? In practice, 45 points are used for CMM measurement of gears. At the same time, with the method presented here, knowing the theoretical tooth surfaces, the points used for inspection can be produced in any desired number and arrangement.

In the study, we focus on bevel gears with curved teeth, but the procedure presented for mathematical modelling can be applied to any gear, or with proper interpretation, it applies to any component.

2. LOCATION OF THE GRID POINTS ON TOOTH SURFACE

The ANSI/AGMA 2009-B01 standard (American Gear Manufacturers Association, 2009) recommends the solution shown in *Figure 1* for the placement of the point grid used for CMM measurement of bevel gears.

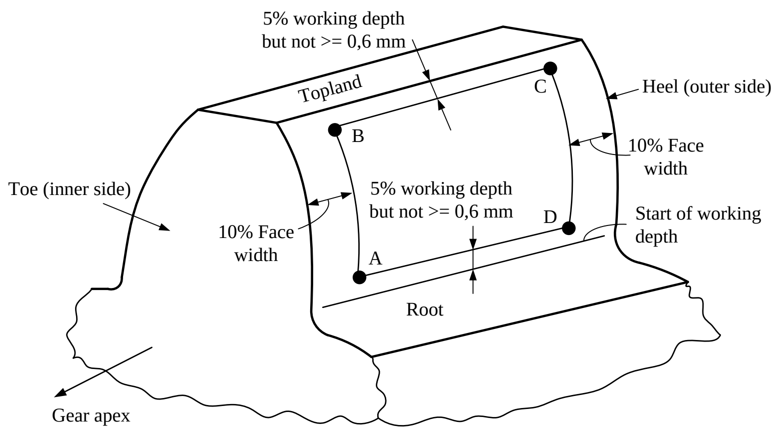

Figure 1. *Placement of mesh points on the tooth surface*

Within the marked area, the points form a 'matrix' consisting of 5 rows and 9 columns according to *Figure 2*. If we want to describe the complete tooth surface, e.g., for finite element analysis, then the points will be recorded on the tooth ends and face cones.

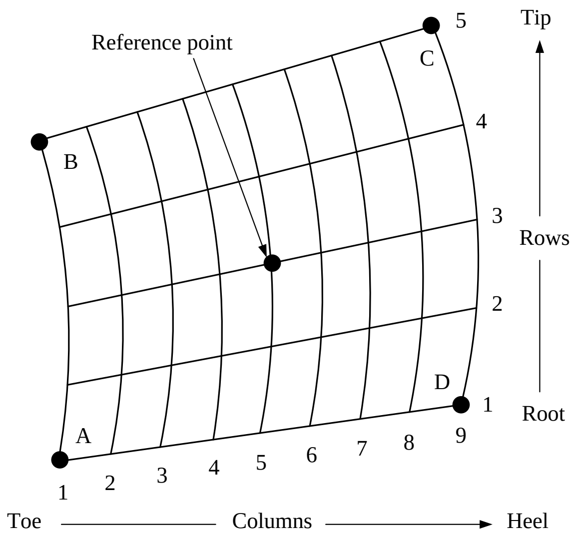

Figure 2. *Arrangement of grid points into rows and columns*

3. DEFINING THE COORDINATES OF THE GRID POINTS

During the present calculations, the point mesh is extended to the useful tooth surface, according to *Figure 3*.

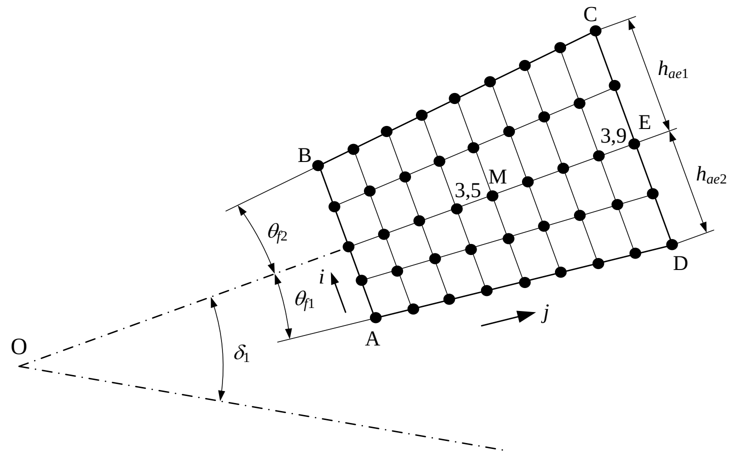

Figure 3. *Placement of grid points on the tooth surface of the smaller bevel gear*

The points forming the surface mesh will be entered in a polar coordinate system. The origin of the coordinate system is the vertex O of the pitch cone of the bevel wheel. Reference point M (3,5) is the point on the pitch cone, belonging to the mean cone distance, whose radial coordinate is $R_{3,5} = R_m \sin\delta_1$, and its axial coordinate is $L_{3,5} = R_m \cos\delta_1$. The distance OE is the outer cone distance R_e. In *Figure 3*, h_{ae1} and h_{ae2} are the addendums of the gear pair, θ_{f1} and θ_{f2} are the dedendum angles, δ_1 is the pitch angle of the bevel gear.

The coordinates of the boundary points are as follows:

$$R_C = R_e \sin\delta_1 + h_{ae1} \cos\delta_1 , \quad L_C = R_e \cos\delta_1 - h_{ae1} \sin\delta_1. \tag{1}$$

$$R_D = R_e \sin\delta_1 - h_{ae2} \cos\delta_1 , \quad L_D = R_e \cos\delta_1 + h_{ae2} \sin\delta_1. \tag{2}$$

$$R_A = R_D + b\left(\tan\theta_{f1} \cos\delta_1 - \sin\delta_1\right), \quad L_A = L_D - b\left(\tan\theta_{f1} \sin\delta_1 + \cos\delta_1\right). \tag{3}$$

$$R_B = R_C - b\left(\tan\theta_{f2} \cos\delta_1 + \sin\delta_1\right), \quad L_B = L_C + b\left(\tan\theta_{f2} \sin\delta_1 - \cos\delta_1\right). \tag{4}$$

The coordinates of the grid points can be calculated as follows:

$$R_{i,j} = R_A + (i-1)\Delta r_i + \frac{j-1}{8}[R_D - R_A + (i-1)(\Delta r_e - \Delta r_i)] \tag{5}$$

$$L_{i,j} = L_A - (i-1)\Delta l_i + \frac{j-1}{8}[L_D - L_A - (i-1)(\Delta l_e - \Delta l_i)] \tag{6}$$

here

$$\Delta r_i = \frac{R_B - R_A}{4}, \quad \Delta r_e = \frac{R_C - R_D}{4}, \tag{7}$$

furthermore

$$\Delta l_i = \frac{L_A - L_B}{4}, \quad \Delta l_e = \frac{L_D - L_C}{4}. \tag{8}$$

To identify the points, i = 1 ... 5, j = 1 ... 9.

4. MATHEMATICAL MODEL OF TOOTH SURFACE

The mathematical modelling of gear tooth surfaces was based on the works of Litvin (Litvin, 1989), (Litvin, 1994), (Litvin & Fuentes, 2004). For the mathematical modelling of the tooth surfaces of the bevel gear, the following data or preliminary calculations are required:

- Geometrical data of the bevel gear.
- Cutter dimensions: cutter radius, blade angle, the relative position of the blades working on the two tooth sides.
- Machine setting data: the relative position of the tool and the workpiece on the machine tool.
- Movement conditions: the coordinated movement of the tool and the workpiece during production.

When manufacturing Gleason-type spiral bevel gears, chipping results from the rotating movement of the cutter head. This movement is independent of other motions that determine the shape of the tooth surface. During rotation, the cutting edges of the tool produce a surface of rotation, which is called generating surface. The position vector of the generating surface can be described with two parameters:

$$\boldsymbol{r}_c(s,\theta) \tag{9}$$

The generating surface results in a family of surfaces in the relative movement. Which has the following form in the coordinate system fixed to the workpiece, with its origin located at the apex of the pitch cone:

$$\boldsymbol{r}(s,\theta,\psi) = \boldsymbol{M}(\psi)\boldsymbol{r}_c(s,\theta) \tag{10}$$

Here, **M** is the matrix of the transformation, ψ is the motion parameter.

To produce the tooth surface, we need to find a relationship between the 3 parameters of the family of surfaces. One possible way to do this is to obtain partial derivatives and solve the following equation:

$$\left(\frac{\partial \boldsymbol{r}}{\partial s} \times \frac{\partial \boldsymbol{r}}{\partial \theta}\right) \cdot \frac{\partial \boldsymbol{r}}{\partial \psi} = f(s, \theta, \psi) = 0 \qquad (11)$$

Here, the first term in brackets on the left side is the normal at the instantaneous point of contact, the second term is the quantity proportional to the relative velocity at the same point. (11) is the equation of meshing.

Using the parameter relation generated from the solution of equation (11), the equation of the tooth surface of bevel gear is available in the following form:

$$\boldsymbol{r}(\theta, \psi) = \boldsymbol{r}(s(\theta, \psi), \theta, \psi) \qquad (12)$$

5. Coordinates of the tooth surface in the grid points

(12) position vector given with the coordinates is the following:

$$\boldsymbol{r}(\theta, \psi) = \begin{bmatrix} x(\theta, \psi) \\ y(\theta, \psi) \\ z(\theta, \psi) \end{bmatrix} \qquad (13)$$

The location of any point on the tooth surface can be determined by solving the following system of non-linear equations.

$$\begin{aligned} &[x(\theta, \psi)]^2 + [y(\theta, \psi)]^2 = R^2, \\ &z(\theta, \psi) = L. \end{aligned} \qquad (14)$$

Here, R is the distance of the point from the axis, and L is the distance of the point from the origin, along the axis.

The parameters θ and ψ are available as a solution to equation system (14), so the coordinates of the point on the tooth surface can be determined.

Now substitute the coordinates of the grid points according to (5) and (6) into equations (14).

$$\begin{aligned} &[x(\theta, \psi)]^2 + [y(\theta, \psi)]^2 = R_{i,j}^2, \\ &z(\theta, \psi) = L_{i,j}. \end{aligned} \qquad (15)$$

Applying equations (15) to all grid points, as a result of i = 1 ... 5, j = 1 ... 9, we have to solve 45 system of equations, which results in 45 θ and 45 ψ values. By substituting $\theta_{i,j}$ and $\psi_{i,j}$, the coordinates of the grid points on the tooth surface are as follows:

$$x_{i,j}\left(\theta_{i,j}, \psi_{i,j}\right), \quad y_{i,j}\left(\theta_{i,j}, \psi_{i,j}\right), \quad z_{i,j}\left(\theta_{i,j}, \psi_{i,j}\right) = L_{i,j}. \qquad (16)$$

6. APPLICATION EXAMPLE

For the practical application of the presented method, we worked out a numerical example. The investigated gear is a spiral bevel gear having circular arc profile along tooth length, whose geometrical data is given in *Table 1*.

Table 1

Dimensions of investigated bevel gear

Designation	Notation	Data
Number of teeth	N	30
Outer transverse module, mm	m_{te}	4,791
Face width, mm	b	40
Pressure angle, °	α_n	20
Mean spiral angle, °	β_m	30
Outer cone distance, mm	R_e	235.01
Mean cone distance, mm	R_m	215.01
Pitch diameter, mm	d_{ae}	143.73
Outer addendum, mm	h_{ae}	3.69
Outer dedendum, mm	h_{fe}	5.16
Cutter radius, mm	r_c	114.3
Cutter point width, mm	P_w	2.286
Pitch angle, °	δ	17.8
Face angle, °	δ_a	17.9667
Root angle, °	δ_f	17.6333
Dedendum angle, °	θ_f	0.1667
Hand of spiral		left

Additional data not included in the table, but necessary for the calculations, were determined based on (American Gear Manufacturers Association, 2006). *Table 2* and *Table 3* summarize the coordinates of the grid points determined by the method presented in Chapter 3.

Table 2

Radial coordinates of grid points

R	1	2	3	4	5	6	7	8	9
1	55.897	57.412	58.926	60.441	61.955	63.47	64.985	66.499	68.014
2	57.677	59.198	60.720	62.241	63.763	65.284	66.806	68.328	69.849
3	59.457	60.985	62.513	64.042	65.570	67.099	68.627	70.156	71.684
4	61.236	62.772	64.307	65.843	67.378	68.913	70.449	71.984	73.520
5	63.016	64.559	66.101	67.643	69.186	70.728	72.270	73.812	75.355

To solve the system of equations (15), we created a program based on mathematical software. *Table 4* and *Table 5* contain the x and y coordinates of the mesh points on the tooth surface. The z coordinate is the same as the L coordinate of the grid points.

Table 3

Axial coordinates of grid points

L	1	2	3	4	5	6	7	8	9
1	186.868	191.633	196.398	201.163	205.928	210.694	215.459	220.224	224.989
2	186.297	191.06	195.822	200.585	205.348	210.111	214.874	219.637	224.4
3	185.725	190.486	195.246	200.007	204.768	209.528	214.289	219.05	223.81
4	185.154	189.912	194.671	199.429	204.187	208.946	213.704	218.463	223.221
5	184.582	189.339	194.095	198.851	203.607	208.363	213.12	217.876	222.632

Table 4

x coordinates of surface grid points

x	1	2	3	4	5	6	7	8	9
1	54.90	56.73	58.53	60.27	61.92	63.46	64.86	66.08	67.07
2	56.57	58.43	60.26	62.02	63.71	65.28	66.71	67.96	68.98
3	58.19	60.09	61.95	63.76	65.48	67.10	68.57	69.86	70.92
4	59.76	61.70	63.61	65.46	67.24	68.90	70.42	71.76	72.87
5	61.28	63.27	65.23	67.14	68.97	70.69	72.26	73.66	74.83

Table 5

y coordinates of surface grid points

y	1	2	3	4	5	6	7	8	9
1	10.52	8.808	6.827	4.567	2.017	–0.835	–4	–7.488	–11.31
2	11.24	9.496	7.484	5.187	2.595	–0.304	–3.522	–7.068	–10.95
3	12.21	10.43	8.375	6.031	3.386	0.4303	–2.849	–6.463	–10.42
4	13.36	11.55	9.444	7.049	4.349	1.331	–2.014	–5.699	–9.734
5	14.68	12.82	10.67	8.22	5.461	2.381	–1.034	–4.793	–8.908

7. Summary

To determine the theoretical tooth surfaces of bevel gears, we created a mathematical model describing the production. Mathematical modelling of tooth surfaces is done by specifying discrete points for practice. The surface can be meshed with any number of points. In practice, the use of 45 points for checking on coordinate measuring machines is widespread. In the grid points, the known coordinates of the theoretical tooth surface can be combined with the measured values to evaluate and classify the accuracy.

For the practical application of the method, we developed a numerical example using the data of a bevel gear with curved teeth. We selected the mesh points and determined their coordinates, then produced the coordinates of the theoretical tooth surface at the given points. These coordinates will be the reference values during the measurement on the coordinate measuring machine.

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