

CALCULATION OF THE FLOW GENERATED IN THE RUNNER OF A BÁNKI TURBINE

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Abstract: The article describes the numerical calculation of a streamline in a liquid jet flowing through the impeller of a Bánki turbine. The importance of the results summarized in the article is that the knowledge of the velocity situation in the liquid jet flowing through the impeller is essential for the calculation of the flow losses occurring in the runner of a Bánki turbine and of the hydraulic efficiency of the runner.

Keywords: *streamline, methodology, Bánki turbine*

1. INTRODUCTION

The Hungarian literature of Bánki turbines has been lacking any improvement of the theory – that has remained the same since the age of Bánki – for a long time. In order to reach any advancement, a completely new way, which diverges from the previously available literature on Bánki turbines, had to be found in the research work. An important partial result of this new way is a new procedure of calculating the physical properties of the flow generated in the runner of a Bánki turbine, the flow losses in the runner of a Bánki turbine, and the hydraulic efficiency of the runner.

2. THE BÁNKI TURBINE

The Bánki turbine utilizes the kinetic energy of the mass flow of the liquid flowing through it. The theoretical head of the runner is equal to the drop in the kinetic energy of the liquid flowing through the runner, related to the unit liquid mass. The liquid fills only a part of the turbine runner. The blade force generated by the interaction between the blades and the liquid diverts the jet. The rotational speed of the runner is determined by the balance of the shaft torque (driving the generator) and the torque generated by the blade force that diverts the jet.

The liquid enters the runner of a Bánki turbine as a free jet. The pressure in a free jet is equal to the environmental pressure. The velocity of the free jet is determined by the available head drop, which means that the quantity of liquid that flows through the runner of a Bánki turbine through unit time is independent of the rotational speed of the runner.

The volumetric flow rate of the liquid that enters the runner is the product of the velocity component perpendicular to the entry surface and the size of the entry surface. The liquid that exits from the guiding channel (*Figure 1*) flows through the blade set of the runner *transversally*, i.e. the entry and the exit surfaces of the liquid are identical. The liquid arriving at the runner circumference flows through the blade set in the *centripetal* direction at first, and then, after passing through the bladeless part inside the runner, enters the blade set again, and flows through it in the *centrifugal* direction before leaving it.

The blade curve has a circular arc shape [2], [3]; the tangents of the blade curve are radial along the inside edge of the bladed area.

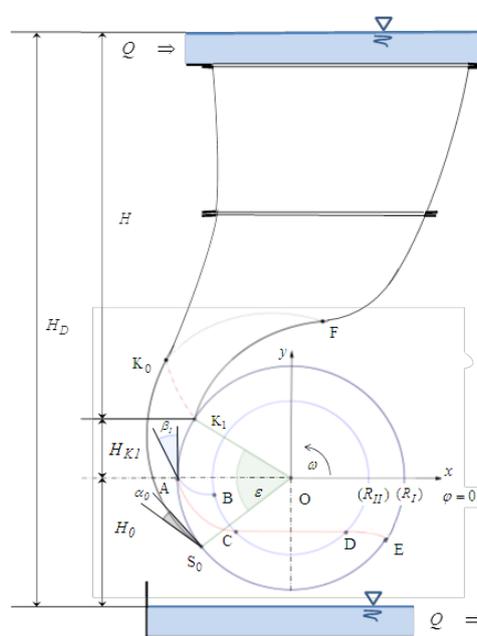


Figure 1. Schematic of a Bánki turbine

The directrix of the guide channel surface is, with the symbols of *Figure 1*, the curve section K_0S_0 , while that of the movable guide channel surface – i.e. of the regulation flap – in the fully open state is the curve segment K_1F . The directrix of the movable guide channel surface – i.e. of the regulation flap – in the completely fully closed state is the curve section K_0F .

The direction angle of the flow entering the runner is the angle between the tangent of the guide channel and that of the circle with the radius R_I , i.e. α_0 . This angle is constant along the runner circumference involved in the liquid entry (K_1S_0). Bánki ensured this by constructing the cylinder-like K_0S_0 and FK_1 surfaces of the guide channel, which are situated between two parallel planes, in a way that their

directrices be logarithmic spirals shifted along the circumference of the runner whose angle of inclination is α_0 . The highest available head (*Figure 1*) is:

$$H = H_D - H_0 - H_{K1} = H_D - H_0 - R_I \sin(\varphi_{S0} - \varepsilon) \quad (1)$$

3. CALCULATION OF THE FLOW GENERATED IN THE RUNNER

A complete absolute streamline along the runner consists of three sections from the entry into the runner (*Figure 1*): the streamline section AC with a centripetal flow direction, the straight streamline CD along the bladeless inside part of the runner and the streamline section DE with a centrifugal flow direction.

Explanation of the indices: the index of velocity and directional angle at the centripetal entry into the bladed runner area is 1; the index of velocity and directional angle at the centripetal exit from the bladed runner area is 2; the index of velocity and directional angle at the centrifugal entry into the bladed runner area is 3; the index of velocity and directional angle at the centrifugal exit from the bladed runner area is 4.

The origins of the Cartesian coordinate system x, y and of the polar coordinate system r, φ are both located in the centre of the runner, O ; the x axis of the Cartesian coordinate system x, y is horizontal. The polar angle φ increases in the counterclockwise direction. The relation between the r, φ polar and the x, y Cartesian coordinates is:

$$x = r \cos \varphi \quad ; \quad y = r \sin \varphi$$

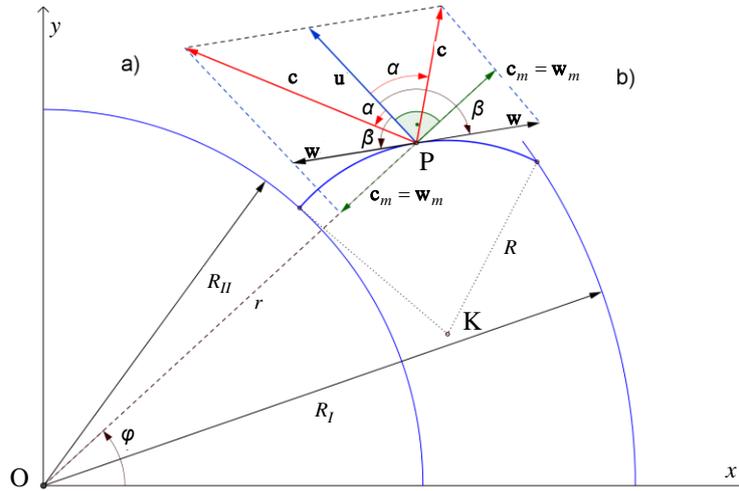


Figure 2

Velocity triangles in one point of the blade curve
 a) along the centripetal section
 b) along the centrifugal section

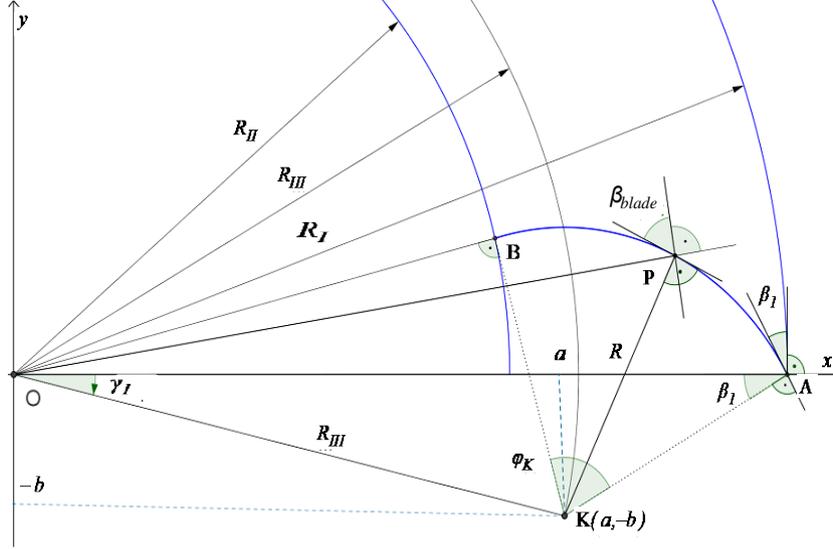


Figure 3. Geometry of a circular blade arc

Figure 2 shows a velocity triangle that applies to a random P point in the bladed area of a Bánki turbine, which is situated on a circle of a radius $R_{II} \leq r \leq R_I$, assuming *infinitely dense and infinitely thin blades*, where the relative flow in the bladed area of the runner is *blade congruent*, i.e. the relative streamlines are identical with the blade curves, which means that the relative velocity vectors are identical with the tangents of the blade curves.

Along the circle with the radius of $R_{II} \leq r \leq R_I$ (the index m here and hereinafter refers to the meridian velocity component)

$$c_m = w_m \quad (2)$$

i.e. along a circle of the same radius, *at the centrifugal- and centripetal-flow sections, the relative velocity vectors \mathbf{w} have the same magnitude, but opposite directions* (Figure 2).

The directional angle β of the relative flow

- along the *centripetal section*: $\beta_1 < \beta \leq \pi/2$ (acute angle);
- along the *centrifugal section*: $\pi/2 \leq \beta \leq \pi - \beta_1$ (obtuse angle).

If assuming a blade congruent relative flow, the outermost streamlines are identical in the jet generated inside the bladed area, which means that the shapes of the outermost streamlines can be rotated around the centre of the runner to cover each other, i.e. the ε angle is constant along the r radius. This means that the theorem of continuity for an incompressible liquid in the bladed area of the runner is described by the equation

$$rc_m = R_I c_{1m} \quad (3)$$

The absolute velocity at the runner entry is defined by the available head H as described by the *Equation (1)*:

$$c_1 = \sqrt{2gH} \quad (4)$$

In the bladed runner area, the relation between circumferential velocity u and meridional velocity c_m is the following:

$$u = c_m(\cot \alpha - \cot \beta)$$

(*Figure 2*). Given the continuity *Equation (3)* and that $u = r\omega = ru_1/R_I$, the above equation can take the following form:

$$u_1 \frac{r}{R_I} = \frac{R_I}{r} c_{1m}(\cot \alpha - \cot \beta) \quad (5)$$

Based on the publication of Czibere [1], the operating state of the runner can, using the velocity relation

$$\psi = \frac{u_I}{c_{1m}} \quad (6)$$

be described by the following form of *Equation (5)*:

$$\cot \alpha = \psi \left(\frac{r}{R_I} \right)^2 + \cot \beta \quad (7)$$

which is the *basic equation* of the flow occurring in the bladed area of the runner of a Bánki turbine. This basic equation – assuming a *blade congruent* relative flow – provides a relation between the directional angle α of the absolute flow and that of the relative flow, β , depending on the operating state of the turbine. The basic *Equation (7)* can be used for calculating a streamline of the liquid jet occurring in the runner.

The blade angle β_{blade} is an acute angle in a point P of the blade curve, between the tangent of the circular arc-shaped blade curve and the tangent of a circle concentric with the runner circumference and containing the point P (*Figure 3*). When assuming a blade congruent relative flow, the directional angle β of the relative velocity in the bladed area of the runner is, along the section with *centripetal* flow, $\beta = \beta_{blade}$ (acute angle), while along the section with *centrifugal* flow, $\beta = \pi - \beta_{blade}$ (obtuse angle). The basic *Equation (7)* of the flow in the bladed area of the runner of a Bánki turbine takes the following form when written with the *blade angle*:

$$\cot \alpha = \psi \left[\frac{r}{R_I} \right]^2 \pm \cot \beta_{blade}(r) \quad (8)$$

where the positive sign applies to the centripetal-flow section ($\beta = \beta_{blade}$), and the negative one to the centrifugal-flow one ($\beta = \pi - \beta_{blade}$).

The angle β_{blade} between the tangent of the circular arc shaped blade curve and the tangent of the circle of a radius r is defined by the following equation based on *Figure 3*:

$$\beta_{blade}(r) = \pi - \arccos\left(\frac{R_{III}^2 - R^2 + r^2}{2r R_{III}}\right) - \arccos\left(\frac{R_{III}^2 + R^2 - r^2}{2R_{III}R}\right); \quad R_{II} \leq r \leq R_I,$$

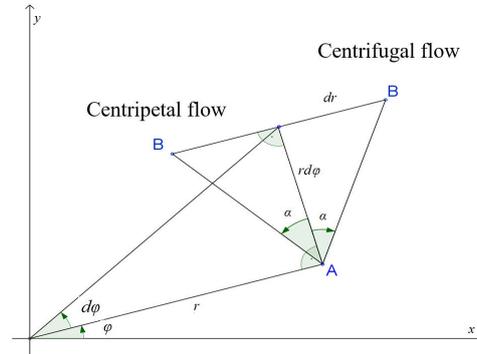


Figure 4. The relationship between the directional angle α of the absolute velocity and the elementary triangle defined by the differentials

Along the blade curve, the directions of the relative velocity along the centripetal-flow and the centrifugal-flow sections are opposite, i.e. the differential equations of the streamline parts corresponding to the bladed areas are, in the polar coordinate system r, φ around the rotational axis, different for the centripetal-flow and the centrifugal-flow sections.

Along the *centripetal-flow section*, the relationship between the directional angle α of the absolute velocity and the differentials applying to the elementary AB section of the streamlines can, according to *Figure 4*, be defined using the basic *Equation (8)*:

$$\cot \alpha = -r \frac{d\varphi}{dr} = \psi \left(\frac{r}{R_I} \right)^2 + \cot \beta_{blade}(r)$$

The differential equation of the absolute streamline along the centripetal-flow area is:

$$\frac{d\varphi}{dr} = -\frac{1}{r} \left[\psi \left(\frac{r}{R_I} \right)^2 + \cot \beta_{blade}(r) \right]$$

By integrating it:

$$\varphi(r) = \int_r^{R_I} \left[\psi \left(\frac{r}{R_I} \right)^2 + \cot \beta_{blade}(r) \right] \frac{dr}{r}; \quad R_{II} \leq r \leq R_I \quad (9)$$

The point of the centripetal section of the streamline, which is located on the runner circumference has, due to $\varphi(R_I) = 0$, the polar coordinates $(R_I; 0)$.

Along the *centrifugal-flow section*, the relationship between the directional angle α of the absolute velocity and the differentials applying to the elementary AB section of the streamlines can, according to *Figure 4*, be defined using the basic *Equation (8)* as written with the blade angle β_{blade} :

$$\cot \alpha = r \frac{d\varphi}{dr} = \psi \left(\frac{r}{R_I} \right)^2 - \cot \beta_{blade}(r)$$

The differential equation of the absolute streamline along the centrifugal-flow area is:

$$\frac{d\varphi}{dr} = \frac{1}{r} \left[\psi \left(\frac{r}{R_I} \right)^2 - \cot \beta_{blade}(r) \right]$$

By integrating it:

$$\varphi(r) = \varphi_D + \int_{R_{II}}^r \left[\psi \left(\frac{r}{R_I} \right)^2 - \cot \beta_{blade}(r) \right] \frac{dr}{r} \quad ; \quad R_{II} \leq r \leq R_I \quad (10)$$

In the *Equation (10)* the angular coordinate φ_D of the streamline point D depends on the position of the straight streamline part CD (*Figure 5*). The angular coordinate φ_C of the point C is calculated using the definite integral from (9).

$$\varphi_C = \varphi(R_{II}) = \int_{R_{II}}^{R_I} \left[\psi \left(\frac{r}{R_I} \right)^2 + \cot \beta_{blade}(r) \right] \frac{dr}{r} \quad (11)$$

The direction of the straight streamline section is defined by the directional angle of the streamline in the point C, α_C . This can be calculated using the form of the basic *Equation (8)*, given that in the point C, the blade angle is $\beta_C = \pi/2$ and thus $\cot \beta_C = 0$:

$$\cot \alpha_C = \psi \left(\frac{R_{II}}{R_I} \right)^2 \quad \Rightarrow \quad \alpha_C = \arccot \left[\psi \left(\frac{R_{II}}{R_I} \right)^2 \right]$$

Based on *Figure 5*, the angular coordinate of the point D, φ_D is defined by that of the point C, φ_C and by the directional angle α_C , as follows:

$$\varphi_D = \varphi_C + 2\alpha_C$$

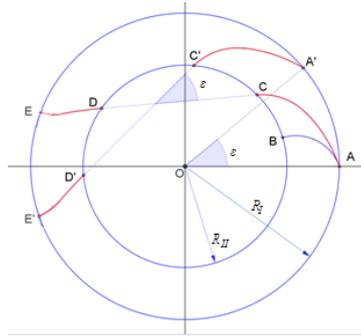


Figure 6. Streamline sections AC and DE , and their projections $A'C'$ and $D'E'$ as rotated by the central angle ε

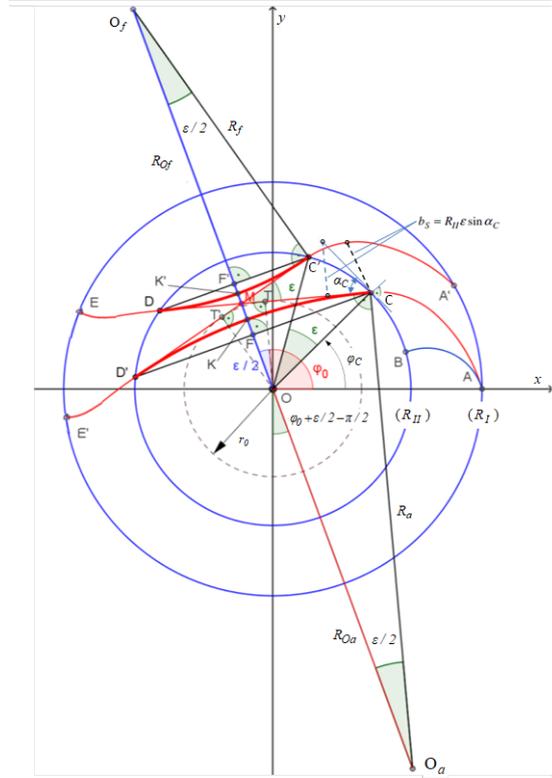


Figure 7. Contraction of the liquid jet flowing through the blade set

Given the continuity equation that applies to the centrifugal-flow runner section, $R_{II}c_{2m} = R_Ic_{1m}$ the formula

$$c_{1m}^2 = c_1^2 \sin^2 \alpha_0 = c_1^2 \frac{1}{1 + \cot^2 \alpha_0},$$

the fact that $\beta_2 = \pi/2$ and thus the basic Equation (7) being

$$\cot \alpha_2 = \psi \left(\frac{R_{II}}{R_I} \right)^2$$

in the directional angle α_2 , and the formula (4), the following equation can be written:

$$c_2 = \frac{c_{2m}}{\sin \alpha_2} = \frac{c_{1m}}{\sin \alpha_2} \frac{R_I}{R_{II}} = \sqrt{2gH} \frac{\sin \alpha_0}{\sin \alpha_2} \frac{R_I}{R_{II}} = \sqrt{2gH} \frac{R_I}{R_{II}} \sqrt{\frac{1 + \psi^2 (R_{II}/R_I)^4}{1 + \cot^2 \alpha_0}},$$

and therefore the specific contraction energy loss as related to the unit mass of the flowing medium takes the following form:

$$e'_K = \zeta_K gH \frac{1 + S_{II}^4 \psi^2}{S_{II}^2 (1 + \cot^2 \alpha_0)},$$

where $S_{II} = R_{II} / R_I$ is the runner radius ratio and α_0 is the directional angle of the guide channel. Finally, the power loss of the mass flow ρQ that flows through the runner due to radius contraction is:

$$P'_K = e'_K \rho Q = \zeta_K \rho gQH \frac{1 + S_{II}^4 \psi^2}{S_{II}^2 (1 + \cot^2 \alpha_0)} \quad (12)$$

The frictional losses caused by the viscosity of the liquid that flows through the runner of a Bánki turbine occur in the runner blade channels in contact with the medium. The number of blade channels in contact with the flow that enters along the section defined by the central angle ε into a partially open runner with N blades is, along the centripetal-flow section of the blade set, $N\varepsilon/2\pi$, and the same along the centrifugal-flow section, i.e. the total number of blade channels in contact with the medium is $N\varepsilon/\pi$. The length of the rectangular cross-section pipe equivalent to the blade channels in contact with the medium is:

$$L_{cs} = L_k N\varepsilon / \pi,$$

where L_k is the length of the circular arc shaped blade. The relative flow that occurs in the blade channels can be approximated to be a flow in a rectangular cross-section pipe, where the flow velocity is the geometric mean of the relative velocity w_1 at the entry into the runner along the circumference of a radius R_I , and the relative velocity w_2 at the exit along the circumference of a radius R_{II} :

$$\bar{w} = \sqrt{w_1 w_2} = c_{Im} \sqrt{\frac{R_I}{R_{II} \sin \beta_1}}.$$

The hydraulic diameter of the rectangular cross-section pipe is:

$$D_H = 4 \frac{A}{K} = 4 \frac{L2\pi R_I / N}{2(L + 2\pi R_I / N)} = \frac{4L\pi R_I}{LN + 2\pi R_I},$$

where A is the cross-section, K is the circumference in contact with the medium, and L is the runner width, which is, in this case, identical with the guide channel width.

The specific frictional loss related to the unit mass of the liquid flowing through the runner is:

$$e'_V = \zeta_V \frac{\bar{w}^2}{2} = \zeta_V gH \frac{1}{S_{II} \sin \beta_1 (1 + \cot^2 \alpha_0)},$$

where ζ_V is the frictional loss factor, which can be calculated using the following formula:

$$\zeta_V = \lambda_{cs} \frac{L_{cs}}{D_H}$$

The pipe friction factor λ can be calculated using the well-known pipe friction law when the Reynolds number $Re = \bar{w}D_H / \nu$ is known. Therefore, the power loss caused by the medium viscosity in a mass flow ρQ is:

$$P'_V = \zeta_V \rho g Q H \frac{1}{S_H \sin \beta_1 (1 + \cot^2 \alpha_0)} \quad (13)$$

The basic *Equation (7)* dictates that the blade angle at the entry, β_1 , and the velocity ratio that defines the operating state of the runner, $\psi = u_1 / c_{1m}$ (where u_1 is the tangential velocity in the runner entry point and c_{1m} is the radial component of the entry velocity in the same point) unambiguously define the absolute directional angle α_1 of the entry velocity c_1 :

$$\cot \alpha_1 = \psi + \cot \beta_1.$$

If this directional angle α_1 is identical with the directional angle α_0 of the Bánki turbine guide channel, the absolute flow will enter the runner without any *directional refraction*. This so-called *impact-free* inflow will, according to the previous equation, occur in the

$$\psi_0 = \cot \alpha_0 - \cot \beta_1$$

operating state of the runner. If the turbine operating state is different ($\psi \neq \psi_0$), the directional angle α_1 of the absolute entry velocity will also differ from the directional angle α_0 of the guide channel – in this case, an energy loss will occur in the liquid jet entering the runner. This specific energy loss, related to the unit mass flow, is proportional to the square of the absolute value of the velocity difference Δc between the absolute velocity at a directional angle $\alpha_1 \neq \alpha_0$ and that at a directional angle α_0 (the latter being the impact-free operating state):

$$e'_0 = \zeta_0 \frac{|\Delta c|^2}{2} = \zeta_0 \frac{|u_1 - u_0|^2}{2} = \zeta_0 \frac{(\psi - \psi_0)^2 c_{1m}^2}{2}$$

where ζ_0 is the impact loss factor and c_{1m} is the meridian component of the velocity of the liquid jet entering the runner, which is defined by the available elevation drop H and the directional angle α_0 of the guide channel:

$$c_{1m} = \sqrt{2gH} \sin \alpha_0 = \sqrt{\frac{2gH}{1 + \cot^2 \alpha_0}}$$

In the fully open state, the impact-related power loss in a mass flow ρQ through the runner is:

$$P'_0 = e'_0 \rho Q = \zeta_0 \rho g Q H \frac{(\psi - \psi_0)^2}{1 + \cot^2 \alpha_0} \quad (14)$$

The power of the liquid jet arriving at the runner of a Bánki turbine in the fully open turbine state is unambiguously defined by the mass flow ρQ arriving at the runner and the available elevation drop H :

$$P_H = \rho g Q H .$$

The *theoretical* elevation drop of the runner is, in the fully open turbine state, equal to the change in the kinetic energy of the unit weight of the liquid that flows through the turbine:

$$H_E = \frac{c_1^2 - c_4^2}{2g} = \frac{c_1^2}{2g} \left(1 - \frac{c_4^2}{c_1^2} \right) \quad (15)$$

Given that $c_1 = \sqrt{2gH}$; $c_{1m} = c_{4m}$; $c_1 = c_{1m} / \sin \alpha_0$; $c_4 = c_{4m} / \sin \alpha_4$ and using the identity $\sin^2 x = 1/(1 + \cot^2 x)$, the velocity ratio c_4 / c_1 can be described by the following formula:

$$\frac{c_4^2}{c_1^2} = \frac{\sin^2 \alpha_0}{\sin^2 \alpha_4} = \frac{1 + \cot^2 \alpha_4}{1 + \cot^2 \alpha_0},$$

and the basic *Equation (7)* dictates, (given that $\beta_4 = \pi - \beta_1$) that

$$\cot \alpha_4 = \psi + \cot \beta_4 = \psi - \cot \beta_1,$$

therefore the *Equation (15)* can be written in the following form:

$$H_E = H \left(1 - \frac{1 + (\psi - \cot \beta_1)^2}{1 + \cot^2 \alpha_0} \right)$$

which means that the theoretical power of the runner is:

$$P_E = \rho g Q H_E = \rho g Q H \left(1 - \frac{1 + (\psi - \cot \beta_1)^2}{1 + \cot^2 \alpha_0} \right).$$

By subtracting the impact loss P'_0 , the contraction loss P'_K and the frictional loss P'_V from the theoretical power P_E of the runner, the runner power P_{jk} is obtained. In the fully open state, the *hydraulic efficiency* of the runner is the ratio of the runner power and the power of the liquid jet arriving at the runner:

$$\begin{aligned} \eta_{jk} &= \frac{P_{jk}}{P_H} = \frac{P_E - (P'_0 + P'_K + P'_V)}{P_H} = \\ &= 1 - \frac{1 + (\psi - \cot \beta_1)^2}{1 + \cot^2 \alpha_0} - \frac{\zeta_0 S_{II}^2 (\psi - \psi_0)^2 + \zeta_K (1 + S_{II}^4 \psi^2)}{S_{II}^2 (1 + \cot^2 \alpha_0)} - \left. \frac{\zeta_V}{S_{II} \sin \beta_1 (1 + \cot^2 \alpha_0)} \right) \end{aligned} \quad (16)$$

The hydraulic efficiency of a stationary runner (in this state, $\psi = 0$) is zero. The freely selectable impact loss factor ζ_0 should be selected so that, in case of $\psi = 0$, the hydraulic efficiency calculated using (16) be zero. By multiplying both sides of the equation $\eta_{jk} = 0$ with the expression $1 + (\psi_0 + \cot \beta_1)^2$ – given that the basic Equation (7) dictates that $\cot \alpha_0 = \psi_0 + \cot \beta_1$ –, the following equation is obtained to the loss factor ζ_0 :

$$\psi_0^2 + 2\psi_0 \cot \beta_1 - \frac{\zeta_0 \psi_0^2 S_{II}^2 + \zeta_K}{S_{II}^2} - \zeta_V \frac{1}{S_{II} \sin \beta_1} = 0$$

From this equation, the impact loss factor ζ_0 can be written in the following formula:

$$\zeta_0 = 1 + \frac{2 \cot \beta_1}{\psi_0} - \frac{\zeta_K}{\psi_0^2 S_{II}^2} - \frac{\zeta_V}{\psi_0^2 S_{II} \sin \beta_1} \quad (17)$$

4. SUMMARY

Assuming a circular arc-shaped blade curve (the tangent to the blade curve is radial at the inside edge of the bladed area) and a blade congruent relative flow, the formulas deduced in the publication make it possible to calculate the r, φ coordinates of the points of a streamline in the free jet crossing the runner from the dimensions R_I, R_{III}, R of a Bánki turbine runner in a required operating state ψ . Based on these coordinates, the complete streamline can be drawn by points in the coordinate plane r, φ . No such calculation has been published in the literature yet. Once the velocity situation in the liquid jet flowing through the runner is known, it becomes possible to determine the frictional losses in a Bánki turbine runner and the hydraulic efficiency of the runner, too.

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