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THE EFFECT OF THE CONTACT ZONE OF CYLINDRICAL HELICAL GEARS ON THE MESHING AND SOME CONSIDERATIONS FOR DETERMINING ITS SHAPE

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Abstract: The study deals with the meshing characteristics of cylindrical helical external gear pairs. The gear pairs, following the nowaday's strength and quality requirements are becoming ever smaller. Accuracy in the background also attracts the importance of vibration and noise reduction. The inclined tooth meshing, in contrast to the straight tooth, due to the specificity of its zone of contact, is the subject of this study. This is of special importance because the meshing stiffness varies for one to more teeth pairs, the meshing contact lines are of continuously varying length during meshing, and as a consequence load sharing and distribution is changing with. This paper deals with the zone of contact and its geometric modification in order to light on a new type of vibrational excitation.

Keywords: *zone of contact, meridian, top land modification*

1. DEVELOPMENTAL MOTIVATORS OF THE MESHING NATURE OF CYLINDRICAL HELICAL GEARS

Technical progress in moving structures has always shown that developers cannot avoid that the structures they build includes a toothed element in the drive chain. The drive chains used have and continue to have a wide variety of shapes, from the simplest to the most advanced solutions used nowadays. This diversity was reflected in the materials used, the teeth geometry and the expected accuracy [1]. The design of the tooth has undergone a long development from, through the carved tooth, to the fine finished tooth form [1, 9].

The development of military technology (on the land, in the air, under water), in parallel with the development of terrestrial civilian means of transport, required more and more precise elements of the drive chains. This was motivated on the one hand by the extension of service life, on the other hand by safety and on the third hand by recognizability. In terms of service life, it can be observed that the gears require less and less care. The quality of the materials used and the refinement of strength calculation and inspection procedures also support safety [2].

Recognition is already a more complex problem. The gear transmission was initially expected to be reliable, today it is expected to be also quiet. In special cases the unidentifibility of the drive chain can be also a requirement, especially by the military equipement.

The development today is clearly directioned as follows:

– the toothed element connection in the drive chains cannot be avoided,
– the accuracy of the motion mapping, thus reducing the variation of the angular velocity to an absolute minimum,
– the vibration-generating sources of the meshing shall be minimized,
– the coupling of the toothed pair should have less acoustic emission in order to become more difficult to recognize it.

2. THE CONTACT ZONE AS THE LOCATION OF THE MESHING IS THE SOURCE OF THE PROBLEMS

The mapping of the contact zone is well defined and described in all gear literature [3, 4, 5, 10], yet let us consider it in a figure *(Figure 1)*. The tooth pairs are meshing in a field (AEA'E'□). Points A and E on the line of action are designated by the head cylinders. The common width (b) of the gear body determines the points A' and E', thus the theoretical zone of the meshing becomes the rectangle AEA'E'.

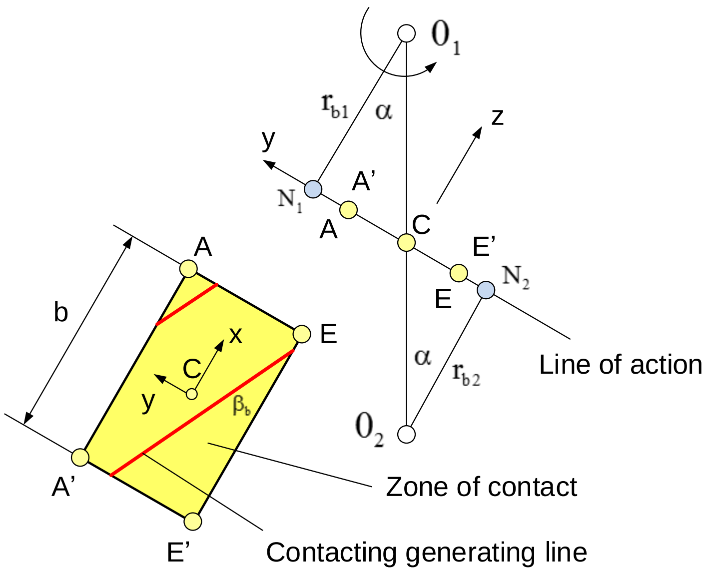

Figure 1

Interpretation of the uncorrected contact zone

The meshing begins at point A and proceeds to point E'. It can be observed that, depending on the base pitch, more than one pair of teeth can connect at the same time, which is also indicated by the number of contacting generating lines. The individual length of the instantaneous contacting generating lines and their sum also change continuously in the zone of contact. These lengths are determined by purely geometrical features. The meshing pairs of teeth also carry a load, the consequence of which is that their stiffness – considering them individually or summarized – is constantly changing. At the same time the load distribution between and along the contacting generating lines also changes [5, 6]. The movement in the zone of contact and the load conditions can be significantly affected by the manufacturing and assembling errors, as well by the errors resulting from the elastic deformation of the drive.

The effects of the errors mentioned above result in vibrations and acoustic phenomena. Research in recent decades has focused on understanding these phenomena, on exploring their impact, and on reducing their influence [7, 13, 14].

3. POSSIBILITIES OF DEFINING THE CONTACT ZONE

The meshing characteristics of helical cylindrical gears are affected by the shape (appearance) of the zone of contact. This statement is of great significance because here appears the characteristic effect and source that determines the connection of each gear.

When designing the gear, a basic geometry is defined, which records the basic input data (gear ratio, basic profile, module, number of teeth), the diameter of the characteristic circles (cylinders), the shaft distance, and the common tooth width. The top land of the teeth is a cylinder whose meridian section is a line parallel to the axis. The zone of contact that can be mapped from this is a rectangle *(Figure 1)*. If we want to form a different geometric shape in addition to the regular rectangular shape for some expedient consideration, three possible ways of defining the zone of contact are conceivable. Variants can be created through keeping the meridian section of the theoretical head cylinders unchanged or changing them. The basic cases are as follows:

– the meridian section of the top land surface remains a line parallel to the axis of rotation *(Figure 1)*, i.e. a rectangular zone,
– the meridian of the top land surface is determined using straight lines or a set of higher order curves (direct method),
– the complete rectangular zone of contact is modified first and the meridian curve or curves of the top land surface are determined from this (indirect method).

The algorithm for determining the possible solutions is illustrated in *Figure 2*, which also points the necessary modification of the drawings of gears.

4. ZONE OF CONTACT GENERATED BY THE MERIDIAN OF TOP LAND SURFACE, INDIRECT PROCEDURE

The indirect solution of the mapping of the contact zone means starting from the given geometry of the meshing gears and not touching directly the zone of contact. The actual geometry here means that all the geometrical data of the gears are known, as well the dimensions related to the center distance. *Figure 3* shows the mapping of the contact zone, strating first from an unmodified top land surface, followed by the mapping after the modification of the topand surfaces, using the indirect procedure. The starting point for the mapping is to disregard the modification of the meridian of the head cylinder. A regular rectangular zone of contact can then be mapped. The geometric basis of this is known from several literature [3, 5]. This is determined on the one hand by the geometric dimensions from the basic geometry:

– normal module,
– number of teeth,
– base profile angles (working, supporting),
– addendum height coefficient,
– clearance coefficient,

on the other hand, the connection characteristics:

– shaft distance,
– addendum modification coefficients,
– addendum circles (uncorrected case),
– dedendum circles,
– tooth width.

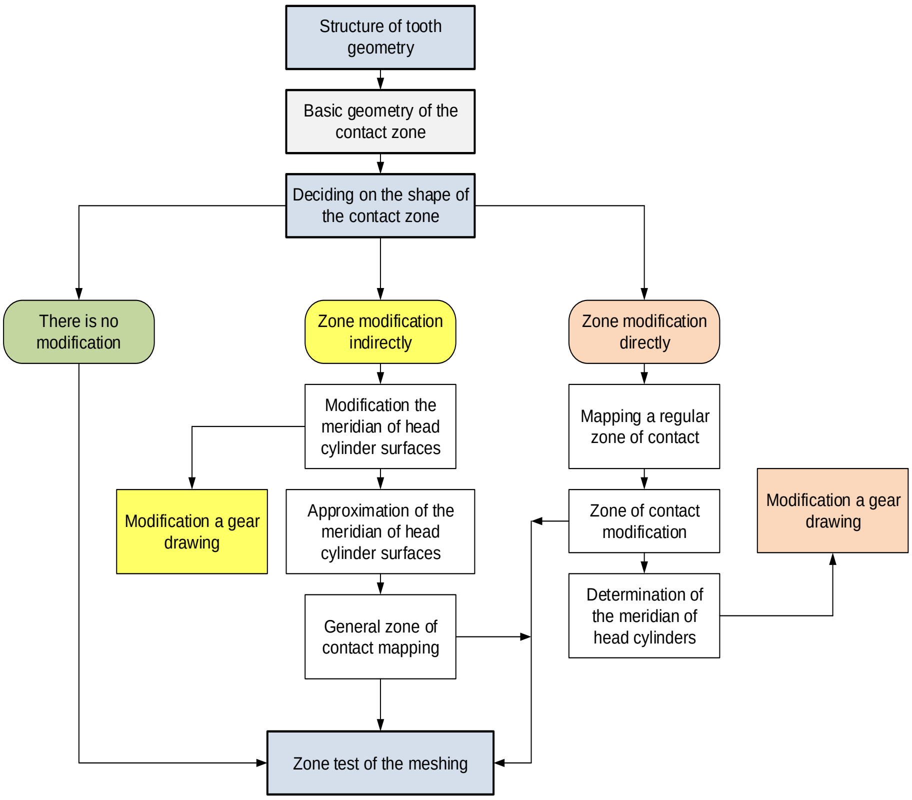

Figure 2

Possible cases of defining the zone of contact (own figure)

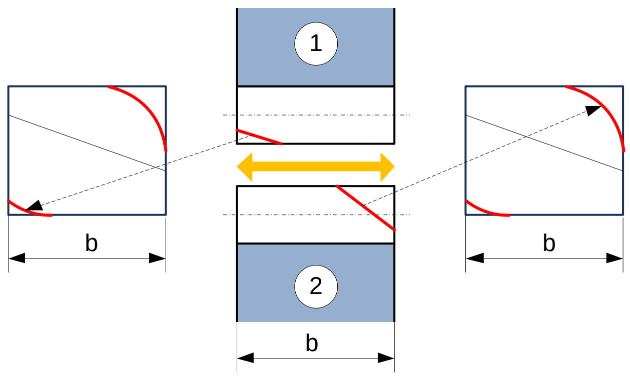

Figure 3
The top land surface generates the zone of contact,
while applying the indirect procedure

The meridian of the top land surface can be modified by breaking the straight line parallel to the axis of rotation, by another straight line or regulus, in a more complex case by a higher order curve. Such possibilities are illustrated in *Figure 4* on a single gear only.

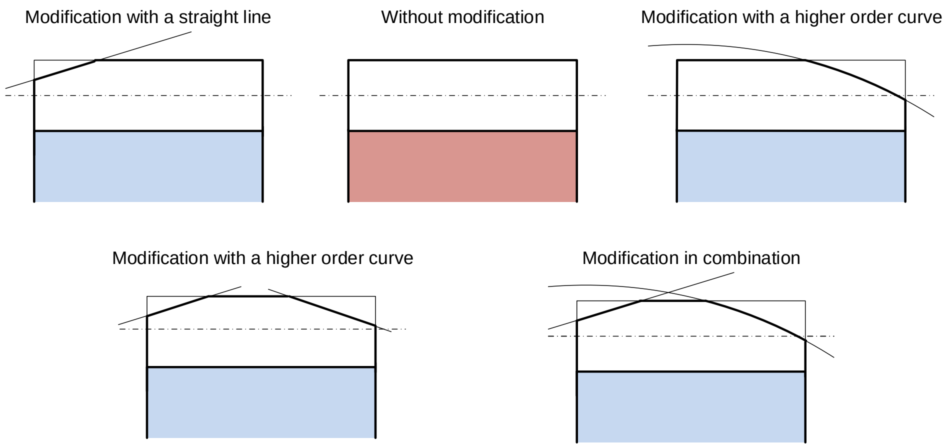

Figure 4
Top land meridian design options

The zone of contact is limited here by the upper and lower zone borders and the common tooth width, as illustrated in *Figure 5*. The top land meridian was modified by taking a straight line for each gear as shown in *Figure 3*. The borders of the contact zone can be determined by the coordinates (x_i^*, r_{i*}) of the points of the meridian curve in the x, y coordinate system, which is connected to the main point C taken in the middle of the common tooth width (*Figures 1* and *5*).

The y coordinates of the zone borders can be determined by *Equations (1)* and *(4)* as a function of x, whose domain is: $-b/2 \le x \le b/2$.

Zone upper border points (red line) are defined by the following equations:

$$y_F^* = \overline{N_2A^*} - \overline{N_2C}, \tag{1}$$

$$\overline{N_2A^*} = \sqrt{\left(\frac{d_{a2}^*}{2}\right)^2 - \left(\frac{d_{b2}}{2}\right)^2}, \tag{2}$$

$$\overline{N_2C} = \sqrt{\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_{b2}}{2}\right)^2}. \tag{3}$$

Zone lower border points (blue line) are given by:

$$y_A^* = \overline{N_1C} - \overline{N_1E^*}, \tag{4}$$

$$\overline{N_1E^*} = \sqrt{\left(\frac{d_{a1}^*}{2}\right)^2 - \left(\frac{d_{b1}}{2}\right)^2}, \tag{5}$$

$$\overline{N_1C} = \sqrt{\left(\frac{d_1}{2}\right)^2 - \left(\frac{d_{b1}}{2}\right)^2}. \tag{6}$$

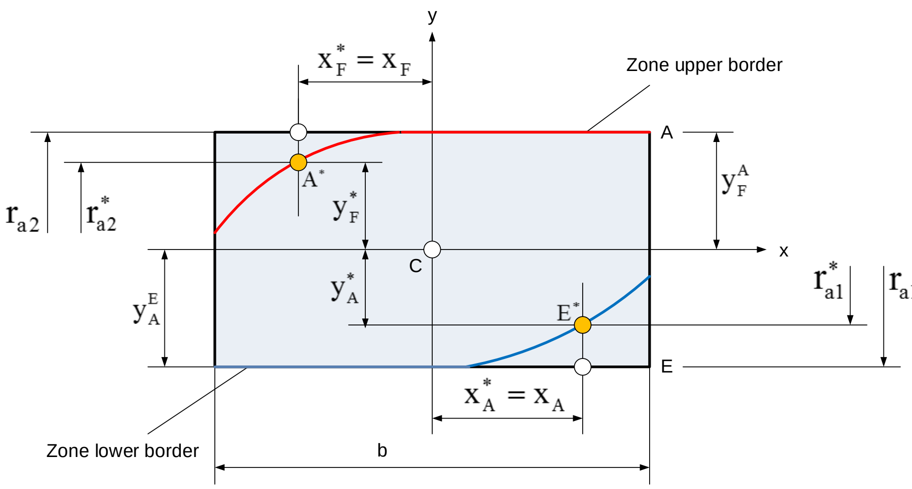

Figure 5

Geometric mapping of the contact zone

In the zone of contact, as shown in *Figure 1*, the contacting generating lines are located according to the inclination of tooth directional angle (β_b) on the base cylinder and thus they traverse in this mode the zone of contact. It can also be seen that the lengths of instantaneous contacting generating lines and their sum also vary. The nature of the change will be influenced by the total tooth width (b) and the shape of the lower and upper zone borders. Here we do not deal with the analytical solution, it will be included in a further presentation.

5. GENERATION OF THE MERIDIAN OF THE TOP LAND SURFACE, DIRECT PROCEDURE

The direct procedure for modifying the contact zone consists in modifying first the regular rectangle zone of contact. The reason for the modification may be to improve a zone property [7, 8]. For example, it may be to reduce the amount of variation of the total length of the contacting generating lines or restrict the migration of tooth forces. *Figure 5*. shows, on the one hand, the gears with an uncorrected top land surface and the corresponding regular rectangular zone of contact while by the other hand, it shows the modified contact zone and its effect on the form of the meridians of the gear top land surface.

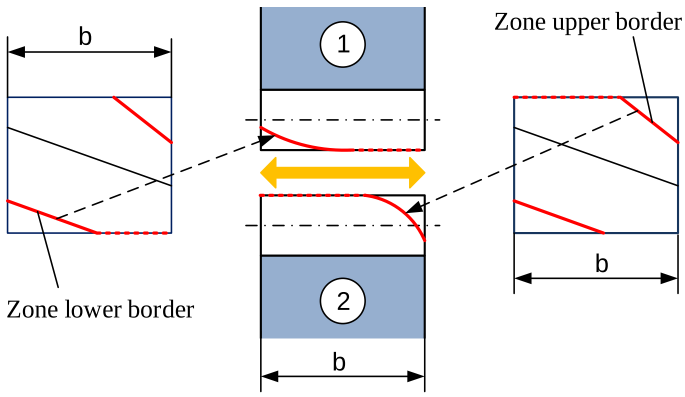

Figure 6
The zone of contact generates the meridian of the top land surface, direct procedure

In the zone on the left, we truncate the zone with a straight line not the full width of the tooth, so we get a new *lower zone border*. That zone border modifies the top land surface of the gear 1, thus determining the shape and expression of the meridian. In the zone on the right side, we have proceeded in a similar manner to determine the *upper zone border* and the meridian of the top land surface of gear 2. Of course, the modification using a straight line is not the unique solution; it can be implemented also through regulus or a higher order curve, resp. a group of curves.

Figure 7 illustrates a modified zone of contact obtained by the direct method. The marked points on the zone borders, are transposed on the top lands as points of the modified meridian curves.

$$r_{a1}^{*} = \sqrt{\left(\frac{d_{b1}}{2}\right)^{2} + \left(\overline{N_1E^{*}}\right)^{2}} \tag{7}$$

$$\overline{N_1E^{*}} = \overline{N_1C} + \left|y_A^{*}\right|, \tag{8}$$

$$\overline{N_1C} = \sqrt{\left(\frac{d_1}{2}\right)^{2} - \left(\frac{d_{b1}}{2}\right)^{2}}\ , \tag{9}$$

and

$$r_{a2}^{*} = \sqrt{\left(\frac{d_{b2}}{2}\right)^{2} + \left(\overline{N_2A^{*}}\right)^{2}}\ , \tag{10}$$

$$\overline{N_2A^{*}} = \overline{N_2C} + \left|y_F^{*}\right|, \tag{11}$$

$$\overline{N_2C} = \sqrt{\left(\frac{d_2}{2}\right)^{2} - \left(\frac{d_{b2}}{2}\right)^{2}}\ . \tag{12}$$

Figure 8 shows where this point is located on the gears. The point taken at the zone border can be transferred to the gear by *Equations (7)* to *(12)* using *Figure 1*. The relations can be applied for all the points of the lower and upper zone borders with arbitrary x^{*}coordinate points. It determines the meridians of the top land surfaces, to which can be finally added a fitting function.

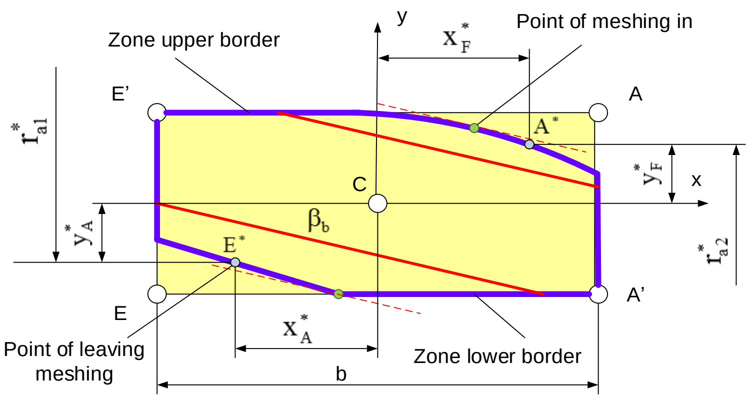

Figure 7. *Defining a zone of contact directly*

In the zone of contact, the lower and upper zone borders will contain also unaffected segments. The top land surfaces of the gears remain unchanged in the width corresponding to this zone, which is also shown in *Figure 8*.

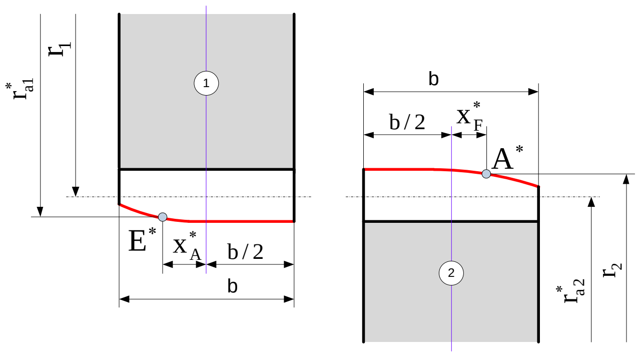

Figure 8
Gears with modified top land surfaces

The corrected tooth top land surface can be provided by machining the wheel bodies before toothing. This is easily feasible in nowadays modern CNC techniques.

6. THE EFFECT OF THE CHANGED CONTACT ZONE ON THE LENGTH OF THE CONTACTING GENERATING LINES

In the zone of contact, the contacting generating lines of the meshing pairs of teeth follow each other at a distance of base pitch in normal section p_{bn}. In the case where the zone of contact is not modified, the length of the contacting generating lines can be determined from the parameters of the contact zone. Examining the pair of teeth entering at point **A** (point of meshing in) and its contacting generating line *(Figure 9)*, it can be observed that its length varies continuously till they reach the point of leaving the meshing **E**. This change in length is also influenced by the common tooth width **b**, which is always the result of a designer decision. The effect of the length variation on the meshing process can be described in dependence with the zone parameter Δy .

The summed contacting generating line results as the sum of each component:

$$L_{\Sigma}(\Delta y) = \sum_{i=1}^{i=n(\Delta y)} L_i(\Delta y). \tag{13}$$

In the design phase, it is expected to reach the maximum load capacity in addition to minimum weight. The defining of the common tooth width also obeys this goal. The literature [1, 3,] was coming here with a recommendation that couldn't be refuted for a long time. According to that, as long the common tooth width is imposed to be an integer multiple of the axial pitch (p_x), the sum of the lengths of the components (contacting generating lines) remains constant and thus torsional excitation can be avoided. This is in fact true, but subsequent research [12, 13] has shown that this cannot be substantiated, since the common tooth width must be determined from different consideration, because other types of excitations are also present. Subsequent research [7, 8, 11] confirmed this hypothesis that significant results in reducing vibration excitation can be achieved by expedient modification of the contact zone.

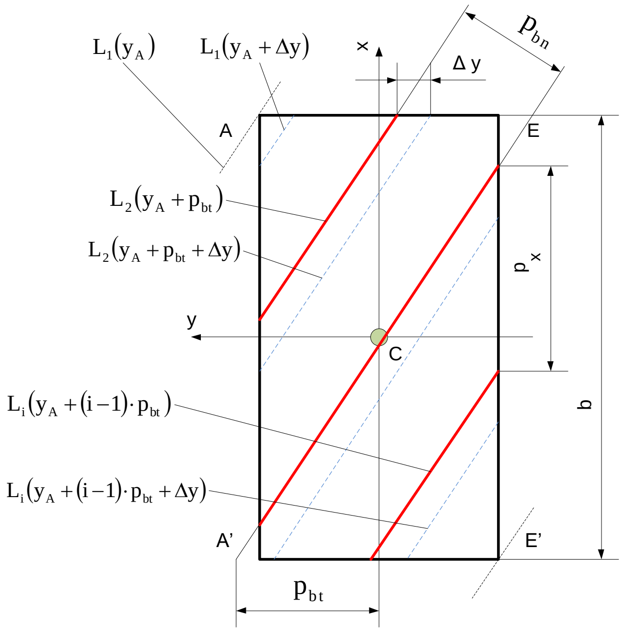

Figure 9
Length change of contacting generating lines

7. Conclusions, results

The article points that the well-known powertrain element in the literature, the helical-toothed cylindrical external gear, raises certain questions that, in the context of nowadays manufacturing technology and the charge under the high-quality requirements oblige the designer to meet them. The solution is hiding in the geometry of the contact zone. The results of our study can be summarized as follows:

- the shape of the contact zone does not have to follow the shape of the classic, regular rectangle, well-known from the literature,
- the zone of contact is characterized by the change of the length of the contacting generating lines located inside it, and this depends on the meshing position,
- modifying the shape of the contact zone can be done in two independent ways,
- the reason for the deformation of the contact zone is to reduce the level of vibrations,
- it is not justified to choose a width of the zone of contact (common tooth width) that equals a multiple of the axial pitch.

A further aim of the research is to describe the changes in the zone of contact and their effect on the meshing characteristics.

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