

## **ANALYSIS OF DMU40 MACHINE CENTRE BY FINITE DEGREES OF FREEDOM**

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**Abstract:** This article is part of a longer research-analytical work, because it outlines the results and conclusions of the study according to the main topic of the research and the applies method. The central theme of the research is the dynamic stiffness of machine tools and the various methods for their determination. The first such (finite element) method is modal analysis, which allows for an analytical test. The purpose of this article is to approach this study from the practical side through a specific example.

**Keywords:** *analytical method, dynamic stiffness, modal analysis, natural frequency, FEM*

### **1. INTRODUCTION**

During the test, the primary goal was to set up the mechanical and mathematical model of the DMU40 machine centre, in order to achieve more realistic results. Machine tools always form a vibrating system due to the generated vibrations and following this line the machine tools can be considered as a multi degree of freedom, damped, linear vibration systems which containing excited vibration [1] [3]. The mathematical model based on the mentioned systems and on the vibration model. After sketching the mechanical model, the mathematical model can be written in the form of a differential equation system, which provides the deformation of each component (machine base, x-slide, y-slide, z-slide) in different directions as a function of time (amplitude-time diagrams).

Dynamic stiffness is basically influenced by the following factors: static stiffness, oscillation frequency, natural frequency and the Lehr-damping. Moreover, the dynamic stiffness and the magnitude of the damping at the local frequencies can be considered as dynamic characteristics of a vibrating system.

Because of the precision, shape accuracy and surface roughness are all important criteria in the production and whereas these are greatly influenced by vibrations, so first of all the above-mentioned natural frequency locations should be determined and this can improve the precision and quality of the manufacturing.

## 2. MECHANICAL MODEL

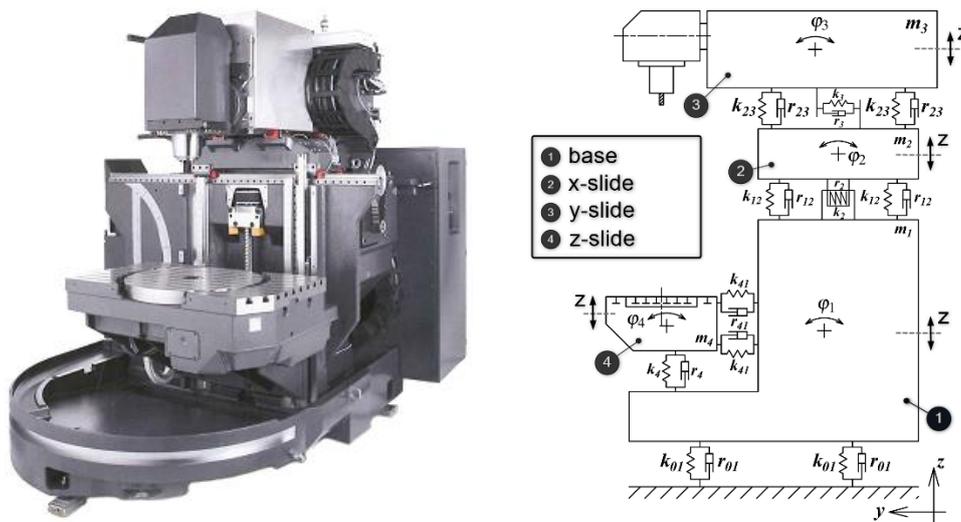
Figure 1 is shown the examined catalog image of a 5-axis CNC controlled machine centre (without cover) which can be found in the department's workshop and aligns it with the mechanical model of the same machine.

On the vibration model beside the four main building units, there are linear guideways, ball screw-ball nut systems which are connecting to the main units. These items have been replaced by springs and dampers on the mechanical model. Later, based on catalog data, these spring stiffness values and damping factors became determinable.

The structure equation of the machine centre:

$$A_I \mathbf{0}_F(s, 4) X(s, 1) Y(s, 2) Z(m, 1) R_y(s, 3) R_z(m, 2)$$

The following picture (Figure 1) contains the examined machine tool (without cover) and this machine's vibration model (with replacement models in case of the linear guideways and the ball screw-ball nut systems):



**Figure 1**  
Catalog image and mechanical model of the machine tool [2]

Where each parameter meaning:

- $k$  [N/m] – spring stiffness
- $r$  [Ns/m] – damping factor
- $\varphi$  – angle of rotation
- $z$  – linear displacement

### 3. MATHEMATICAL MODEL

The above mechanical model (*Figure 1*) can be used to describe the system's motion equation using the Lagrange's Equations of Motion of Second Kind [4]. This allows both linear displacements and angle of rotations to be taken into account. Taking the damping into account, the basic context can be written as:

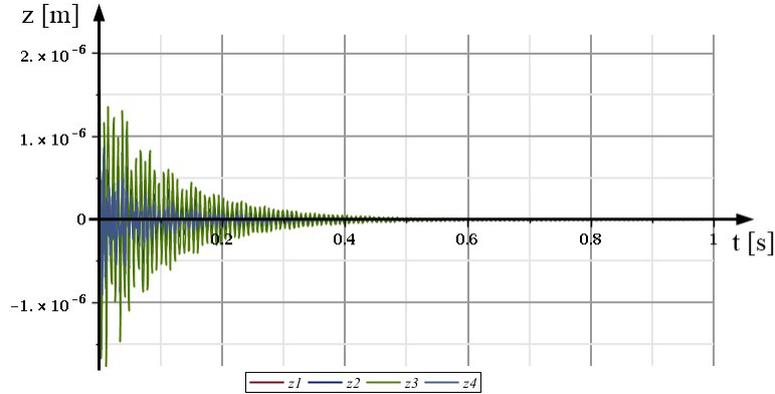
$$\frac{d}{dt} \left( \frac{\partial \alpha}{\partial \dot{q}_i} \right) - \frac{\partial \alpha}{\partial q_i} + \frac{\partial Q_f}{\partial \dot{q}_i} = Q_f \quad (1)$$

By writing the (1) Lagrange-function and performing the mathematical operations in the above context, we obtain the following differential equation system:

$$\begin{aligned} \ddot{z}_1 &= \frac{1}{m_1} \left[ -\frac{z_1}{c_{01}} - \frac{z_1 - z_2}{c_{12}} - \frac{z_1 - z_4}{c_{41}} - \frac{(l_{12} - l_{11})\varphi_1}{2c_{01}} - \frac{(l_{21} - l_{22})\varphi_2}{2c_{12}} + \frac{(l_{42} - l_{41})\varphi_4}{2c_{41}} \right] \\ \ddot{\varphi}_1 &= \frac{1}{J_1} \left[ -\frac{(l_{12} - l_{11})z_1}{2c_{01}} - \frac{(l_{12}^2 + l_{11}^2)\varphi_1}{2c_{01}} \right] \\ \ddot{z}_2 &= \frac{1}{m_2} \left[ -\frac{z_2 - z_3}{c_{23}} - \frac{z_2 - z_1}{c_{12}} - \frac{(l_{31} - l_{32})\varphi_3}{2c_{23}} - \frac{(l_{22} - l_{21})\varphi_2}{2c_{12}} \right] \\ \ddot{\varphi}_2 &= \frac{1}{J_2} \left[ -\frac{(l_{21} - l_{22})z_1}{2c_{12}} - \frac{(l_{22} - l_{21})z_2}{2c_{12}} - \frac{(l_{21}^2 + l_{22}^2)\varphi_2}{2c_{12}} \right] \\ \ddot{z}_3 &= \frac{F_g}{m_3} + \frac{1}{m_3} \left[ -\frac{z_3 - z_2}{c_{23}} - \frac{(l_{32} - l_{31})\varphi_3}{2c_{23}} \right] \\ \ddot{\varphi}_3 &= \frac{1}{J_3} \left[ -\frac{(l_{31} - l_{32})z_2}{2c_{23}} - \frac{(l_{32} - l_{31})z_3}{2c_{23}} - \frac{(l_{31}^2 + l_{32}^2)\varphi_3}{2c_{23}} \right] \\ \ddot{z}_4 &= \frac{1}{m_4} \left[ -\frac{z_4 - z_1}{c_{41}} - \frac{(l_{42} - l_{41})\varphi_4}{2c_{41}} \right] \\ \ddot{\varphi}_4 &= \frac{1}{J_4} \left[ -\frac{(l_{42} - l_{41})z_4}{2c_{41}} + \frac{(l_{42} - l_{41})z_1}{2c_{41}} - \frac{(l_{42}^2 + l_{41}^2)\varphi_4}{2c_{41}} \right] \end{aligned} \quad (2)$$

This (2) DE system was solved using the Maple (this is a mathematical software), based on the Runge-Kutta method. The diagram in *Figure 2* shows the degree of deformation in the z-direction for each component, for the above mechanical system. In the undamped case, oscillatory motion of the mass is observed, while in the damped case (*Figure 2*) the damping absorbs energy, causing the vibration of the system to disappear after a certain period of time [5].

The maximum displacement is related to the y-slide and its magnitude  $z \approx (-2 \div 2) \mu m$ .



**Figure 2**

*Amplitude-time diagram – in damped case*

Modal analysis deals with the study of vibration characteristics and allows the determination of the natural frequencies of any physical system.

The basic equation of the modal analysis [6]:

$$\underline{\underline{M}} \cdot \underline{\dot{q}} + \underline{\underline{K}} \cdot \underline{q} = \underline{f} \quad (3)$$

Modifying the above *Equation (3)*, the resulting relation can be traced back to an eigenvalue task, which problem is the next:

$$\underline{\underline{M}}^{-1} \cdot \underline{\underline{K}} = \alpha^2 = \lambda \quad (4)$$

If we solve this *Equation (4)*, we will get the following natural frequencies, which based on the above-mentioned eight degree of freedom vibration system (*Figure 3*):

mode	$\lambda \left[ \frac{\text{rad}^2}{\text{s}^2} \right]$	$\alpha \left[ \frac{\text{rad}}{\text{s}} \right]$	T [s]	f [Hz]
1	$4.553 \cdot 10^5$	674.740	0.0093	107.3882
2	$7.112 \cdot 10^5$	843.328	0.0075	134.2199
3	$1.011 \cdot 10^6$	1005.586	0.0062	160.0439
4	$1.932 \cdot 10^6$	1389.829	0.0045	221.1982
5	$3.955 \cdot 10^6$	1988.684	0.0032	316.5089
6	$5.662 \cdot 10^6$	2379.484	0.0026	378.7066
7	$1.153 \cdot 10^7$	3396.107	0.0019	540.5072
8	$1.716 \cdot 10^7$	4142.302	0.0015	659.2678

**Figure 3**

*A summary of the results obtained*

#### 4. SUMMARY

In this article we have presented a possible method for determining the dynamic stiffness (natural frequencies) of a machine centre. This method is extremely time consuming due to the high computing and work requirements of multi degree of freedom systems. However, in simpler cases it may be faster and easier than other methods. The results obtained were approximately the same as for other methods.

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