Design of Machines and Structures, Vol. 15, No. 1 (2025), pp. 5–15.
https://doi.org/10.32972/dms.2025.001

POSITION CONTROL OF PERMANENT MAGNET DC MOTOR

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Abstract: This paper presents a study of a DC motor position control system, a critical component in many industrial applications. A detailed mathematical model is developed to represent the motor's dynamics. Three advanced control strategies, namely Linear Quadratic Regulator (LQR), Linear Quadratic Integral (LQI), and Model Predictive Control (MPC), are designed and implemented to control the motor position. An open-loop simulation is conducted to verify the DC motor model. The performance of these controllers is evaluated and compared through closed-loop step response and closed-loop ramp response scenarios. The comparison is based on several matrices, such as the rise time, settling time, and overshoot. The simulation results showed that the LQR has the fastest response, but the MPC is the most energy efficient.

Keywords: *DC motor, position control, LQR, LQI, MPC*

1. INTRODUCTION

Direct current (DC) motors are commonly used today due to their cost-efficiency, ease of use, high performance, and quiet operation (Abut, 2016). DC motors are essential components in various mechanical systems, ranging from industrial automation to consumer products (Praveen, et al., 2012), (Aung, 2007), (Ohishi, Nakao, Ohnishi, & Miyachi, 1987), (Aljawabrah & Lovas, 2021), (Aljawabrah & Lovas, 2022), (Aljawabrah, 2024), (Jneid, Harth, & Ficzere, 2020). Accurate position control of DC motors is crucial for ensuring optimal performance and reliability. Also, developing a good control system will reduce the energy consumption, and lead to a sustainable system. This supports the direction of Industry 5.0 (Ficzere, 2024). Several control strategies have been proposed in the literature, including Linear Quadratic Regulator (LQR), Linear Quadratic Integral (LQI), and Model Predictive Control (MPC).

LQR, a classic optimal control method, minimises a quadratic cost function to balance state deviations and control effort. It offers robustness to disturbances and guaranteed stability. LQI extends LQR by incorporating integral action, effectively eliminating steady-state errors. MPC, a more modern approach, predicts future system behaviour and optimises the control input over a horizon to minimise a cost function. This strategy allows for handling constraints, making it suitable for complex DC motor applications.

Previous studies have investigated the application of LQR, LQI, and MPC for DC motor position control. Xiang and Wei (Xiang & Wei, 2021) designed a DC motor position tracking system using Linear Quadratic Regulator (LQR) control. They compared the LQR controller's performance with other standard control methods- fractional PID- using MATLAB simulations. The results show that the LQR controller provides superior tracking accuracy, minimal overshoot, and short adjustment times compared to fractional PID. Abut (Abut, 2016) focused on modelling and optimising the speed control of DC motors using Kalman filter-based LQR and Kalman filter-based PID controllers. The Kalman filter is added to the environment to enhance performance in noisy conditions where the process and computation noise are present. These controllers were designed and tested in MATLAB/SIMULINK. Kalman filter-based LQR was more successful where it has much lower overshoot, and less oscillations.

Yang, Wu, Hu and Li (Yang, Wu, Hu, & Li, 2019) introduced a robust predictive speed regulation method for DC motors driven by DC-DC buck converters using a discrete-time MPC algorithm. It features a novel reduced-order GPIO to estimate system states and disturbances, optimising the converter's duty ratio while considering control input constraints. Implemented on a DSPACE-based setup, the method shows superior robustness and accuracy in handling parametric uncertainties, load torque variations, and time-varying disturbances compared to traditional PID controllers. They proposed a finite-states model predictive control (FS-MPC) method to minimise commutation torque ripple in permanent magnet brushless DC motors (BLDCM) across all speeds. This method simplifies implementation by directly selecting the optimal conduction status, avoiding complex current controllers or modulation models. A discrete-time predictive model for the non-commutated phase current during commutation ensures torque ripple minimisation by matching the slope rates of incoming and outgoing phase currents. Simulations and experiments validate the FS-MPC method's effectiveness in reducing torque ripple in both dynamic and steady states.

Chimborazo-Taipe, Torres-Hinojosa and Montalvo-Lopez (Chimborazo-Taipe, Torres-Hinojosa, & Montalvo-Lopez, 2023) presented a study aimed at optimising the Q and R matrices of the Linear Quadratic plus Integral (LQI) controller using meta-heuristic algorithms such as Genetic Algorithms (GA), Bacterial Foraging Optimization (BFO), and Ant Colony Optimization (ACO) to enhance the performance of a permanent magnet DC motor. The efficiency of the optimized LQI controller is validated using the Integral of the Absolute Value of the Time Weighted Error (ITAE) and the Wilcoxon statistical method, with BFO showing the best performance by eliminating overshoot, steady-state error, and oscillations. The study compared the performance of GA, BFO, and ACO algorithms, finding that BFO and ACO provide better results in settling time and stability, with BFO being the most effective.

This paper aims to contribute to the existing body of knowledge by presenting a comprehensive study of DC motor position control using LQR, LQI, and MPC. The objective is to compare the performance of these controllers in terms of tracking accuracy and response time.

2. DC MOTOR SYSTEM

2. 1. Dynamic model

The dynamic model of the single-phase DC motor contains two parts: the electrical and mechanical parts. The differential equation for the electrical part is shown below:

$$L\frac{di}{dt} + Ri = V + K_e\omega \tag{1}$$

Here, L is the motor inductance, R is the motor resistance, V is the supply voltage, K_e is the back electromotive force (BEMF) constant, ω is the angular speed, and the term $K_e\omega$ represents the BEMF. The differential equation for the mechanical part is shown below:

$$J\frac{d\omega}{dt} + B\omega = K_t i \tag{2}$$

Here J represents the sum of the Motor Inertia (JM) and the load inertia (Jl), and B is the viscous friction coefficient. K_t represents the torque constant, and the term $K_t i$ is the motor electromagnetic torque. As seen, the two equations are coupled with the angular speed and the current, so these two equations are solved simultaneously.

The goal here is to control the motor angular position (θ). To develop the position controller, the state space model is needed and provided as below:

$$x_1 = \theta$$
$$x_2 = \dot{x}_1 = \dot{\theta} = \omega$$
$$x_3 = i$$

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$$
$$y = C\boldsymbol{x} + Du$$

$$x = [x_1 \quad x_2 \quad x_3]^T \tag{3}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & -\frac{K_t}{J} \\ 0 & -\frac{R}{L} & -\frac{K_e}{L} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad u = V$$

$$C = [1 \quad 0 \quad 0], \qquad D = \boldsymbol{0}$$

As seen in the above equation, the state vector is $[x_1 \quad x_2 \quad x_3]'$ or $[\theta \quad \omega \quad i]'$ while the output, or controlled variable, is θ. Three controllers are used: LQR, LQI, and MPC. LQR is a classic optimal control method that is widely used for linear systems. It aims to find a control input that minimises a quadratic cost function, which typically includes a balance between state deviations and control effort. LQI is an extension of LQR that incorporates integral action to eliminate steady-state errors. While LQR focuses on minimising state deviations and control efforts, LQI also considers the integral of the tracking error. This allows LQI to effectively handle constant disturbances and achieve zero steady-state error. MPC is a more advanced control strategy that predicts future system behaviour based on a model and optimises the control input over a horizon to minimise a cost function. This approach allows MPC to handle constraints, nonlinearities, and multi-variable systems. The structure, design, and calibration are well illustrated in (Xiang & Wei, 2021), while MPC is illustrated in (Alkurawy & Khamas, 2018).
During the controller design and calibration, one main requirement is kept in mind, which is the overshoot. Overshoot is quantified as the amount by which a system's response exceeds its final steady-state value. It is typically expressed as a percentage of the steady-state value.

$$\frac{Peak\ value\ -\ Final\ Value}{Final\ value} \times 100\% \tag{4}$$

Later on, the controllers are compared regarding different matrices such as response time, oscillation, and energy efficiency.
Moreover, they are compared using mathematically defined matrices according to:

- Rise time: this is the time needed to rise from 10% to 90% of the steady-state final value.
- Settling time: the time needed from the request time so that the relative error between the actual value and the steady-state final value drops below 2%.
- Mean Squared Error (MSE) is a widely used metric in control systems to evaluate the performance of a controller. It measures the average squared difference between the desired output (the target or setpoint) and the actual output (the real-time performance of the system) at all time points. A lower MSE indicates better performance, meaning the controller is more accurately following the desired trajectory.
- Integrated Squared Error (ISE) is a performance metric commonly used in control systems to evaluate the controller's overall accuracy and effectiveness over a given period. Unlike Mean Squared Error (MSE), which calculates the average squared error, ISE integrates the squared errors over time, providing a cumulative measure of the system's deviation from the desired trajectory.
- Integrated Squared Controller Effort (ISCE) is a performance metric used in control systems to evaluate the effort or energy expended by a controller to maintain the desired output of a system. Unlike error-based performance metrics like ISE, which focuses on how well the system follows the desired trajectory, ISCE measures the cost associated with the controller's actions, particularly regarding the magnitude and intensity of control signals. ISCE integrates the squared controller input (voltage) over time.

3. RESULTS

3. 1. Open loop response

Firstly, the system has been supplied with 24V constant voltage. The utilised motor parameters are shown in Table 1.

Table 1.
DC motor parameters

Parameter	Symbol	Value	Parameter	Symbol	Value
Motor Inertia	JM	$720 \cdot 10^{-9} \text{kg}m^2$	**Load Inertia**	JL	$224.43 \cdot 10^{-6} \text{kg}m^2$
BEMF constant	Ke	$\frac{0.1125\text{Vs}}{\text{rad}}$	**Torque constant**	Kt	$\frac{0.1125\text{Nm}}{\text{A}}$
Inductance	L	$3.2 \cdot 10^{-3} H$	**Resistance**	R	0.9Ω
Viscous Friction	B	$1.23 \cdot \frac{10^{-3} Nms}{rad}$			

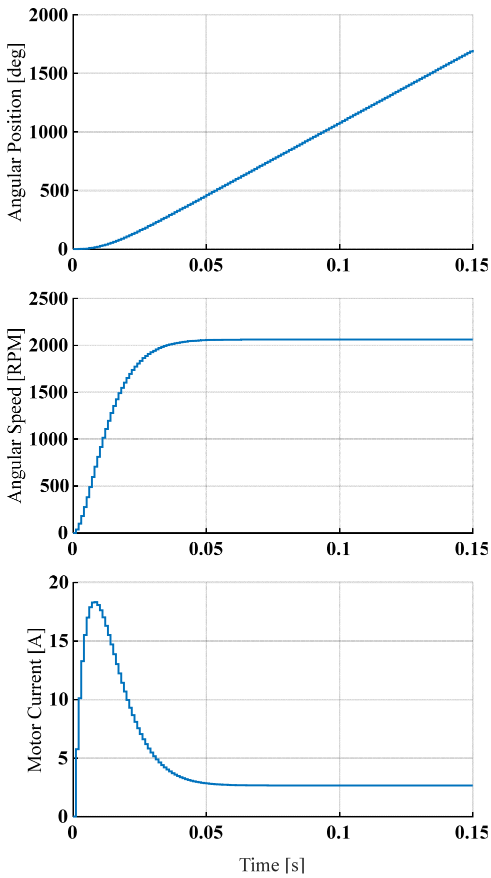

Figure 1. *Open loop response*

Figure 1 shows the open loop response. At the start of the motion, the motor exhibits nonlinear behaviour in both speed and position. Initially, the motor accelerates from rest, and the speed increases nonlinearly due to the rotor's inertia. This nonlinear rise in speed occurs as the motor takes time to overcome its inertia, along with the effects of frictional forces and the inductance of the motor windings. Since the angular position is the integral of the speed, the nonlinearity in speed is reflected in the position plot, causing the position to increase nonlinearly. Once the motor reaches a steady state, the speed stabilises at 0.05 s with a constant value, 2063 RPM, and as a result, the angular position increases linearly over time. The steady-state speed is achieved when the motor reaches equilibrium, where the torque produced by the applied voltage matches the resistive forces, such as viscous friction. As the speed increases, the BMEF increases, which reduces the available voltage for the motor, as the left side in equation (1) shows. This reduces the maximum current the motor can draw, reducing the electromagnetic torque that the motor can produce.
As for the current, it increases at the start of the motion when the motor is trying to overcome inertia and accelerate. As the speed increases, the current decreases due to the back-EMF opposing the applied voltage, and the current reaches a maximum value of 18 A before it starts to decrease.

3. 2. Closed loop step response

Figure 2 presents a comparative analysis of three controllers, LQR, LQI, and MPC, used to control the angular position's step response by regulating the motor's input voltage. In the first subfigure, the angular position response is shown, where all three controllers are compared against the reference position, represented by the black dashed line. The LQR controller provides a fast response with the highest overshoot, though it exhibits. The LQI controller aims to eliminate the steady-state error by incorporating integral control, but this comes at the cost of a slightly slower rise time compared to LQR. The MPC demonstrates a smooth and balanced performance, closely following the reference.
The impact of these control strategies is further reflected in the second subfigure, which shows the angular speed of the motor. The LQR controller produces a rapid change in speed, consistent with its fast angular position response, but this comes with larger oscillations. The LQI controller provides a smoother speed response, reducing oscillations and moderating the speed change due to the slower rise time seen in the position response. Conversely, MPC maintains a steady and smooth speed trajectory, balancing between rapid response and oscillation control.
Finally, the third subfigure highlights the input voltage applied by each controller to achieve the desired motor behaviour. The LQR controller demands a more

aggressive input voltage, corresponding to its fast response but indicating higher energy consumption, pushing the system towards the supply voltage limit of 24 V. The LQI controller uses a more moderate input voltage, balancing performance and energy efficiency. MPC once again delivers the most controlled and efficient use of the input voltage, exhibiting a smoother profile that results in more efficient control with fewer fluctuations.

Table 2.
Performance comparison of controllers for step response

	Rise time (ms)	Settling time (ms)	Overshoot (%)	MSE	ISE	ISCE
LQR	32	87	4.763	84.46	25.42	15.96
LQI	42	100	3.758	106.10	42.23	10.06
MPC	67	187	4.757	153.31	61.32	5.78

In summary, the LQR controller is better in fast response but at the cost of potential steady-state error and higher energy use. LQI prioritises steady-state accuracy with smoother, slower responses, while MPC provides a balanced performance across all metrics –closely tracking the reference, minimising oscillations, and efficiently managing input voltage. The values for the overshoot, rise time, settling time, MSE, ISE, and ISCE are shown in Table 2.

Table 3.
Controller performance comparison for ramp response

	MSE	ISE	ISCE
LQR	58.54	30.44	29.05
LQI	200.50	120.10	27.92
MPC	400.57	240.34	27.21

Figure 3 shows the system response for ramp response. The reference angular position is generated at a rate of 900 RPM while it is saturated to 1440 degrees (4 revolutions) to allow the system to accelerate to the required speed of 900 RPM. In the first subfigure, it can be seen that all the curves run parallel to the reference curve, but the LQR is closest while the MPC is the farthest. This difference in response is similar to the step response comparison. The difference in response can be seen better in the second subfigure for the motor angular speed. The LQR has the most rapid change in speed, but the MPC is the lowest. Moreover, LQR took 45 ms to reach 900 RPM, LQI took 74 ms, and MPC took 113 ms. The values for the MSE, ISE, and ISCE are shown in Table 3.

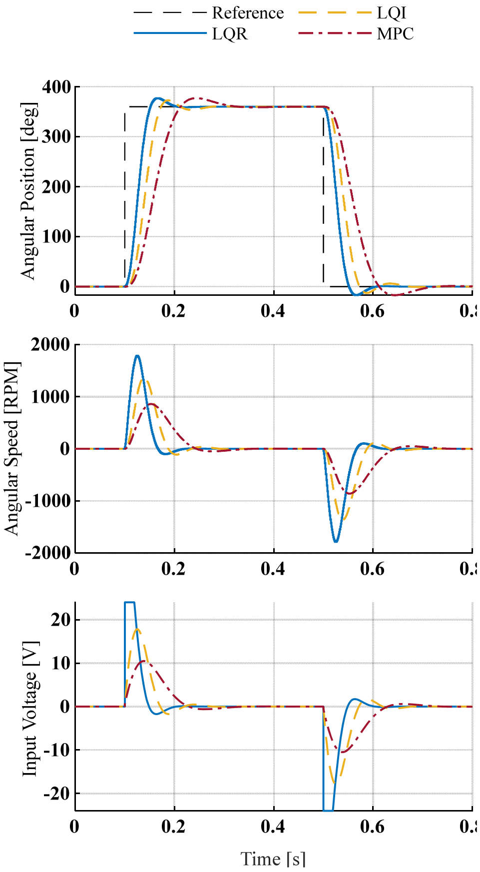

Figure 2. *Closed loop step response*

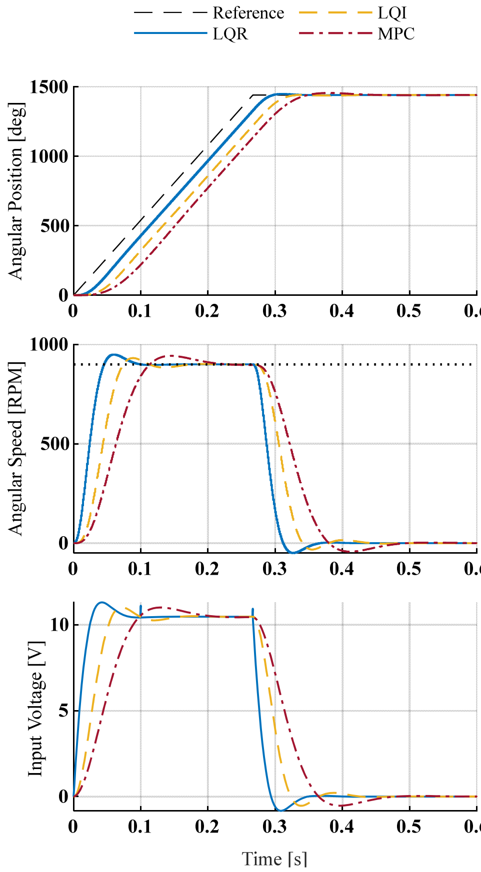

Figure 3. *Closed loop ramp response*

4. SUMMARY

This study analysed LQR, LQI, and MPC methods for DC motor position control. LQR, while offering the fastest response and minimal rise time, exhibits higher overshoot and energy consumption. LQI effectively eliminates steady-state errors but compromises response speed slightly. Though slower in response, MPC provides a balanced performance, demonstrating smooth trajectories, minimal oscillations, and the most efficient energy use.

The findings underscore the trade-offs between speed, accuracy, and energy efficiency inherent in each control strategy. LQR is best suited for applications demanding rapid response, while LQI is ideal for scenarios prioritising steady-state precision. MPC stands out for its flexibility in handling complex control scenarios and optimising energy use, making it the preferred choice for applications with stringent performance and efficiency requirements.

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