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DETERMINATION OF IDEAL CURVE HAVING CONSTANT WEDGE ANGLE FOR ROLLER FREEWHEELS

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Abstract. The task of the starter motors is to rotate the combustion engines to the necessary rotational speed. An element of the mechanism is a free running clutch. It has two functions. The first one is the torque transmission from the driving member to the driven shaft, and to speed up the combustion engine. The9 other function is to disconnect the elements, when the combustion engine has been already turned over, and the driven shaft rotates faster than the driver.

In this paper the pressure angle as the most important parameter of operation is investigated. A new curve is developed for inner profile of housing, which operates to constant wedge angle.

Keywords: free running clutch, roller freewheel, logarithmic spiral

1. Introduction

The task of the starter motors is to rotate the combustion engines to the necessary rotational speed. An element of the mechanism is a free running clutch. It has two functions. The first one is the torque transmission from the driving member to the driven shaft and to speed up the combustion engine. The other function is to disconnect the elements, when the combustion engine has already turned over, and the driven shaft rotates faster than the driver.

In a previous paper we have discussed the operation and geometry of the roller freewheels, and the effect of load, deformation and wear on the operation of clutches has been analysed [4]. In paper [5] we have defined the geometry, which occurs constant wedge angle. In this paper the fundamental knowledge is presented and a new method, which helps us to determine the inner profile curve at the housing of clutch. This method gives a chance to reconstruct roller freewheels having unknown geometry.

2. Elements and operation of roller freewheels

Figure 1 shows a sketch from a roller freewheel having a so called outer star-wheel. It has four components: the housing, the hub, the rollers and the springs.

Figure 2 shows a detail of a roller freewheel, where the roller connects with the housing and the hub. The figure contains the necessary dimensions. The shape of the rollers and the hub is cylinder, and the housing has a curved surface usually based on logarithmic spiral.

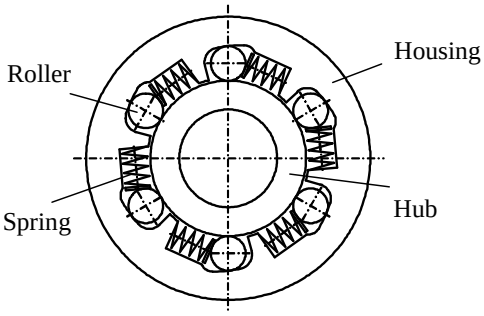


Figure 1. Roller freewheel assembl

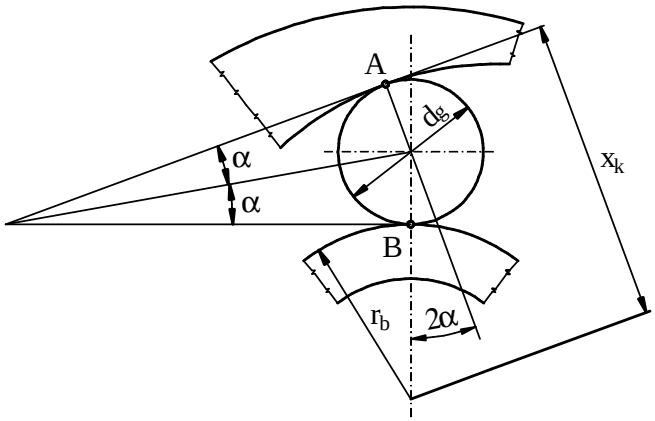


Figure 2. The geometry of the contact

For the operating of the roller freewheel it is very important requirement, that the profile of the housing and the hub should produce a taper gap. The tangent lines at the contact points determine the angle 2α, which defines the dimension of the gap. 2α is called as wedge angle. r_b is the radius of the hub, and d_g is the diameter of the roller in *Figure 2*. The operating of the roller freewheels is shown in *Figure 3*.

During the connection, the driving element is the housing, which is rotated by torque M_1 into the shown direction. To generate the equilibrium, the torque M_t on the hub equals M_1, but the directions of the torques are opposite. M_1 means the torque on one roller. When uniform load distribution is assumed between rollers, then

$$M_1 = \frac{M}{z}, \tag{2.1}$$

where torque M is the total load on the clutch and z is the number of rollers. The calculated tangential force F_t from the torque at the contact point of the roller and the hub is

$$F_t = \frac{M_1}{r_b}. \tag{2.2}$$

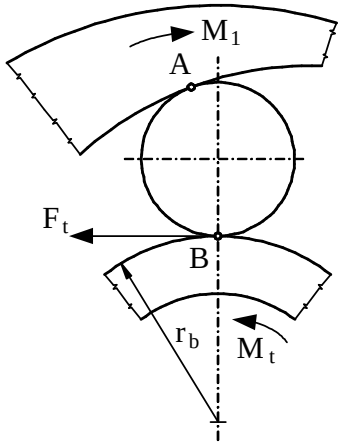


Figure 3. The operating principle of the roller freewheels

The tangential force presses the roller into the taper gap, as long as this tangential force is smaller than the friction resistance, that is $F_t < F_s$. The operating condition of the clutches is the self-locking. If the inequality is not realized, there is no self-locking and the hub slides on the roller. In this case the clutch cannot work. The equilibrium of forces is represented in *Figure 4*. From vector triangle we can write the following expression:

$$F_t = F_n \cdot tan\,\alpha\ . \tag{2.3}$$

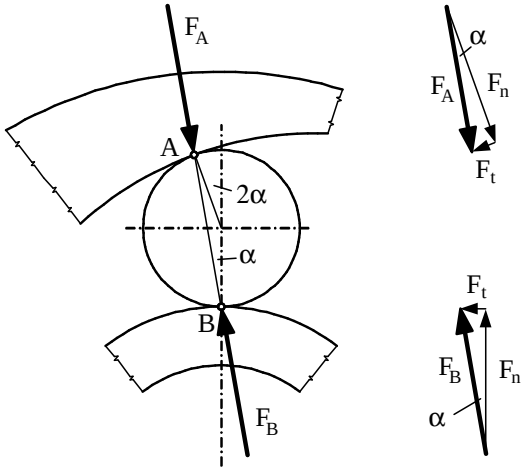


Figure 4. The equilibrium of forces acting to the roller

The working condition is obtained by simplification from the above relation:

$$tan\,\alpha < \mu \ \text{ and } \ \alpha < arctan\,\mu\ . \tag{2.4}$$

It is found, that the operation of the roller freewheels is defined by the relation between the wedge angle and the friction coefficient, and it does not depend on the magnitude of the load. The clutch will slip in that case only, if the inequality (2.4) is unrealized.

3. Profile of housing in accordance with the literature

Roller freewheels with outer star wheel are generally used by automotive industry. They have logarithmic spiral profile according to the literature. The logarithmic spiral has a special property: there is a constant angle β between the radius and the tangent line in any point of the curve. Using this curve as a profile of star wheel the wedge angle 2α has slight changes at different contact points. It becomes almost constant.
The task is to find a new curve, which has constant wedge angle 2α in any arbitrary contact point. This is granted when the centre point of the roller having variable radius r_g moves on a logarithmic spiral path.

4. The equation of the profile having constant wedge angle

During determination of profile the radius of the inner ring (r_b), and the wedge angle (2α) are considered as constant. In this case we can fit rollers with different radius (r_g) on any points of the inner ring and the profile of outer star wheel. Therefore, r_g can be interpreted as a function of θ in the following form:

$$r_g = r_g(\theta). \tag{4.1}$$

An illustration from the structure and the used notations are shown in *Figure. 5*.

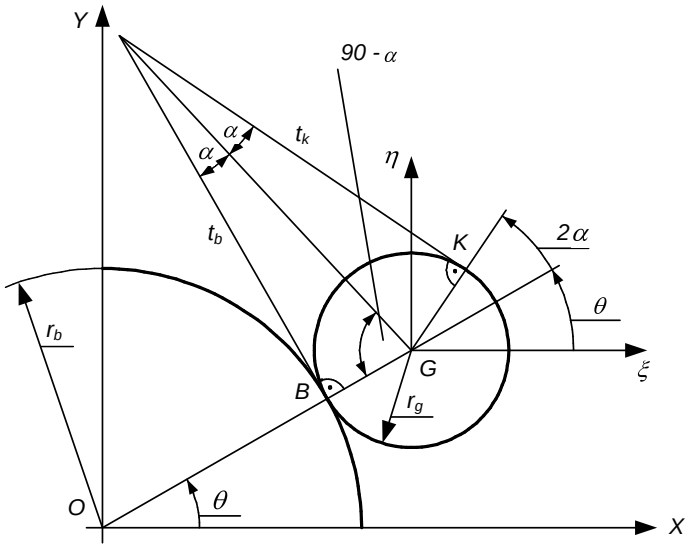


Figure 5. Layout draft of the roller freewheel

The coordinates *x* and *y* of the point „K" on the curve can be written using auxiliary coordinate system ξ - η as follows:

$$x = x_G + \xi_K \,, \tag{4.2}$$

$$y = y_G + \eta_K \,. \tag{4.3}$$

All the parameters of the above equations are the function of angle θ:

$$x_G = (r_b + r_g) \cdot \cos\theta \,, \tag{4.4}$$

$$y_G = (r_b + r_g) \cdot \sin\theta \,, \tag{4.5}$$

$$\xi_K = r_g \cdot \cos(\theta + 2\alpha) \,, \tag{4.6}$$

$$\eta_K = r_g \cdot \sin(\theta + 2\alpha) \,. \tag{4.7}$$

Substituting expressions from (4.4) to (4.7) into (4.2) and (4.3) contexts, coordinates *x* and *y* of the point „K" are obtained as a function of angle θ:

$$x = (r_b + r_g) \cdot \cos\theta + r_g \cdot \cos(\theta + 2\alpha) \,, \tag{4.8}$$

$$y = (r_b + r_g) \cdot \sin\theta + r_g \cdot \sin(\theta + 2\alpha) \,. \tag{4.9}$$

Derive the coordinates *x* and *y* according to θ to determine the slope of the tangent line:

$$x' = \frac{dx}{d\theta} = -(r_b + r_g) \cdot \sin\theta + r_g' \cdot \cos\theta - r_g \cdot \sin(\theta + 2\alpha) + r_g' \cdot \cos(\theta + 2\alpha) \,, \tag{4.10}$$

$$y' = \frac{dy}{d\theta} = (r_b + r_g) \cdot \cos\theta + r_g' \cdot \sin\theta + r_g \cdot \cos(\theta + 2\alpha) + r_g' \cdot \sin(\theta + 2\alpha) \,. \tag{4.11}$$

The slope of the tangent line (t_k) at the point „K" is:

$$\frac{dy}{dx} = \frac{r_g' \cdot [\sin\theta + \sin(\theta + 2\alpha)] + r_g \cdot [\cos\theta + \cos(\theta + 2\alpha)] + r_b \cdot \cos\theta}{r_g' \cdot [\cos\theta + \cos(\theta + 2\alpha)] - r_g \cdot [\sin\theta + \sin(\theta + 2\alpha)] - r_b \cdot \sin\theta} \,. \tag{4.12}$$

which can be written as follows (see *Figure 5*.)

$$tg(\frac{\pi}{2} + \theta + 2\alpha) = \frac{\sin(\frac{\pi}{2} + \theta + 2\alpha)}{\cos(\frac{\pi}{2} + \theta + 2\alpha)} \,. \tag{4.13}$$

After further modifications based on the trigonometric relationships we have

$$\frac{\sin(\frac{\pi}{2}+(\theta+2\alpha))}{\cos(\frac{\pi}{2}+(\theta+2\alpha))}=\frac{\sin\frac{\pi}{2}\cdot\cos(\theta+2\alpha)+\cos\frac{\pi}{2}\cdot\sin(\theta+2\alpha)}{\cos\frac{\pi}{2}\cdot\cos(\theta+2\alpha)-\sin\frac{\pi}{2}\cdot\sin(\theta+2\alpha)}=\frac{\cos(\theta+2\alpha)}{-\sin(\theta+2\alpha)}. \quad (4.14)$$

The slope of the tangent line using equations (4.12) and (4.14) is described by the following relation:

$$\frac{r_g{}'\cdot[\sin\theta+\sin(\theta+2\alpha)]+r_g\cdot[\cos\theta+\cos(\theta+2\alpha)]+r_b\cdot\cos\theta}{r_g{}'\cdot[\cos\theta+\cos(\theta+2\alpha)]-r_g\cdot[\sin\theta+\sin(\theta+2\alpha)]-r_b\cdot\sin\theta}=\frac{\cos(\theta+2\alpha)}{-\sin(\theta+2\alpha)}. \quad (4.15)$$

Multiplying both sides of the equation with the members of denominator we get the following formula:

$$\begin{aligned}&-r_g{}'\cdot\sin\theta\cdot\sin(\theta+2\alpha)-r_g{}'\cdot\sin^2(\theta+2\alpha)-r_g\cdot\cos\theta\cdot\sin(\theta+2\alpha)+\\&-r_g\cdot\cos(\theta+2\alpha)\cdot\sin(\theta+2\alpha)-r_b\cdot\cos\theta\cdot\sin(\theta+2\alpha)=\\&=r_g{}'\cdot\cos\theta\cdot\cos(\theta+2\alpha)+r_g{}'\cdot\cos^2(\theta+2\alpha)-r_g\cdot\sin\theta\cdot\cos(\theta+2\alpha)+\\&-r_g\cdot\sin(\theta+2\alpha)\cdot\cos(\theta+2\alpha)-r_b\cdot\sin\theta\cdot\cos(\theta+2\alpha)\end{aligned} \quad (4.16)$$

Sorting each member of the equation according to the coefficients we obtain the following form:

$$\begin{aligned}&r_g{}'\cdot[+\sin\theta\cdot\sin(\theta+2\alpha)+\cos\theta\cdot\cos(\theta+2\alpha)]+\\&r_g{}'\cdot[+\sin^2(\theta+2\alpha)+\cos^2(\theta+2\alpha)]+\\&r_g\cdot[+\cos\theta\cdot\sin(\theta+2\alpha)-\sin\theta\cdot\cos(\theta+2\alpha)]+\\&r_b\cdot[+\cos\theta\cdot\sin(\theta+2\alpha)-\sin\theta\cdot\cos(\theta+2\alpha)]=0.\end{aligned} \quad (4.17)$$

We can recognize trigonometric identities so the equation would be as follows:

$$\begin{aligned}&r_g{}'\cdot\cos[\theta-(\theta+2\alpha)]+\\&r_g{}'\cdot1+\\&r_g\cdot\sin[(\theta+2\alpha)-\theta]+\\&r_b\cdot\sin[(\theta+2\alpha)-\theta]=0.\end{aligned} \quad (4.18)$$

After simplifying:

$$r_g{}'\cdot\cos(-2\alpha)+r_g{}'+r_g\cdot\sin(2\alpha)+r_b\cdot\sin(2\alpha)=0. \quad (4.19)$$

Because of the cosine function is symmetrical ($\cos(x) = \cos(-x)$) therefore

$$r_g' \cdot \cos(2\alpha) + r_g' + r_g \cdot \sin(2\alpha) + r_b \cdot \sin(2\alpha) = 0 \tag{4.20}$$

can be prescribed. Expressing r_g' from the above equation:

$$r_g' = -\frac{(r_g + r_b) \cdot \sin 2\alpha}{1 + \cos 2\alpha} = -\frac{\sin 2\alpha}{1 + \cos 2\alpha} \cdot (r_g + r_b) . \tag{4.21}$$

Examining the constant coefficient (before the bracket) of the above equation, we have a simple formula:

$$-\frac{\sin 2\alpha}{1 + \cos 2\alpha} = -\frac{2 \cdot \sin\alpha \cdot \cos\alpha}{1 + \cos^2\alpha - \sin^2\alpha} = -\frac{2 \cdot \sin\alpha \cdot \cos\alpha}{2 \cdot \cos^2\alpha} = -tg\,\alpha = A = const. \tag{4.22}$$

The result is a first-order, variable-separable differential equation:

$$r_g' = A \cdot (r_g + r_b) , \tag{4.23}$$

namely

$$\frac{dr_g}{d\theta} = A \cdot (r_g + r_b) . \tag{4.24}$$

After rearranging the equation, it has to be integrated, then both sides have to be involved with "e":

$$\frac{dr_g}{r_g + r_b} = A \cdot d\theta \qquad /\int \tag{4.25}$$

$$\ln \cdot (r_g + r_b) = A \cdot \theta + C \qquad /^e . \tag{4.26}$$

Organizing the result and introducing a simplification $e^C = K$, we obtain the general solution of the first-order, variable-separable differential equation (4.29)

$$r_g + r_b = e^{(A \cdot \theta + C)} = e^{A \cdot \theta} \cdot e^C , \tag{4.27}$$

$$r_g + r_b = K \cdot e^{A \cdot \theta} , \tag{4.28}$$

$$r_g = K \cdot e^{A \cdot \theta} - r_b = K \cdot e^{-\theta \cdot tg\,\alpha} - r_b . \tag{4.29}$$

If we suppose that r_b = constant and we demand the value of $r_g = r_{g0}$ on the place of $\theta = \theta_0$, we can calculate the value of „K" as particular solution of the differential equation.

$$K = \frac{r_g + r_b}{e^{-\theta \cdot tg\,\alpha}} = (r_g + r_b) \cdot e^{\theta \cdot tg\,\alpha} \Rightarrow K_0 = (r_{g0} + r_b) \cdot e^{\theta_0 \cdot tg\,\alpha} \quad . \tag{4.30}$$

For the sake of verification we have to replace the result into the original differential equation:

$$r_g{}' = A \cdot (r_g + r_b)$$

$$A \cdot K \cdot e^{A \cdot \theta} = A \cdot [(K \cdot e^{A \cdot \theta} - r_b) + r_b] = A \cdot K \cdot e^{A \cdot \theta} - A \cdot r_b + A \cdot r_b \tag{4.31}$$

Because of the equality existing between the two sides of equation, the result is acceptable. Formula (4.29) is an equation of the curve, at which the angle α (and the wedge angle 2α) does not change at any possible contact point.

5. The proof of the permanence of the wedge angle 2α

Although the calculation presented above clearly shows that the wedge angle 2α is constant in any contact point, let's consider an other approach. This hypothesis proves clearly that the wedge angle 2α is permanent using another condition. A result of the method is also the parametrical equation of the ideal curve.

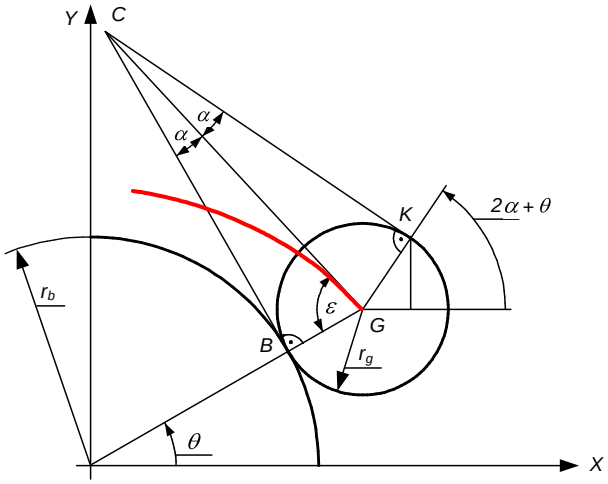


Figure 6. Moving of the roller central point

Let's consider the situation shown in *Figure 6*. If the red curve is a logarithmic spiral, than the parameter of the equation is ε. In this case of logarithmic spiral angle ε is constant. Therefore the angle α at apex "C" from right-angled triangle BGC must be also constant.

So in this case of the curve when constant wedge angle is produced the centre of the roller must be run along a logarithmic spiral. The current position of the point G(θ) is described as:

$$G = R_0 \cdot e^{-\theta \cdot ctg\,\varepsilon}, \tag{5.1}$$

where

$$\varepsilon = \frac{\pi}{2} - \alpha . \tag{5.2}$$

The value of the inner ring radius (r_g) at the place $\theta = 0$ be r_{g0} and the angle α is a design parameter of the freewheel, which can be determined using the friction relationships. The coordinates of an arbitrary point on the ideal curve can be described using the following formula (see *Figure 6.*):

$$\begin{aligned} x_K &= x_G + r_g \cdot \cos(2\alpha + \theta), \\ y_K &= y_G + r_g \cdot \sin(2\alpha + \theta), \end{aligned} \tag{5.3}$$

namely

$$\begin{aligned} x_K &= R_0 \cdot e^{-\theta \cdot ctg\,\varepsilon} \cdot \cos\theta + r_g \cdot \cos(2\alpha + \theta), \\ y_K &= R_0 \cdot e^{-\theta \cdot ctg\,\varepsilon} \cdot \sin\theta + r_g \cdot \sin(2\alpha + \theta). \end{aligned} \tag{5.4}$$

Compared to equations (4.8) and (4.9) we find the following

$$R_0 \cdot e^{-\theta \cdot ctg\,\varepsilon} = r_b + r_g . \tag{5.5}$$

Using this equation we can calculate the roller radius r_g:

$$r_g = R_0 \cdot e^{-\theta \cdot ctg\,\varepsilon} - r_b . \tag{5.6}$$

The general solution of the solved differential equation in (4.29) and the equation (5.6) describe the same curve, if the coefficients are identical.
Substitute the constant K_0 (see equation (4.30)) into relation (4.29):

$$r_g = K \cdot e^{A \cdot \theta} - r_b = K \cdot e^{-\theta \cdot tg\,\alpha} - r_b = (r_{g0} + r_b) \cdot e^{\theta_0 \cdot tg\,\alpha} \cdot e^{-\theta \cdot tg\,\alpha} - r_b , \tag{5.7}$$

namely

$$r_g = (r_{g0} + r_b) \cdot e^{(\theta_0 - \theta) \cdot tg\,\alpha} - r_b . \tag{5.8}$$

We must prove agreement of the following coefficients while comparing the equations (5.6) and (5.8):

$$R_0 = r_{g0} + r_b \tag{5.9}$$

and

$$-\theta \cdot ctg\,\varepsilon = (\theta_0 - \theta) \cdot tg\alpha . \tag{5.10}$$

Fulfilment of condition (5.9) is easy to show. In the initial position (when $\theta_0 = 0$) the value of R_0 can be defined as the sum of the constant inner ring radius (r_b) and the initial roller radius (r_{g0}). In case of condition (5.10), when $\theta_0 = 0$ the equation ctg ε = tan α must be satisfied.
After conversion using formula (5.2) the following equation is indeed satisfied:

$$ctg\,\varepsilon = ctg\,(\frac{\pi}{2} - \alpha) = tg\alpha\,. \tag{5.11}$$

The conclusion is that both methods have reached the same solution. This proves that the derivation is correct. It is proved, that if the path of the roller centre is a logarithmic spiral, the described ideal curve cannot be a logarithmic spiral because of the changing roller radius. The shape of inner ring, the ideal curve and the path of the roller centre are shown in *Figure 7*.

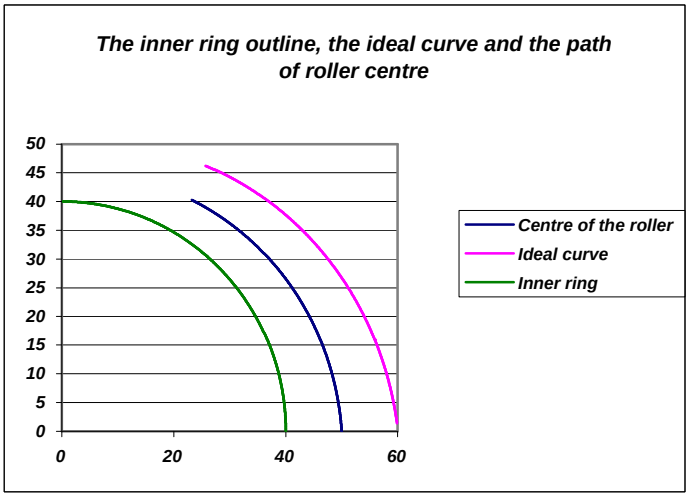


Figure 7. The inner ring outline, the ideal curve and the path of roller centre

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