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DEVIATION FROM STANDARDS IN RELATION TO CYLINDRICAL GEARS WITH EXTERNAL STRAIGHT TEETH

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Abstract. In propulsion technique applied in every day practice the load of gears is usually one directional or two directional, but the stresses on the tooth sides are not the same degree. Numerous researches verify that departing from standard symmetric toothed wheels, the loading capacity of gears can be increased by asymmetric teeth. Effects of deviation from standard have numerous consequences, which have to be examined.

Keywords: tooth shape, tooth root curve, tooth root stress, asymmetric teeth

1. Application of gears

Functional units and gear drives, which modify parameters of movement, take a great part in energetic chain realized in mechanical products. In gear drives continuous one-directional rotation of the toothed element pairs can be ensured by a suitably integrated rotation reversing facility. These transmission elements of movement realize either exact map of movement or large load transfer. The tooth shape plays a great role in determination of transmission parameters. Deviation from symmetric tooth shape generates questions about design, manufacturing and certification (proportioning, reliability). [9] The most important function of gear drives applied in driving mechanisms is steady transmission of movement during working. [1] Considering the single forms of motion, rotation would be changed, for example by reduction of revolutions.

Gearings can essentially be divided into two groups, kinematical and load transfer drives. [1] These functions mean different requirements for the teeth. Kinematical drives ensure precise angle rotation. These drives are characterized by small pitch. In load transfer drives pitch is larger, which helps accommodation to the fluctuation of temperature during working respectively inaccuracy of gears.

In the drive train of mechanical systems there is a usual functional claim, where the energy in a unit of time, which is the source of power, should be available for any working machine. This availability taking the energy components into consideration can be changed discreetly or continuously in a range. These functional units are driving mechanisms.

One element of the system is denoted functionally by the structure element in *Figure 1.*, which can be examined from inside as if it would be an unknown construction.

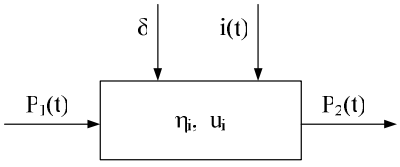


Figure 1. Functional unit describing driving mechanism

The input is described by the effect $P_1(t)$, with its components in the energy chain that are the moment $M_1(t)$ and the angular speed $\omega_1(t)$. This characterizes the source of energy at the same time.

The output is marked by the effect $P_2(t)$, which due to construction of the system element depends on the system controller information. These are the stage contact (i), which can be discreet or time control and it gives the system transmission, and the functional δ, which denotes the direction of rotation inside the system.

Based on the above mentioned it can be written that

$$P_1(t) = M_1(t) \cdot \omega_1(t), \tag{1}$$

$$P_2(t) = M_i(t) \cdot \omega_i(t), \tag{2}$$

$$\frac{\omega_1(t)}{\omega_i(t)} = u_i = f(i), \tag{3}$$

together with appropriation the formulas (1), (2), (3).

$$M_i(t) = \eta_i \cdot M_1(t) \cdot u_i, \tag{4}$$

where η_i is the efficiency which belongs to the controlled settings (stages) of the system element. Examining the system so that in point $\omega_i(t)$ regulated rotating movement should be mapped as well, from formula (3) it follows that

$$\omega_i(t) = \delta \cdot u_i \cdot \omega_1(t), \tag{5}$$

where δ is a { –1,0,1 }-value function, which is a further directional input and defines the rotation direction character of the input (given direction, stillness, change of direction).

To the outgoing moment of the system generally it can be written that

$$M_2(t) = \delta \cdot \eta_i \cdot M_1(t) \cdot u_i. \tag{6}$$

Figure 1. can be transformed in view of formula (6) so that it can be seen in *Figure 2*., which shows that one group of the driving mechanism (for example gradual) elements can characterised by the unchanging rotation direction movement, while the reversing can be solved by insert a functional element.

This recognition lead the researchers for example at the driving gears of the vehicles or wind turbines that the unvaried rotation direction for instance at loading capacity, efficiency of gears can result regular working, if deviating from symmetric tooth shape non-symmetric tooth form is applied. This motivated rows of researches for the practical application, on the one hand to the description of geometry, on the other hand towards the static and the dynamic examinations, and the manufacturability as well.

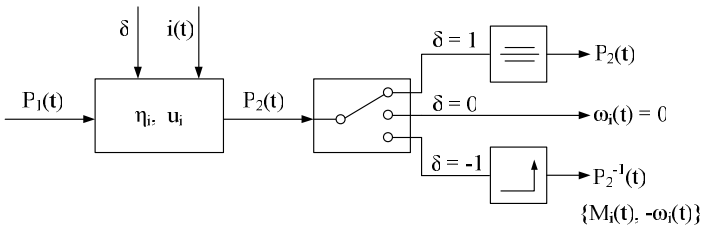


Figure 2. Reversing function built in the driving mechanism

2. Damage of gears and modification of loading capacity

Damage forms of gears can be divided into two groups. Tooth breaks caused by fatigue or aggressive origin breaks belong to one of the largest group. [1, 2] When it is caused by fatigue in connecting point perpendicular to the tooth surface tangential direction component of the normal tooth force results in complex stresses (bending and shearing) in the tooth root. Through bending stress mainly in the tooth root on the loaded tooth side i.e. on the drive side tensile stress wakes, while on the unloaded side i.e. on the coast side pressure rises. These pulsating fatigue stresses are responsible for development of cracks, which can cause break later.

Another large part of damages are caused by casualties of the tooth surface. [1, 2] Abrasion, pitting, different scratches and scoring, cracking and other tooth surface damages belong here. Damage of the tooth surface can be caused by several reasons, but here we would like to talk about damages caused by fatigue first of all. During contact of curved surfaces because of surface pressure Hertzian-stress is consorted with sharing stresses, therefore fatigue of tooth surface occurs above a loading period number (as in case of tooth root fatigue). These stresses are pulsating pressure stresses and this process causes pitting of the tooth surface.

The shape of tooth influences remarkably loading capacity of gears, extent of stresses in the tooth, which develop in meshing because of loading. Having the tooth manufacturing realized by extracting principle in sight by changing the tool basic profile modified teeth can be produced. [9] Shaping the tool cutting edge or changing it we can affect the shape of tooth root. With increase or decrease the tool basic profile angles we can modify the radius of curvature of the tooth evolve profile arcs, thus extent of Hertzian-stresses as well.

3. Concerning standards

The established manufacturing processes and the requirements of application define basic profile symmetric, which is standard. It is because of economy; integrate of cutting tools, changeability of gears, given kinematics of manufacture instruments, standard forms of calculating procedures, demand of changes of direction in drives.

The basic profile determines the exact geometry of gear teeth, which is a fictive rack profile. This can mesh with teeth of a given gear-family. [1] On the middle line of the basic profile the tooth thickness and the tooth cut width are the same. The acute angle between parts of the straight profiles, determine the sidelong edges, and the straights perpendicular to the middle line is the basic profile angle (α). Value of the standard basic profile angle is usually 20° but in special applications it can be different from this as well. We can express the odd sizes of the basic profile with module (m), which is nationally standardized too.

The basic profile of the evolve teeth determines the basic profile of the cutting tool, which is called tool basic profile. [1] If we put these two kinds of basic profile to each other, then their middle lines cover each other. The head line of the tool basic profile corresponds to the foot line of the teeth basic profile. It means that the shape of the tool head edge determines the tooth root profile of the gear teeth. In case of little module (smaller than 1 mm) the foot line of the tool basic profile agrees with the head line of the teeth basic profile. In the other cases the tooth height of the tool basic profile is larger than the tooth height of the teeth basic profile.

In our country the basic profile of cylindrical gears with evolves teeth and its tool basic profile to manufacture are described by standards MSZ 433 and MSZ 17154. Standard MSZ 310 decrees module series.

4. Increasing the loading capacity and the lifespan as a new possibility

Researches tending to increase loading capacity of gears are in connection with asymmetric tooth shape mainly. In these studies tooth sides are distinguished. One of them is the loaded tooth side (drive side) another is the unloaded side (coast side). The profile angles of both sides are different, that causes asymmetry particularly. Various advantageous or disadvantageous properties come to the front depending on whether profile angle on the drive side or on the coast side is larger. Greater parts of the researchers prefer that case, when the profile angle on the drive side is larger. Circumstances and marginal conditions of the researches have to be taken into consideration in each case.

Kapelevich [3] realizes computer aided simulation of the gear design in his method. This way special pair of gears can be solved widely. Increase of loading capacity of gears can be reached by application of larger profile angle of the drive side beside size and mass reduce. Furthermore he showed decrease of vibration level, which was available by decreasing meshing stiffness.

Working gears at big revolutions per minute dynamic load and vibration cause the main problem. Karpat et al. in [5] investigated behaviour of asymmetric gears during dynamic load. With enlarging the profile angle on the drive side dynamic factor of the asymmetric toothed gears was grown. However this factor reduced remarkably by increase of addendum, item the area on the tooth surface was grown where one pair of tooth meshes with each other and extent of static transmission errors reduced at the same time. Design of asymmetric teeth can be optimized by decrease of dynamic effects. These results are significant mainly in relation with aircraft industry, automotive industry and wind turbines.

Results of Pedersen [6] showed that in order to decrease the bending stresses in the tooth root it is necessary to apply asymmetric teeth. The greatest effect can be reached by this way. Reduction of bending stresses can be reached by two methods: either by thickening of tooth root or changing the shape of tooth root where the stress density is the largest one.

In Senthil Kumar et al.'s study [4] the asymmetry enlarges the loading capacity of the tooth root in contradiction to traditional symmetric gears. The asymmetry means larger profile angle on the drive side in this case.

Researches can be found, in which the authors propose shaping of the cutting tool (rack), because the shape affects the gear teeth (evolve profile and tooth root curve). Rack profiles proposed by Senthil Kumar et al. [4] have one circle arc on its head edge. In this case by application of profile angles differ from each other the radius of tool head edge decreases to a lesser degree than in case of symmetric. The positive effect on the bending

stresses is in connection with widening of the tooth. In this study at designing of non-standard tools the authors considered only standard module and standard profile angle on the coast side, which makes the process of the tool engineering easier.

Pedersen [6] proposed new shape of standard rack. He made a difference in between the parts of the tool head edge according to which root of the tooth profile side it forms. On the coast side of the tool head edge there is circle arc, on the drive side there is ellipse arc. He guaranteed the continuity of the tool profile with an initiated factor μ, which modifies the width of the tooth at the same time.

Alipiev [7] generated variations of rack tool profiles in case of different profile angles on the drive and on the coast side, while he changed the size of the head edge radius and its positions.

These results are not complete, only a short summary can be read above.

5. Generalization of the tool basic profile

In order to study of effects of asymmetry, one of the first steps is to determine the basic profile of the tool, which generates the gear teeth. [10] Later changing possibilities of the meshing characteristics can be examined with this. [8] Points of the basic profile are arranged in an xy orthogonal coordinate-system in such a way that the middle line of the basic profile overlaps with the x axis of the coordinate-system, the y axis divides longitudinal the tooth of the basic profile into halves. This arrangement was shown in *Figure 3*.

The basic profile is constituted by straights and curves connected to one another, equations determining them must be described in the xy coordinate-system. The straights construct the edges at the sides, head and at foot line of the basic profile. The curves are the rounding between the straight parts. The rounding between the side edge and the head edge has a great influence on the shape of the tooth root profile. [1] The equations of the straights can be given as a function of x in the xy coordinate-system. These straights are marked I. and II. in *Figure 3*. The straight I. which forms the edge at the side of the tool basic profile crosses the point P_3 on the x axis in a given direction. The straight I. encloses $(90°+\alpha)$ with the x axis, where α means profile angle. The coordinates of the point P_3 (7) can be read from *Figure 3.* and by this the equation of straight I. can be written (8).

$$P_3(x_3; y_3) = P_3(-\frac{\pi \cdot m}{4}; 0)\,, \tag{7}$$

$$y_I = -\frac{1}{\tan\alpha}(x + \frac{\pi \cdot m}{4})\,. \tag{8}$$

The straight II., which defines the straight part of the tool basic profile, is parallel with the x axis, and the distance between them is equal to the sum of clearance and module. The equation of the straight II. can be given by formula (9).

$$y_{II} = -(m + c) = -(m + c^* \cdot m) = -m \cdot (1 + c^*)\,. \tag{9}$$

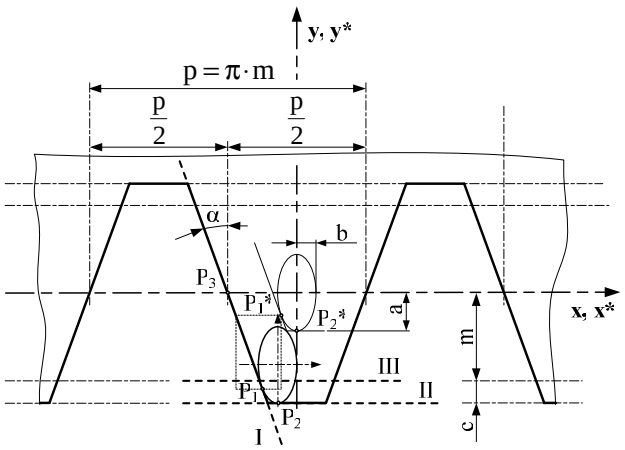


Figure 3. Tool basic profile with ellipse curve [10]

To the rounding of the head edge and the side edge for example an arch of a round or an ellipse can be taken. Another curve can be used as well, which is attached to a suitable reference point. Another x*y* coordinate-system can be used to write down the equation of the curve. The origin of this new coordinate-system overlaps the origin of the xy coordinate-system. As an example, let the rounding of head edge and side edge be an ellipse arc, which's major axis (2a) to the y axis, and minor axis (2b) to the x axis is fitted. From well known equation of the ellipse value of y can be expressed (10).

$$y^* = \sqrt{(1 - \frac{(x^*)^2}{b^2}) \cdot a^2}\ . \tag{10}$$

After tangents of the curve are defined (11.) on the understanding that gradients of the tangents be equal with gradients of the straights I. and II. Thus the tangents are parallel with the straights I. and II. The tangents on the points P_1* and P_2* are tangential to the curve (12). After differentiation formula (13) is offered.

$$\frac{dy^*}{dx^*} = (y^*)'\,, \tag{11}$$

$$\frac{dy^*}{dx^*} = \frac{dy_{I,II}}{dx} \rightarrow P_{1,2}{}^*(x_{1,2}{}^*; y_{1,2}{}^*)\,, \tag{12}$$

$$\frac{dy^*}{dx^*} = (y^*)' = \frac{-\frac{a^2}{b^2} \cdot 2x^*}{2 \cdot \sqrt{a^2 - \frac{a^2}{b^2} \cdot (x^*)^2}}\ \cdot \tag{13}$$

If the formula (13) equals with gradient of the straight I. and x* is expressed from the equation, then formula (15) is given. From the result of the extraction of root the negative value is considered. This equals to the coordinate x_1* of the usual point P_1* later.

$$(y^*)' = -\frac{1}{\tan\alpha}, \tag{14}$$

$$x^* = -\frac{1}{\sqrt{\frac{1}{b^2}\left(\frac{a^2}{b^2}\cdot\tan^2\alpha + 1\right)}}\cdot \tag{15}$$

After this the origin of the ellipse curve is moved (shifting in the direction of x and y) in the xy coordinate-system so that the tangents and the straights I. and II. are covered with one another. At this time the rounding between the head edge and the side edge which are straights, is generated by the arc of the given curve now between the contact points P_1 and P_2 as it can be seen in *Figure 3*.

The shifting in the direction y of the ellipse (point P_2* is moved to the straight II. → point P_2 is offered) is given by the difference between the y coordinates of the definite tangent and the straight II. The gradient of the straight I. influences the shifting in the direction x (point P_1* is moved to the straight I. → point P_1 is offered). Given from geometry the coordinates of the point P_2* can be read from *Figure 3*. On the basis of this the correct shifting in the y direction of the ellipse can be given by formula (16).

$$O'y = -(m + c) + a \tag{16}$$

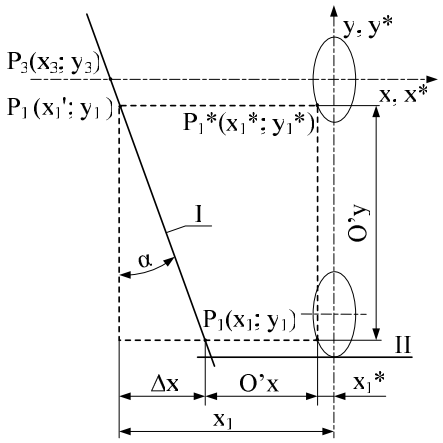


Figure 4. Tool basic profile with ellipse curve [10]

To the determination of the shift in the x direction through the y_1' coordinate of the point P_1' on straight I., which equals to the y_1* coordinate of the point P_1*, the x_1' coordinate can be written by formula (17). Considering the *Figure 4*., formula (18) gives the correct shift in the y direction. By adding the correct shift values under formulas (16) and (18) to the corresponding coordinates of the point P_1* we get the coordinates of the point P_1 on straight I. In case of the point P_2 it can be done in similar way.

$$x_1' = y_1' \cdot (-\tan\alpha) - \frac{\pi \cdot m}{4}, \tag{17}$$

$$O'x = x_1' + \Delta x + x_1^*. \tag{18}$$

By the definition of these straights (head edge, side edge), the optional arc (rounding) and the coordinates of the connection points of these curves the half of the tool basic profile can be determined. The whole profile can be given exactly by reflection to the y axis the profile got this way or by application of a different curve.

By exact description of the tool basic profile definite teeth can be generated. In the course of generalization by changing the parameters of the basic profile for example the profile angle, the rounding of the head edge, the combination of the rounding curves, just in different way on the two sides of the profile, rows of asymmetric non standard teeth can be originated.

6. Determination of the maximum stress location at the tooth root

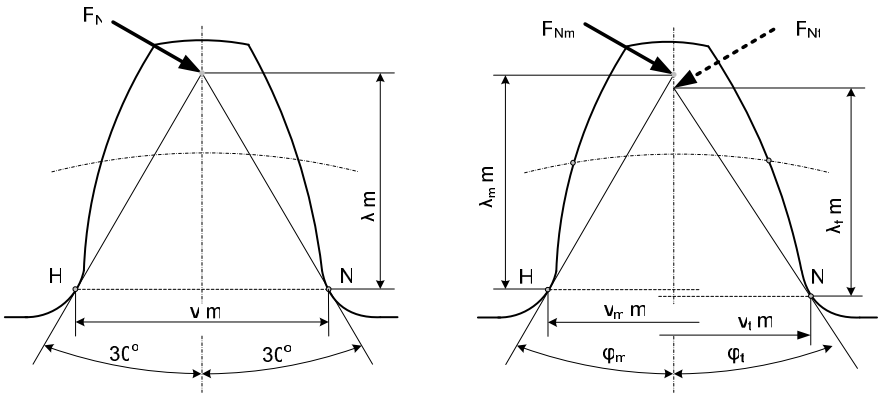
The international and national standards (DIN, ISO and so on) give precise directives to the static examination of cylindrical gear pairs in case of symmetric teeth. At research of tooth root fatigue the calculation methods set out always from a base model. These models present deviation in each standard (for example account the friction). These differences are not significant but they can not be left out of consideration. The models define parameters, which depend on the geometry of the model. The established parameters determine factors suitably. These factors help describing the maximum of the tooth root stress in a maximum stress location supposed and fixed in the model. A base model can be seen in *Figure 5. a)* in case of symmetric tooth shape.

Base models can usually handle specific geometrical constructions only. That is why the designer runs into difficulty of the calculation method immediately, mainly when the axial distance is smaller or larger than the elementary axial distance. At this time teeth has to be done free from clearance so profile removals has to be applied. Determining of the profile removals can be solved by dividing of the common tooth height (h_w) when it is defined by some kind of principles (equal tooth root fatigue endurance, minimal slip lost effect and so on). Because of the differences among tooth heights the static examinations supporter software are not able to handle the tooth root stress modifier factors.

In case of non-symmetric tooth shape [*Figure 5. b)*] the accuracy of the model itself can be questioned because it is not sure that the maximum stress locations take place in the contact points on the 30 degrees tangents of the tooth root curve.

In the tooth root the maximum tension place can be defined by experimental model methods beside classical mechanical procedures. These can be the followings.

- Fatiguing experiment affected on real model, from which after tooth break the position of the broken piece cross section let conclude the maximum stress location but not the effective magnitude of the maximum tension.
- The examination is possible on CAD virtual model with FEM analysis where the maximum stress place and the maximum tension can be calculated.
- With optical stress analysis where with the help of suitable matter on real model near static load the maximum stress location and tension can be defined.



a) symmetric shape b) non-symmetric shape

Figure 5. Geometric base model

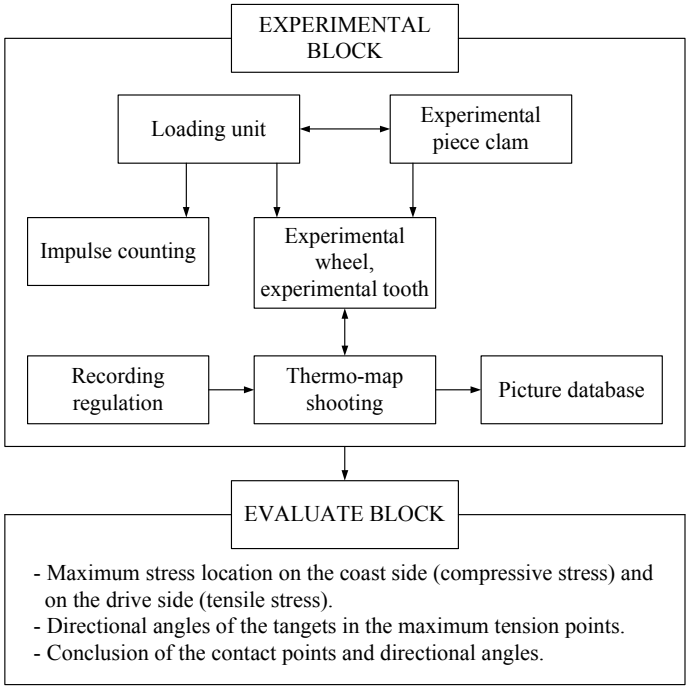


Figure 6. Shoot of thermo-map

– Thermo-map shoot on real model effect of fatigue load. Essence of the method is that the tooth exposed to the fatigue load under effect of the developed stresses because of the transformation limit formed in the tooth root the loss of energy originating in the internal friction changes the thermo-map of the tooth. The thermo-

map gives the critical stress locations thus the maximum stress location, too. *Figure 6.* presents a principled solving on this measuring method.

Locating the maximum stress position and determination of the maximum tension is necessary to give a fast and confirmed proportioning and controlling method to the designers' hand in case of non-symmetrical tooth shape as well, which method is a nominal stress determination of an approximate model and a joint of geometry dependent stress modifying parameters.

7. Summary

In propulsion chain of mechanical constructions the toothed element pairs, mainly gear pairs with evolve tooth profile play an important role. Constructional solving come to foreground increasingly tending to fulfil the function. Becoming the one directional energy transfer (there is no rotation direction change) in the centre of interest proposed the request at tooth shapes the deviation from symmetrical form. This study is tending to sum up several results that have been achieved so far. It presents the generalization of the deductive base profile to mapping the non-symmetrical teeth. At static examinations it calls the attention to the difficulties caused by the divergence from symmetrical tooth shape, for example at applying the standards recommendations. It shows an adoption of thermo-map like new method, which can make it easier to create the approximate models.

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