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MATHEMATICAL MODELS FOR TOOTH SURFACES OF GEAR COUPLING

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Abstract. Gear couplings are used to eliminate the misalignments of the connected shafts. Most important components of the gear coupling are the hub and the sleeve. The hub is an external gear having crowned teeth. The sleeve is an internal spur gear. Both gears have equal number of teeth. In this paper the manufacturing methods are presented for the hub and sleeve and mathematical models are investigated for the tooth surfaces of both components.

Keywords: gear coupling, internal gear, crowning, gear hobbing, gear shaping

1. Introduction

Main components of the gear coupling (*Figure 1.*) are the sleeve and the hub. The sleeve is an internal gear and the hub is an external gear which has crowned teeth.

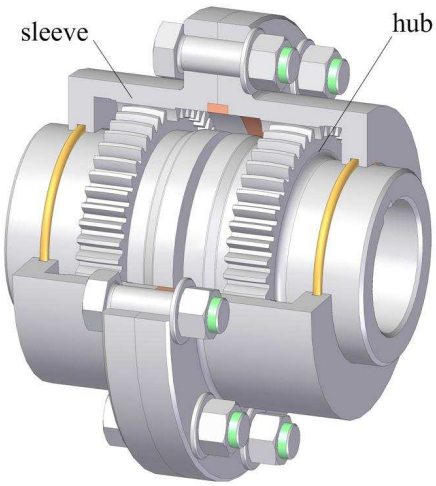


Figure 1. Gear coupling

The two toothed components compose a special gear pair, wherein both number of teeth are the same. The gear coupling is able to compensate the misalignment of the coupled shafts by the tooth crowning and backlash. Using a single hub and sleeve, the effect of angular misalignment may be eliminated. In the practice, generally two hub-sleeve pairs are built up as it is shown in Fig.1. In this case the compensation of the offset misalignment is possible in addition to the angular misalignment. Henceforward the possible manufacturing methods of these special gears will be examined. Mathematical models of the tooth surfaces

will be set up, which can provide the basis of the further investigation for the operation of gear coupling.

2. Manufacturing of the crowned gear

The crowned teeth of the hub are produced by hobbing according to *Figure 2*.

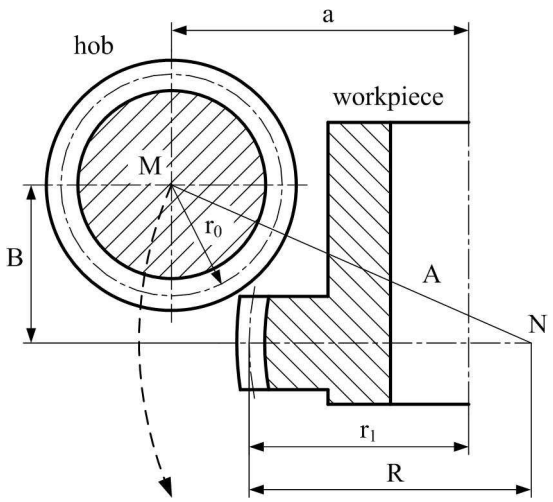


Figure 2. A conceptual sketch for manufacture of crowned tooth surfaces

In the hobbing of cylindrical gears the tool and the workpiece rotate permanently and the tool has a slow feed parallel to the axis of the workpiece. In the hobbing process the tool is called hob. To produce the crowned tooth surfaces the tool moves along a circular path as it is shown in *Figure 2*. The unique structure of the hobbing machine usually does not allow this motion of the tool, so the necessary relative movement is obtained by the radial motion of the workpiece-table and the axial movement of the tool. During production the centre distance varies continually. The maximum value of centre distance is:

$$a_{\max} = r_0 + r_1, \tag{1}$$

where r_0 and r_1 are the radii of pitch circle for the hob and the workpiece respectively.

The circular arc of the relative movement between the tool and the workpiece can be characterized by the radius $A = \overline{MN}$, depending on the pitch radius of the hob and the distance R which is the typical size of tooth crowning (*Figure 2*):

$$A = r_0 + R. \tag{2}$$

In addition, the centre distance is determined by the current axial position of the hob denoted by B in *Figure 2*. Consequently, the actual centre distance:

$$a = \sqrt{A^2 - B^2} - R + r_1. \tag{3}$$

3. The mathematical model of hobbing

The mathematical model of hobbing was presented by Litvin [1, 2] as an envelope with two independent parameters. This solution is suitable to describe the ideal tooth surfaces, but it includes some approximation, since the two parameters are not independent perfectly. Mitome [3] has reported a very expressive method for hobbing of conical involute gears. This method in modified form is suitable to determine the real tooth surfaces of cylindrical gears [4].

A conceptual sketch of the hobbing and the connection between coordinate systems are shown in *Figure 3.*

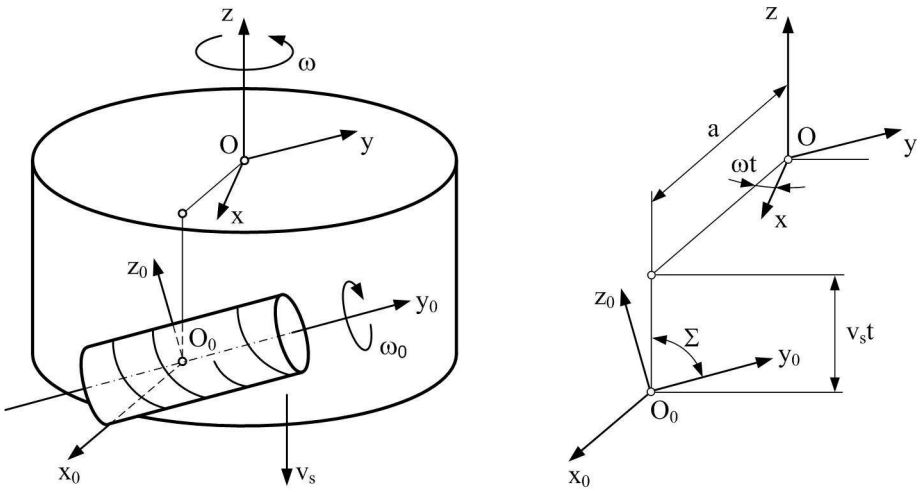


Figure 3. The sketch of hobbing and the used coordinate systems

The hob is considered as an involute worm wherein involute helicoid is fitted to the cutting edge of the hob. Thread surface of the worm has a virtual translation along the axis y_0 in coordinate system x_0, y_0, z_0 because of the angular velocity of rotation ω_0**.**

The equations of the resulted surface-series are

$$\begin{aligned} x_0 &= x_0(u,v), \\ y_0 &= y_0(u,v) + p\omega_0 t, \\ z_0 &= z_0(u,v), \end{aligned} \tag{4}$$

where u and v are the parameters of screw surface, t is the time within one revolution of the workpiece, p is the parameter of screw and ω_0 is the angular velocity of the hob.

During one revolution of the workpiece the tool generates the tooth space F_1. F_2, ... F_{k+1} denote from the second to (k+1)-th tooth spaces which are cut during the second to (k+1)-th revolution of the workpiece. s is the feed of hob during one revolution of the workpiece.

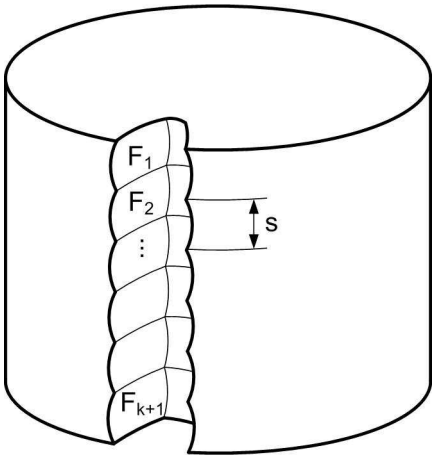


Figure 4. Real tooth surfaces of a cylindrical spur gear

Let T be the time during one rotation of the workpiece. When the surface F_{k+1} is cut the position of the origin O_0 along the axis z is

$$z = -v_s(t + kT) \tag{5}$$

where v_s is the velocity of feed. The workpiece turns the angle $\omega(t + kT)$, which corresponds to the angle $(\omega t + 2k\pi)$. For cutting the surface F_{k+1} the equation system has the following form:

$$\begin{aligned} x_0 &= x_0(u, v), \\ y_0 &= y_0(u, v) + p\omega_0(t + kT), \\ z_0 &= z_0(u, v). \end{aligned} \tag{6}$$

Transform the surface-series given by equations (6) into the coordinate system xyz:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}. \tag{7}$$

The transfer matrix M is as follows:

$$M = \begin{bmatrix} \cos\omega t & \sin\Sigma \sin\omega t & -\cos\Sigma \sin\omega t & a\cos\omega t \\ -\sin\omega t & \sin\Sigma \cos\omega t & -\cos\Sigma \cos\omega t & -a\sin\omega t \\ 0 & \cos\Sigma & \sin\Sigma & -v_s(t + kT) \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

An arbitrary surface F_n can be determined by solving Eq. (7) when k = n – 1 is substituted and at the same time relationship is produced between the parameters *u*, *v*, *t*. One possible way to determine the relation between parameters when the determinant *D* becomes zero:

$$D = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial t} \end{bmatrix} = 0 . \tag{9}$$

Expressions (7) and (9) together define any surface element F_n of the tooth surface.

Examined case of hobbing concerns to the production of cylindrical gears. The presented formulas will valid for the manufacture of crowned teeth by the following conditions:

– in addition to the axial feed velocity v_s should be considered a radial velocity v_r,

– this motion occurs continuous changing in centre distance.

The proportion of radial and axial velocities is expressed by the following equation based on the prescribed path of tool

$$\frac{v_r}{v_s} = \frac{B}{\sqrt{A^2 - B^2}} . \tag{10}$$

$B = b/2$ belongs to the position of tool $z = 0$ and the relationship $B = z + \frac{b}{2}$ is valid where *b* is the face width of the gear. The change of centre distance is described by Eq. (3). Substituting the relation between *B* and *z*

$$a(z) = \sqrt{A^2 - \left(z + \frac{b}{2} \right)^2} - R + r_1 \tag{11}$$

is obtained. It describes the change of centre distance, while the cutter passes along a prescribed path and the temporary position of the tool is determined by the coordinate *z*. All these indicate that the mathematical model for cylindrical gears is suitable to describe the crowned teeth, if the changing of centre distance is considered in the last column of transform matrix (8). Substituting (5) into (11) the changing of centre distance as a function of the time is:

$$a(t) = \sqrt{A^2 - \left[\frac{b}{2} - v_s \left(t + kT \right) \right]^2} - R + r_1 . \tag{12}$$

This relationship should be taken into account in calculating the matrix *M*.

4. Equations of the crowned tooth surfaces

Based on the above-mentioned description, it is found that the resulted tooth surface depend on several parameters. Thus it is influenced by the size of the hob (r_0) and the feed. In fact, it is also true for hobbing of cylindrical gears, that one gear produced by different hob or different feed has several tooth surfaces. The cylindrical gears with involute tooth surfaces are idealized surfaces.

The idealized tooth surface for crowned gearing will be derived so that involute tooth surfaces having variable profile shifting in parallel transverse planes are assumed.

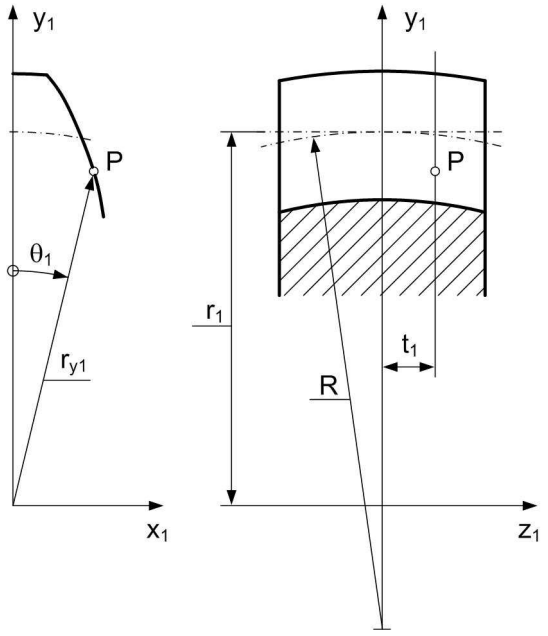


Figure 5. Crowned tooth surface

Equations of the tooth surface are:

$$\left.\begin{aligned} x_1 &= r_{y1}\sin\theta_1, \\ y_1 &= r_{y1}\cos\theta_1, \\ z_1 &= t_1. \end{aligned}\right\} \tag{13}$$

where r_{y1} is the arbitrary radius along the tooth profile, and θ_1 is the tooth angle. To calculate it the following expression is used:

$$\theta_1 = \frac{s}{2r_1} + inv\alpha - inv\alpha_{y1} \tag{14}$$

where s is the tooth thickness along the pitch cylinder, r_1 is the pitch radius, α is the standard pressure angle, α_{y1} is the pressure angle at radius r_{y1}. α_{y1} can be determined by the following equation:

$$\cos\alpha_{y1} = \frac{r_{b1}}{r_{y1}} . \tag{15}$$

Here r_{b1} is the radius of base circle. In Eq. (14) the *inv* means the involute function, which is interpreted as inv α = tan α – α.

The tooth thickness along the pitch cylinder is

$$s = s_0 - 2(R - \sqrt{R^2 - z_1^2})\tan\alpha , \tag{16}$$

where s_0 is the tooth thickness in the plane $z_1 = 0$.

All these indicate that θ_1 depend on the radius r_{y1} and the coordinate z_1=t_1, i.e. in Eq. (13)

$$x_1 = x_1(t_1, r_{y1})$$

and (17)

$$y_1 = y_1(t_1, r_{y1}).$$

5. Manufacture methods for internal gears

Manufacture methods of the internal gears may be classified into two categories, which are the forming and generating procedures. The forming processes include the form milling and broaching.

The form milling is realized by hobbing machine using form milling head and finger type or disk type milling cutter. The teeth are formed one by one without generating motion. Form milling may be used economically for machining the gears which have large diameter and high module. Disk type cutters having carbide bit realize appropriate productivity. The disadvantage of the procedure to be less accurate than the generating methods and large ring gears can be manufactured only. The tip diameter of gear should be many times as large as the milling head. Form milling is not suitable for preparation of helical teeth.

The broaching is the most productive method for manufacture of internal gears, but also the most expensive as well. In consideration of the prime and foundation costs of broaching machines and the high price of the broach, the broaching should only be used economically in quantity production. To produce helical teeth using special machine is possible, but the fabrication and sharpening of tool and the guiding of tool along helical path are very difficult tasks.

The generating processes are the gear shaping, gear skiving and gear hobbing.

The gear shaping was the first generating process which is suitable to produce internal gears too. This procedure is still the best known and most widely used method.

Since the gear shaping has low productivity due to intermittent operation, several attempts have been made to develop efficient production methods. Such methods were the gear skiving and gear hobbing for internal teeth generation.

The gear skiving was created as a special blend of the gear shaping and hobbing. The cutter comes from gear shaping while the movements come from gear hobbing. The productivity of gear skiving is similar to the gear hobbing of external gear teeth. It can be mentioned as an advantage that the helix angle may be set between wide limits, compared to other procedures that are either unsuitable for the manufacture of helical teeth, or just defined helix angle values can be produced. The special tool holder ("flying cutter") did not provide sufficient rigidity, therefore the gear skiving did not come in general use.

Gear hobbing for internal gears can be done on conventional hobbing machine using special tool clamping device. In the course of production barrel-shape hob is used. The spread of procedure was obstructed by the cost of complicated hob geometry, the convenient solution to a rigid tool holder and the size limit, which arises from the fact that the tool holder device must have fit to the internal ring.

Henceforward we consider the gear shaping because it is the only generating process using the manufacture of internal gear, which is widely used, reliable, and has adequate precision.

6. Gear shaping

The gear shaper and shaper cutter were developed and patented by Fellows in 1897. The position of workpiece and cutter and the characteristic movements of gear shaping are illustrated in *Figure 6*.

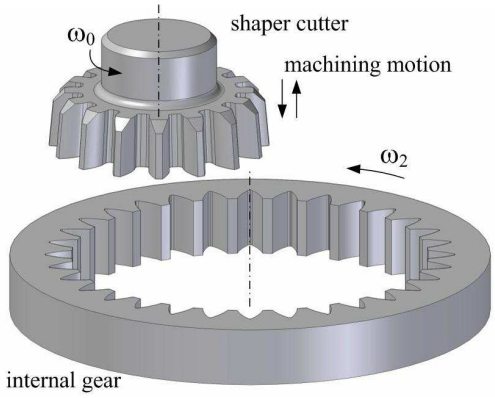


Figure 6. Gear shaping of an internal gear

Axes of workpiece and cutter are parallel to each other. The generation is produced by the harmonized rotation between the cutter and gear blank. The relationship between the angular velocities can be expressed as the gear ratio:

$$\frac{\omega_0}{\omega_2} = \frac{z_2}{z_0} = u\,. \tag{18}$$

The cutting motion is a vertical (at certain types of machine is horizontal) reciprocating movement of the cutting tool. In the machining process there are two type of feed in radial

and tangential direction. The radial feed is realized by cam mechanism or threaded spindle. The tangential feed is the rotation in mm referred to one stroke and measured on the pitch circle of cutter. During cutting neither cutting tool nor workpiece does not rotate. The generating movement that is a slight rotation is carried out during the return motion of cutter.

By gear shaping spur and helical gears can be generated too. Spur gears are produced by straight-toothed tool and helical gears are manufactured by helical shaper cutter. Since the gear couplings contain spur internal gear, hereafter deal with straight teeth only.

7. Equations of the tooth surfaces for internal gear

Theoretical tooth surfaces of the internal gears are involute cylinders. *Figure 7.* shows the tooth profile and the parameters of tooth surface.

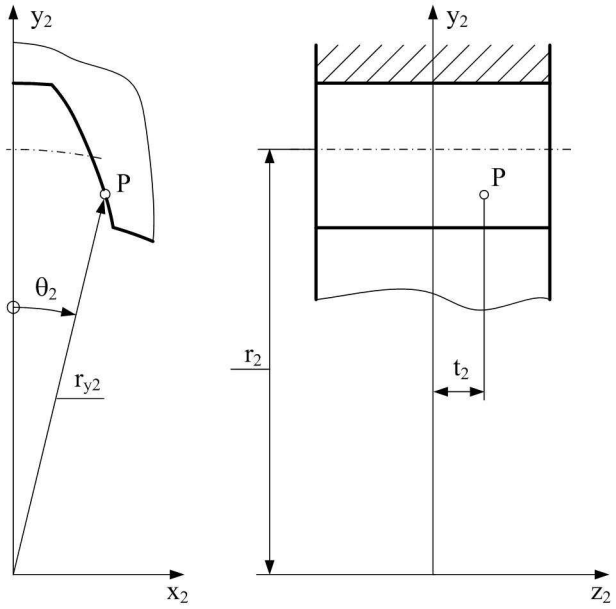


Figure 7. Tooth surface of internal gear

Equations of the tooth surface are:

$$\left.\begin{aligned} x_2 &= r_{y2}\sin\theta_2, \\ y_2 &= r_{y2}\cos\theta_2, \\ z_2 &= t_2. \end{aligned}\right\} \tag{19}$$

Where r_{y2} is the arbitrary radius along the tooth profile, and θ_2 is the angle of tooth space. To calculate this angle the following expression is used:

$$\theta_2 = \frac{e}{2r_2} + inv\alpha - inv\alpha_{y2} \tag{20}$$

where e is the tooth width of space along the pitch circle, r_2 is the pitch radius, α is the standard pressure angle, and α_{y2} is the pressure angle at radius r_{y2}. It can be determined by the following equation:

$$\cos\alpha_{y2} = \frac{r_{b2}}{r_{y2}}. \tag{21}$$

Here r_{b2} is the radius of base circle. In Eq. (20) the *inv* is the involute function, which is interpreted as inv α = tan α – α.

The tooth surface is described by two independent parameters r_{y2} and t_2:

$$\left.\begin{aligned} x_2 &= x_2(r_{y2}), \\ y_2 &= y_2(r_{y2}), \\ z_2 &= z_2(t_2). \end{aligned}\right\} \tag{22}$$

Acknowledgements

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