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ASYMMETRICAL TEETH MESHING NEAR GENERAL CENTRE DISTANCE

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Abstract. In propulsion technique applied in every day practice the load of gears is usually one directional or two directional, but the stresses on the tooth flanks are not the same degree. Numerous researches verify that departing from standard symmetric toothed wheels, the load carrying capacity of gears can be increased by asymmetric teeth. Effects of deviation from standard have numerous consequences, which have to be examined. For example geometric results, especially, when the gear pair is not fitted in elementary centre distance, but general centre distance. We would like to answer that, what kind of addendum modifications have to be applied to ensure backlash-free engagement.

Keywords: tooth shape, tooth root curve, tooth root stress, asymmetrical teeth, general teeth, backlash-free engagement.

1. APPLICATION OF GEARS

Functional units and gear drives, which modify parameters of movement, take a great part in energetic chain realized in mechanical products. In gear drives continuous one- directional rotation of the toothed element pairs can be ensured by a suitably integrated rotation reversing facility. These transmission elements of movement realize either exact map of movement or large load transfer. The tooth shape plays a great role in the determination of transmission parameters. Deviation from symmetric tooth shape generates questions about design, manufacturing and certification (dimensioning, reliability) [3]. The most important function of gear drives applied in driving mechanisms is steady transmission of movement during working [1]. Considering the single forms of motion, rotation would be changed, for example by the reduction of revolutions.

Gearings can essentially be divided into two groups, kinematical and load transfer drives [1]. These functions mean different requirements for the teeth. Kinematical drives have to ensure precise angle rotation. These drives are characterized by small backlash. In load transfer drives the backlash is larger, which helps accommodation to the fluctuation of temperature and reliability of gears during working respectively.

In the drive train of mechanical systems there is a usual functional claim, where the energy in a unit of time, which is the source of power, should be available for any working machine. This availability taking the energy components into consideration can be changed discreetly or continuously in a range. These functional units are the driving mechanisms.

One element of the system is denoted functionally by the structure element in *Figure 1*, which can be examined from inside as if it would be an unknown construction.

The input is described by the power $P_1(t)$, with its components in the energy chain that are the moment $M_1(t)$ and the angular speed $\omega_1(t)$. This characterizes the source of energy at the same time.

The output is marked by the power $P_2(t)$, which due to the construction of the system element depends on the system controller information. These are the stage contact (i), which can be discreet or time controlled - it gives the system transmission - and the functional δ, which denotes the direction of rotation inside the system.

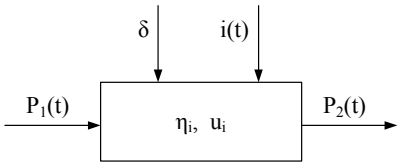


Figure 1. Functional unit describing driving mechanism

Based on the above mentioned it can be written that

$$P_1(t) = M_1(t) \cdot \omega_1(t), \tag{1}$$

$$P_2(t) = M_i(t) \cdot \omega_i(t), \tag{2}$$

$$\frac{\omega_1(t)}{\omega_i(t)} = u_i = f(i), \tag{3}$$

together with the appropriation of the formulas (1), (2), (3)

$$M_i(t) = \eta_i \cdot M_1(t) \cdot u_i, \tag{4}$$

where η_i is the efficiency which belongs to the controlled settings (stages) of the system element. Examining the system so that in point $\omega_i(t)$ regulated rotating movement should be mapped as well, from formula (3) it follows that

$$\omega_i(t) = \delta \cdot u_i \cdot \omega_1(t), \tag{5}$$

where δ is a { -1,0,1 }-value function, which is a further directional input and defines the rotation direction character of the input (given direction, stillness, change of direction).
To the outgoing moment of the system generally it can be written that

$$M_2(t) = \delta \cdot \eta_i \cdot M_1(t) \cdot u_i. \tag{6}$$

It can be concluded from the above that improving the strength of asymmetric teeth, their geometrical examination has to be performed, too. The question is raised so that what kind of geometry can be applied, and how it can be changed and controlled in order that the meshing conditions, the teeth margins can be completed.

2. GENERALIZATION OF THE TOOL BASIC PROFILE

In order to study of effects of asymmetry, one of the first steps is to determine the basic profile of the tool, which generates the gear teeth [4]. Later changing possibilities of the meshing characteristics can be examined with this [2]. Points of the basic profile are arranged in an xy orthogonal coordinate-system in such a way that the middle line of the basic profile overlaps with the x axis of the coordinate-system, the y axis divides longitudinal the tooth of the basic profile into halves. This arrangement was shown in *Figure 2*.

The basic profile is constituted by straights and curves connected to one another, equations determining them must be described in the xy coordinate-system. The straights construct the edges at the sides, head and at foot line of the basic profile. The curves are the roundings between the straight parts. The rounding between the side edge and the head edge has a great influence on the shape of the tooth root profile [1]. The equations of the straights can be given as a function of x in the xy coordinate-system. These straights are marked I. and II. in *Figure 2*. The straight I. which forms the edge at the side of the tool basic profile crosses the point P_3 on the x axis in a given direction. The straight I. encloses $(90°+\alpha)$ with the x axis, where α means profile angle. The coordinates of the point P_3 (7) can be read from *Figure 2* and by this the equation of straight I can be written (8).

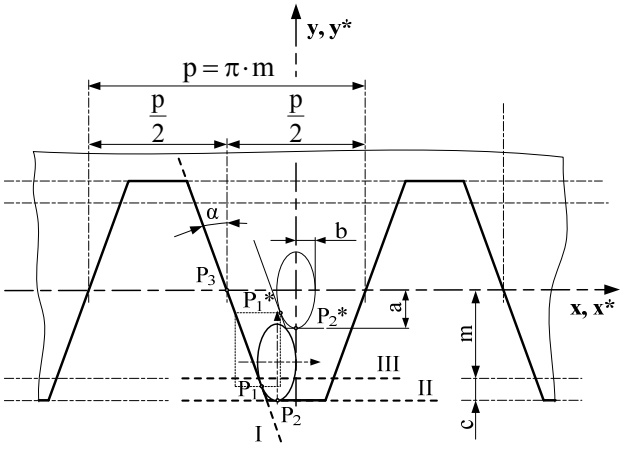


Figure 2. Tool basic profile with ellipse curve [4]

$$P_3(x_3; y_3) = P_3(-\frac{\pi \cdot m}{4}; 0), \tag{7}$$

$$y_I = -\frac{1}{\tan\alpha}(x + \frac{\pi \cdot m}{4}). \tag{8}$$

The straight II, which defines the straight part of the tool basic profile, is parallel with the x axis, and the distance between them is equal to the sum of clearance and module. The equation of the straight II. can be given by formula (9).

$$y_{II} = -(m + c) = -(m + c^* \cdot m) = -m \cdot (1 + c^*) . \tag{9}$$

To the rounding of the head edge and the side edge for example an arch of a round or an ellipse can be taken. Another curve can be used as well, which is attached to a suitable reference point. Another x*y* coordinate-system can be used to write down the equation of the curve. The origin of this new coordinate-system overlaps the origin of the xy coordinate-system. As an example, let the rounding of head edge and side edge be an ellipse arch, whose major axis (2a) is fitted to the y axis while its minor axis (2b) is fitted to the x axis. From well known equation of the ellipse value of y can be expressed (10).

$$y^* = \sqrt{(1 - \frac{(x^*)^2}{b^2}) \cdot a^2} . \tag{10}$$

After tangents of the curve are defined (11) on the understanding that gradients of the tangents be equal with gradients of the straights I. and II. Thus the tangents are parallel with the straights I. and II. The tangents on the points P_1* and P_2* are tangential to the curve (12). After differentiation formula (13) is offered.

$$\frac{dy^*}{dx^*} = (y^*)' , \tag{11}$$

$$\frac{dy^*}{dx^*} = \frac{dy_{I,II}}{dx} \rightarrow P_{1,2}^* (x_{1,2}^*; y_{1,2}^*) , \tag{12}$$

$$\frac{dy^*}{dx^*} = (y^*)' = \frac{-\frac{a^2}{b^2} \cdot 2x^*}{2 \cdot \sqrt{a^2 - \frac{a^2}{b^2} \cdot (x^*)^2}} , \tag{13}$$

If the formula (13) equals with gradient of the straight I. and x* is expressed from the equation, then formula (15) is given. From the result of the extraction of root the negative value is considered. This equals to the coordinate x_1* of the usual point P_1* later.

$$(y^*)' = -\frac{1}{\tan \alpha} , \tag{14}$$

$$x^* = -\frac{1}{\sqrt{\frac{1}{b^2} (\frac{a^2}{b^2} \cdot \tan^2 \alpha + 1)}} \cdot \tag{15}$$

After this the origin of the ellipse curve is moved (shifting in the direction of x and y) in the xy coordinate-system so that the tangents and the straights I. and II. are covered with one another. This time the rounding between the head edge and the side edge which are

straights, is generated by the arch of the given curve now between the contact points P_1 and P_2 as it can be seen in *Figure 2.*

The shifting in the direction y of the ellipse (point P_2* is moved to the straight II. → point P_2 is offered) is given by the difference between the y coordinates of the definite tangent and the straight II. The gradient of the straight I. influences the shifting in the direction x (point P_1* is moved to the straight I. → point P_1 is offered). Given from geometry the coordinates of the point P_2* can be read from *Figure 2*. On the basis of this the correct shifting in the y direction of the ellipse can be given by formula (16).

$$O'y = -(m + c) + a\,. \tag{16}$$

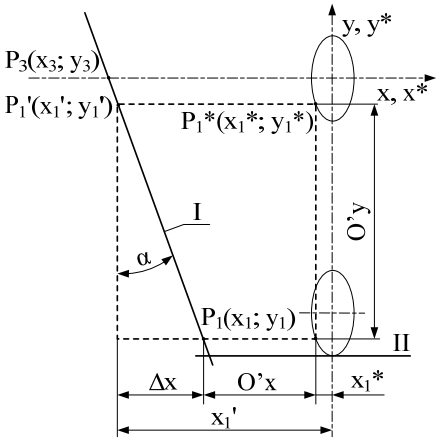


Figure 3. Tool basic profile with ellipse curve [4]

To the determination of the shift in the x direction through the y_1' coordinate of the point P_1' on straight I., which equals to the y_1* coordinate of the point P_1*, the x_1' coordinate can be written by formula (17). Considering the *Figure 3*, formula (18) gives the correct shift in the y direction. By adding the correct shift values under formulas (16) and (18) to the corresponding coordinates of the point P_1* we get the coordinates of the point P_1 on straight I. In case of the point P_2 it can be done in a similar way.

$$x_1' = y_1' \cdot (-\tan\alpha) - \frac{\pi \cdot m}{4}, \tag{17}$$

$$O'x = x_1' + \Delta x + x_1^*\,. \tag{18}$$

By the definition of these straights (head edge, side edge), the optional arch (rounding) and the coordinates of the connection points of these curves the half of the tool basic profile can be determined. The whole profile can be given exactly by reflecting the profile got this way to the y axis or by application of a different curve.

By exact description of the tool basic profile definite teeth can be generated. In the course of generalization by changing the parameters of the basic profile for example the profile angle, the rounding of the head edge, the combination of the rounding curves, in different ways on the two sides of the profile, rows of asymmetric non standard teeth can be originated.

3. PROFILE CHARACTERISTICS OF ASYMMETRIC TEETH FOR MESHING NEAR GENERAL CENTRE DISTANCE

In gear pair design the tooth shape has a great importance. On the one hand it has its importance in the precision of the movement mapping, on the other hand in the determination the load carrying capacity. The base profile parameters do not have an effect on kinematic precision, it is affected by the boundaries of the production technology possibilities.

Beside kinematic movement mapping the dynamic behaviour of the driving has a significant role. Because of the changing tooth stiffness the meshing process oscillates periodical vibration. In the consequence of the oscillation the contacting surfaces in mesh can separate, which causes that not the functional surfaces are touching. This time the meshing continuity is broken, which can be regulated by the degree of backlash release.

However the load carrying capacity depends largely on the geometric determination of the contacting surfaces, which influences the tooth surface fatigue. The fatiguing stress forming in the tooth root is defined by the tooth root geometry – namely, at a how large degree the nominal tooth root stress is increased by the stress correction factors from geometry. The tooth root geometry depends on the tooth generating base profile. Departure from the symmetry can prosperously influence the geometry of the tooth surface and the tooth root.

The meshing gears determine elementary teeth from the basic geometrical data. In the event when the axis distance of the driving is not elementary but general then addendum modification is needed for the backlash-free engagement. In case of symmetric tooth shape known calculation methods are available [1]. By non-symmetric tooth shape relations like this are not yet disposable. In order to determine the addendum modifications some basic geometrical ideas must be introduced.

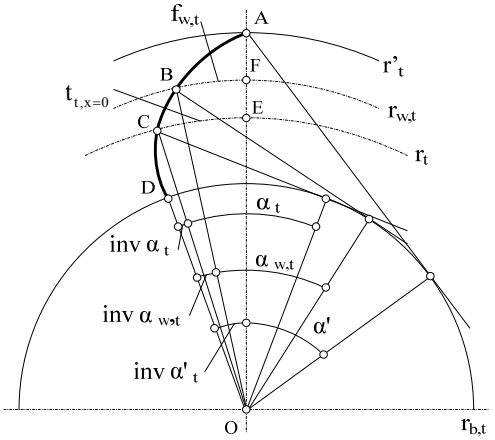


Figure 4. Explanation the tooth profile part thickness belonging to the tooth

3.1. The tooth thickness of the asymmetric teeth

The condition of backlash-free engagement is that the pitch on the rolling circle and the sum of the tooth thickness on the rolling circle are the same. The tooth thickness on the rolling circle can be determined with the help of the two tooth profiles. See *Figure 4*, which explains for a non-symmetric case one tooth profile side. This side is on the basis of indexing the coast one (t). The tooth thickness part belonging to the tooth profile is $f_{w,t}$. In order to define $f_{w,t}$ it can be reached by the following way.

The arch on the pitch circle between the points C and E can be determined from the pitch (p) explained on the base profile. First of all the pitch p has to be divided half-and-half between the tooth and the tooth space (groove) defined on the basic profile (19)

$$s_{x=0} = \frac{m \cdot \pi}{2} \cdot \tag{19}$$

The different arched tooth profiles can be placed on the pitch circle *so* that the tooth thickness (19) is divided between the arches. The proportion is expressed by the factor λ, which suitably be now 0,5. Thus the tooth thickness belonging to the coast side without addendum modification ($t_{t,x=0}$)

$$t_{t,x=0} = \lambda \cdot s_{x=0} = \frac{m \cdot \pi}{4} \qquad (\lambda = 0,5) \cdot \tag{20}$$

Take the profile displacement (21) into consideration, the tooth thickness on the coast side ($f_{w,t}$) can be written by formula (22), then developing it relation (23) is got

$$t_{t,x\neq 0} = t_{t,x=0} + m \cdot x \cdot tg\alpha_t = \frac{m \cdot \pi}{4} + m \cdot x \cdot tg\alpha_t \,, \tag{21}$$

$$f_{w,t} = \left(\frac{t_{t,x\neq 0}}{r} + inv\,\alpha_t - inv\,\alpha_{w,t} \right) \cdot r_w \,, \tag{22}$$

$$f_{w,t} = m \cdot \frac{\cos\alpha_t}{\cos\alpha_w} \left(\frac{\pi}{4} + x \cdot tg\,\alpha_t + \frac{z}{2} \left(inv\,\alpha_t - inv\,\alpha_{w,t} \right) \right) \cdot \tag{23}$$

The tooth thickness part on the drive side ($f_{w,m}$) on the pitch- and rolling circles can be written in a similar way, only the signs of the indexes change. The two tooth thickness parts express the real tooth thickness, namely

$$s_w = f_{w,t} + f_{w,m} \,. \tag{24}$$

3.2. Special circles of the tooth profiles

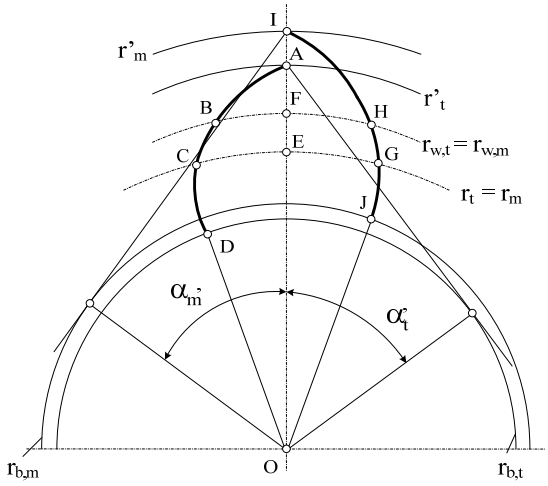


Figure 5. Positions of the drive and the coast tooth profiles

The tooth thickness parts have to be determined on optional circles, because at the teeth margin examination the top land thickness is necessary. It can be given by the sum of the tooth thickness parts. The determination the tooth thicknesses, within the tooth thickness parts on an optional circle is the condition of that the permissive mark of the addendum modifications to the general centre distance be definable. At the same time for the construction it is similarly important that the top land thickness on the addendum circle be determinable as a condition inside the teeth margin.

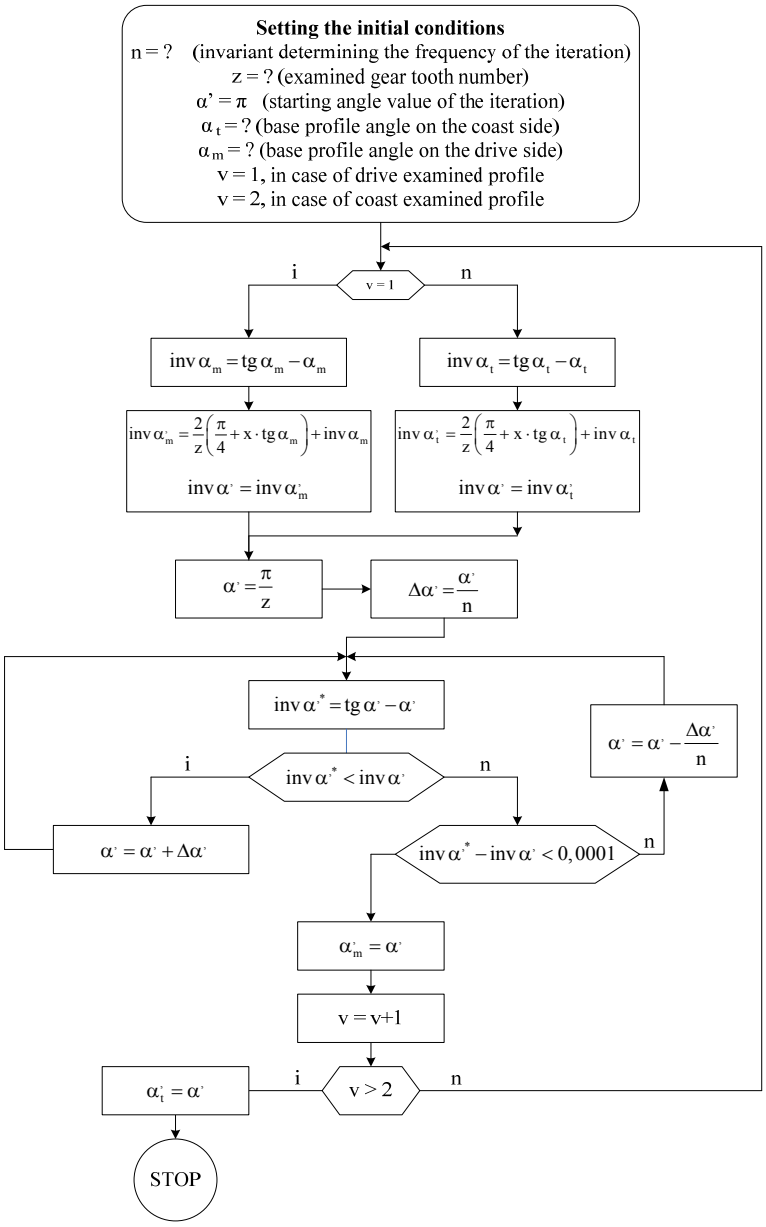


Figure 6. Angles belonging to the closing of the tooth profile sides

The tooth profile on the drive side has similar structure, but because of the basic profile angle deviation its base circle is different, so its arch is dissimilar, too (*Figure 5*). Both tooth profiles intersect the virtual tooth centre line passing the points O and A. These points of intersection determine one-one radius (r'_t, r'_m). For the examination of the teeth compliance

condition we need to know the radiuses belonging to the intersections A and I on the straight OA (virtual centre line).

The radiuses can determined only by the knowledge of the angles α'_m and α'_t. On these radiuses the dimension of the tooth thickness parts is zero. The tooth thickness is written on the coast side and the

$$f'_t = m \cdot \frac{\cos\alpha_t}{\cos\alpha'}\left(\frac{\pi}{4} + x \cdot \operatorname{tg}\alpha_t + \frac{z}{2}\left(\operatorname{inv}\alpha_t - \operatorname{inv}\alpha'\right)\right) \tag{25}$$

relation is given, on which the zero value solving, then it is expressed to inv α'_t

$$\frac{\pi}{4} + x \cdot \operatorname{tg}\alpha_t + \frac{z}{2}\left(\operatorname{inv}\alpha_t - \operatorname{inv}\alpha'_t\right) = 0\,, \tag{26}$$

$$\operatorname{inv}\alpha'_t = \frac{2}{z}\left(\frac{\pi}{4} + x \cdot \operatorname{tg}\alpha_t\right) + \operatorname{inv}\alpha_t \tag{27}$$

equation is received. From this formula and the known data the involute angle can be determined. To the determination the involute angle the formula (28) is available,

$$\operatorname{inv}\alpha'_t = \operatorname{tg}\alpha'_t - \alpha'_t\,, \tag{28}$$

which contains the value of the α'_t in implicit form. We have worked out an iteration algorithm for determination the α'_t and α'_m, which can be seen in *Fisure 6*. In the knowledge of α'_t and α'_m the radiuses r'_m and r'_t belonging to the points A and I can be given

$$r'_m = \frac{r_{b,m}}{\cos\alpha'_m}, \quad r'_t = \frac{r_{b,t}}{\cos\alpha'_t}\cdot \tag{29}$$

4. FEATURES OF NON-SYMMETRIC TOOTH PROFILE FROM THE MESHING CONDITIONS

In case of non-symmetric teeth the important condition of the mesh is the identity of the pitch on the rolling circle. While supposing symmetric teeth the rotation direction can be left out of consideration, in case of asymmetric teeth this can not be done. However, the pitch on the rolling circle has to be the same regardless of the rotation direction, this can be obtained from the meshing conditions. *Figure 7* with the different base circles shows, that the meshing near rotation reversing connection can be established among the pitches on the rolling circle, on the pitch circle and on the base circle.

From the permanence of the base circle radius between the pitches on the pitch circle and on the rolling circle it can be written on the drive side that

$$p_w \cdot \cos\alpha_{wm} = p \cdot \cos\alpha_m , \tag{30}$$

and on the coast side that

$$p_w \cdot \cos\alpha_{wt} = p \cdot \cos\alpha_t . \tag{31}$$

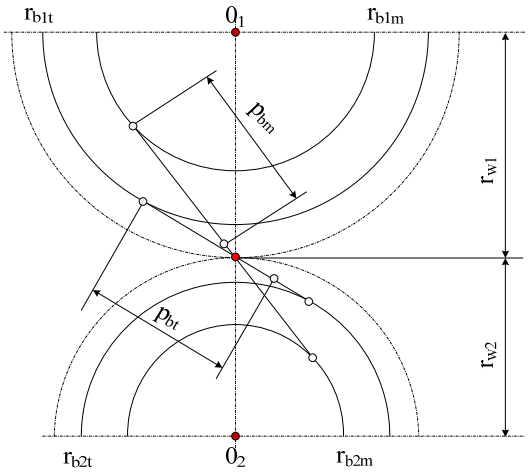


Figure 7. Connection of the base circles and the pitch on the rolling circle

On the ratio of the pitches on the rolling circle and on the pitch circle it is true that

$$\frac{p_w}{p} = \frac{\cos\alpha_m}{\cos\alpha_{wm}} = \frac{\cos\alpha_t}{\cos\alpha_{wt}} . \tag{32}$$

Relation (32) is suitable to guarantee the backlash-free condition referring to the meshing for the backlash release of the engaging teeth.

5. TOOTH THICKNESS DETERMINATION FROM THE BASE PROFILE ANGLE VALUES

When gears are in mesh one essential is that they engage without backlash clearance. Naturally this does not mean that there cannot be any gap between the tooth pairs at all. The minimal thickness of the grease, the demand for the gap originating from teeth deformation appears in the regulation of the base tangent length tolerance. Deviating from symmetric tooth shape at the asymmetric teeth the involute arches building up the tooth flanks are determining the tooth thickness belonging to the given circle of the wheel. The base tangent length between not identical tooth flank measurable determined intercepted teeth gives a

measurable distance. The tooth thickness is a determining component of the base tangent length.

With the help of the initiated relations in *Figure 4* at non-symmetric tooth shape the tooth thickness consists of two parts. One part is an arch belonging to a given circle, which is between the drive side and a seeming symmetry line dividing the tooth. Another part is an arch belonging to a given circle too, which is between the coast side and the seeming symmetry line of the tooth.

The tooth thickness part belonging to the drive side is marked by $f_{i,m}$, the tooth thickness part belonging to the coast side is marked by $f_{i,t}$, where i means the optional radius. In case of rolling circle instead of index i index w is used. On the rolling circle the drive sided tooth thickness part applying relation (21)

$$f_{w,m} = \frac{r_w}{r}\left\{\frac{m\cdot\pi}{4} + r\left(\operatorname{inv}\alpha_m + x\cdot\operatorname{tg}\alpha_m - \operatorname{inv}\alpha_{wm}\right)\right\} \tag{33}$$

can be written. The coast sided tooth thickness part can be written in a similar way as well.

$$f_{w,t} = \frac{r_w}{r}\left\{\frac{m\cdot\pi}{4} + r\left(\operatorname{inv}\alpha_t + x\cdot\operatorname{tg}\alpha_t - \operatorname{inv}\alpha_{wt}\right)\right\} \tag{34}$$

The tooth thickness on the rolling circle can be given by the sum of the drive sided (33) and the coast sided (34) tooth thickness parts.

$$s_w = \frac{r_w}{r}\left\{\frac{m\cdot\pi}{2} + r\left[\operatorname{inv}\alpha_t + \operatorname{inv}\alpha_m + x\cdot\left(\operatorname{tg}\alpha_t + \operatorname{tg}\alpha_m\right) - \left(\operatorname{inv}\alpha_{wt} + \operatorname{inv}\alpha_{wm}\right)\right]\right\} \tag{35}$$

6. SUM OF THE PROFILE DISPLACEMENT FACTOR FOR ANTI-BACKLASH GEARS

To the backlash-free engagement the sums of the tooth thicknesses on rolling circle have to correspond to the pitch on the rolling circle

$$p_w = s_{w1} + s_{w2}\,. \tag{36}$$

The pitch on the rolling circle independently from the rotation of direction must be equal. The equation (32) makes a connection among the pitch (p) explained on the basic profile, the asymmetry angles (α_t, α_m) explained on the basic profile, and the pressure angles ($\alpha_{w,t}$, $\alpha_{w,m}$) depending on the direction of rotation. It gives the left side of the relation (36). The tooth thicknesses, which are interpreted on the rolling circles of the meshing gears, can be written as the sum of the relations (33) and (34) referring to the given tooth, then the sum relating to the meshing teeth. The simpler form can be seen as follows

$$p_w = \left(f_{w,m,1} + f_{w,t1}\right) + \left(f_{w,m,2} + t_{w,t,2}\right). \tag{37}$$

In the knowledge of the referred formulas relation (37) can be written, then simplified. After reducing and expressing the sum of the addendum modification shifts it can be given as

$$\sum_{i=1}^{i=2} x_i = x_1 + x_2 = \frac{z_1 + z_2}{2} \cdot \frac{\text{inv}\,\alpha_m + \text{inv}\,\alpha_t - \left(\text{inv}\,\alpha_{w,m} + \text{inv}\,\alpha_{w,t}\right)}{\text{tg}\,\alpha_m + \text{tg}\,\alpha_t}. \tag{38}$$

On the basis of relation (38) it can be laid down that in case of asymmetrical teeth near general centre distance the sum of the addendum modification coefficients accordance with the direction of rotation can be determined by a simple calculation method.

The sum of the addendum modification defined this way shows that coefficients can be divided between the gear pairs, by the possession of the knowledge of the expected properties from the gear pairs.

7 CONCLUSION

In propulsion chain of mechanical constructions the toothed element pairs, mainly gear pairs with involute tooth profile play an important role. Constructional solving come to foreground increasingly tending to fulfil the function. Now that the one directional energy transfer (there is no rotation direction change) is in the centre of interest, it proposed the request how tooth shapes can be deviated from the symmetrical form. This study presents the generalization of the deductive base profile to mapping the non-symmetrical teeth. It introduces the calculation method of the tooth thickness and the determination of the profile displacement factors, which ensure the backlash-free engagement near general centre distance working.

Acknowledgement

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