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ANALYSIS OF GEAR MESHING FOR GEAR COUPLING

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Abstract. Gear couplings are mechanical components to connect shaft ends and eliminate the misalignments. Most important elements of the gear coupling are the hub and the sleeve. The hub is an external gear having crowned teeth and the sleeve is an internal spur gear. Both gears have equal number of teeth. In this paper the manufacturing methods are presented for the hub and sleeve and mathematical models are investigated for the tooth surfaces of both components. An approximation is presented to determine the contact points and to analyze the gear meshing.

Keywords: gear coupling, internal gear, crowning, gear hobbing, gear shaping, gear meshing

1. INTRODUCTION

Main components of the gear coupling (*Figure 1*) are the sleeve and the hub. The sleeve is an internal gear and the hub is an external gear which has crowned teeth.

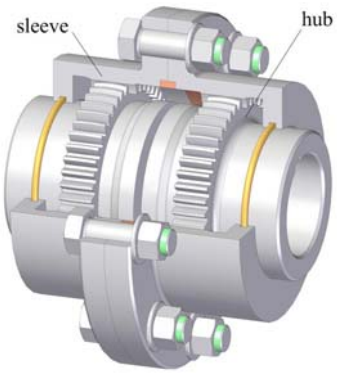


Figure 1. Gear coupling

The two toothed components compose a special gear pair, wherein both number of teeth are the same. The gear coupling is able to compensate the misalignment of the coupled shafts by the tooth crowning and backlash. Using a single hub and sleeve, the effect of angular misalignment may be eliminated. In the practice, generally two hub-sleeve pairs are built up as it is shown in *Figure 1*. In this case the compensation of the offset misalignment is possible in addition to the angular misalignment. Henceforward the possible manufacturing methods of these special gears will be examined. Mathematical models of the tooth surfaces will be set up, which can provide the basis of the further investigation for gear meshing and operation of gear coupling.

2. DETERMINATION OF CROWNED TOOTH SURFACE OF THE HUB

2.1. Manufacture of the crowned gear

The crowned teeth of the hub are produced by hobbing according to *Figure 2*.

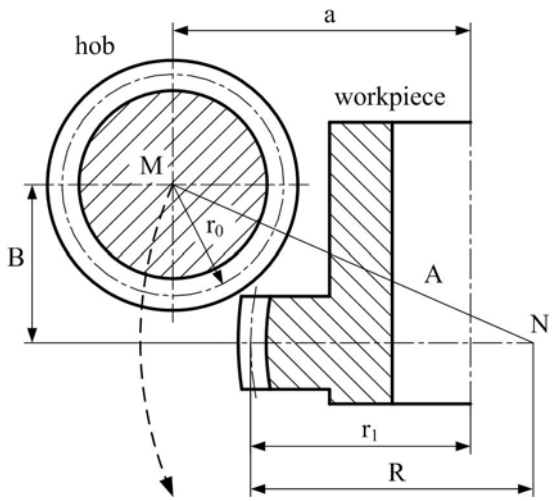


Figure 2. A conceptual sketch for manufacture of crowned tooth surfaces

In the hobbing of cylindrical gears the tool and the work piece rotate permanently and the tool has a slow feed parallel to the axis of the work piece. In the hobbing process the tool is called hob. To produce the crowned tooth surfaces the tool moves along a circular path as it is shown in *Figure 2*. The unique structure of the hobbing machine usually does not allow this motion of the tool, so the necessary relative movement is obtained by the radial motion of the work piece-table and the axial movement of the tool. During production the centre distance varies continually. The maximum value of centre distance is:

$$a_{\max} = r_0 + r_1, \tag{1}$$

where r_0 and r_1 are the radii of pitch circle for the hob and the work piece respectively.

The circular arc of the relative movement between the tool and the work piece can be characterized by the radius $A = \overline{MN}$, depending on the pitch radius of the hob and the distance R which is the typical size of tooth crowning (*Figure 2*):

$$A = r_0 + R. \tag{2}$$

In addition, the centre distance is determined by the current axial position of the hob denoted by B in *Figure 2*. Consequently, the actual centre distance:

$$a = \sqrt{A^2 - B^2} - R + r_1. \tag{3}$$

2.2. Mathematical model for crowned tooth surfaces

Based on the above-mentioned description, it is found that the resulted tooth surface depend on several parameters. Thus it is influenced by the size of the hob (r_0) and the feed.

In fact, it is also true for hobbing of cylindrical gears, that one gear produced by different hob or different feed has several tooth surfaces. The cylindrical gears with involute tooth surfaces are idealized surfaces.

The idealized tooth surface for crowned gearing will be derived so that involute tooth surfaces having variable profile shifting in parallel transverse planes are assumed (*Figure 3*).

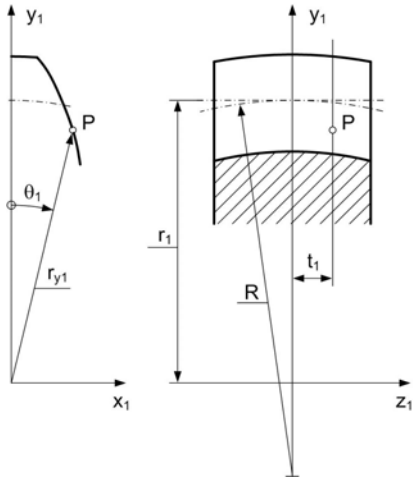


Figure 3. Crowned tooth surface

Equations of the tooth surface are:

$$\left.\begin{aligned} x_1 &= r_{y1}\sin\theta_1, \\ y_1 &= r_{y1}\cos\theta_1, \\ z_1 &= t_1. \end{aligned}\right\} \tag{4}$$

where r_{y1} is the arbitrary radius along the tooth profile, and θ_1 is the tooth angle. To calculate it the following expression is used:

$$\theta_1 = \frac{s}{2r_1} + inv\alpha - inv\alpha_{y1} \tag{5}$$

where s is the tooth thickness along the pitch cylinder, r_1 is the pitch radius, α is the standard pressure angle, α_{y1} is the pressure angle at radius r_{y1}. α_{y1} can be determined by the following equation:

$$\cos\alpha_{y1} = \frac{r_{b1}}{r_{y1}}. \tag{6}$$

Here r_{b1} is the radius of base circle. In Eq. (5) the *inv* means the involute function, which is interpreted as inv α = tan α - α.

The tooth thickness along the pitch cylinder is

$$s = s_0 - 2(R - \sqrt{R^2 - z_1^2})\tan\alpha\,, \tag{7}$$

where s_0 is the tooth thickness in the plane $z_1 = 0$.

All these indicate that θ_1 depend on the radius r_{y1} and the coordinate z_1=t_1, i.e. in Eq. (4)

$$\begin{aligned} x_1 &= x_1(t_1, r_{y1}), \\ y_1 &= y_1(t_1, r_{y1}). \end{aligned} \tag{8}$$

3. DETERMINATION OF TOOTH SURFACE OF THE SLEEVE

3.1. Manufacture methods for internal gears

Manufacture methods of the internal gears may be classified into two categories, which are the forming and generating procedures. The forming processes include the form milling and broaching.

The form milling is realized by hobbing machine using form milling head and finger type or disk type milling cutter. The teeth are formed one by one without generating motion. Form milling may be used economically for machining the gears which have large diameter and high module. Disk type cutters having carbide bit realize appropriate productivity. The disadvantage of the procedure to be less accurate than the generating methods and large ring gears can be manufactured only. The tip diameter of gear should be many times as large as the milling head. Form milling is not suitable for preparation of helical teeth.

The broaching is the most productive method for manufacture of internal gears, but also the most expensive as well. In consideration of the prime and foundation costs of broaching machines and the high price of the broach, the broaching should only be used economically in quantity production. To produce helical teeth using special machine is possible, but the fabrication and sharpening of tool and the guiding of tool along helical path are very difficult tasks.

The generating processes are the gear shaping, gear skiving and gear hobbing.

The gear shaping was the first generating process which is suitable to produce internal gears too. This procedure is still the best known and most widely used method.

Since the gear shaping has low productivity due to intermittent operation, several attempts have been made to develop efficient production methods. Such methods were the gear skiving and gear hobbing for internal teeth generation.

The gear skiving was created as a special blend of the gear shaping and hobbing. The cutter comes from gear shaping while the movements come from gear hobbing. The productivity of gear skiving is similar to the gear hobbing of external gear teeth. It can be mentioned as an advantage that the helix angle may be set between wide limits, compared to other procedures that are either unsuitable for the manufacture of helical teeth, or just defined helix angle values can be produced. The special tool holder ("flying cutter") did not provide sufficient rigidity, therefore the gear skiving did not come in general use.

Gear hobbing for internal gears can be done on conventional hobbing machine using special tool clamping device. In the course of production barrel-shape hob is used. The spread of procedure was obstructed by the cost of complicated hob geometry, the convenient solution to a rigid tool holder and the size limit, which arises from the fact that the tool holder device must have fit to the internal ring.

Henceforward we consider the gear shaping because it is the only generating process using the manufacture of internal gear, which is widely used, reliable, and has adequate precision.

The gear shaper and shaper cutter were developed and patented by Fellows in 1897. The position of work piece and cutter and the characteristic movements of gear shaping are illustrated in *Figure 4*.

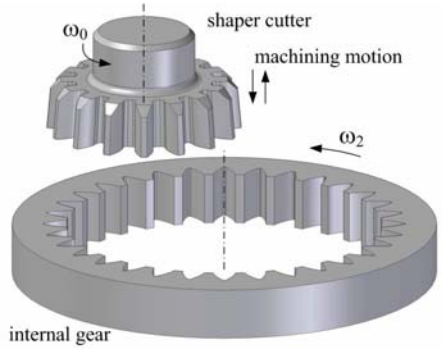


Figure 4. Gear shaping of an internal gear

Axes of work piece and cutter are parallel to each other. The generation is produced by the harmonized rotation between the cutter and gear blank. The relationship between the angular velocities can be expressed as the gear ratio:

$$\frac{\omega_0}{\omega_2} = \frac{z_2}{z_0} = u\,. \tag{9}$$

The cutting motion is a vertical (at certain types of machine is horizontal) reciprocating movement of the cutting tool. In the machining process there are two type of feed in radial and tangential direction. The radial feed is realized by cam mechanism or threaded spindle. The tangential feed is the rotation in mm referred to one stroke and measured on the pitch circle of cutter. During cutting neither cutting tool nor work piece does not rotate. The generating movement that is a slight rotation is carried out during the return motion of cutter.

By gear shaping spur and helical gears can be generated too. Spur gears are produced by straight-toothed tool and helical gears are manufactured by helical shaper cutter. Since the gear couplings contain spur internal gear, hereafter deal with straight teeth only.

3.2. Mathematical model for tooth surfaces of the sleeve

Theoretical tooth surfaces of the internal gears are involute cylinders. *Figure 5* shows the tooth profile and the parameters of tooth surface.

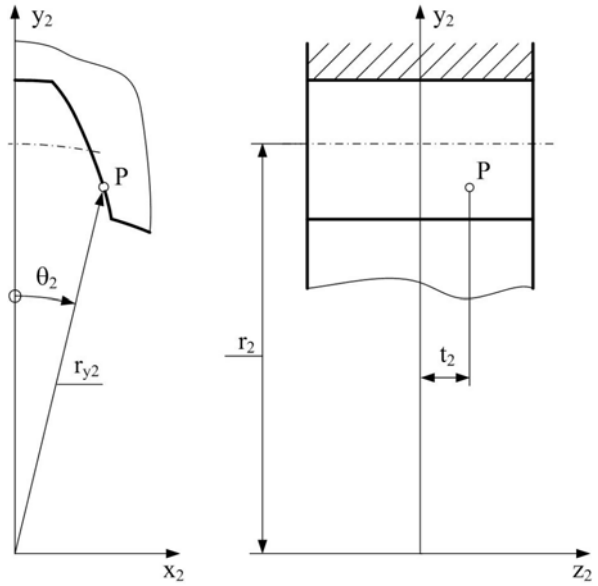


Figure 5. Tooth surface of internal gear

Equations of the tooth surface are:

$$\left.\begin{aligned} x_2 &= r_{y2}\sin\theta_2, \\ y_2 &= r_{y2}\cos\theta_2, \\ z_2 &= t_2. \end{aligned}\right\} \tag{10}$$

Where r_{y2} is the arbitrary radius along the tooth profile, and θ_2 is the angle of tooth space. To calculate this angle the following expression is used:

$$\theta_2 = \frac{e}{2r_2} + inv\alpha - inv\alpha_{y2} \tag{11}$$

where e is the tooth width of space along the pitch circle, r_2 is the pitch radius, α is the standard pressure angle, and α_{y2} is the pressure angle at radius r_{y2}. It can be determined by the following equation:

$$\cos\alpha_{y2} = \frac{r_{b2}}{r_{y2}}. \tag{12}$$

Here r_{b2} is the radius of base circle. In Eq. (11) the *inv* is the involute function, which is interpreted as inv α = tan α - α.

The tooth surface is described by two independent parameters r_{y2} and t_2:

$$\left.\begin{aligned} x_2 &= x_2(r_{y2}), \\ y_2 &= y_2(r_{y2}), \\ z_2 &= z_2(t_2). \end{aligned}\right\} \tag{13}$$

4. ANALYSIS OF GEAR MESHING

The gear couplings having angular misalignment compose a special gear pair with intersecting axes (*Figure 6*).

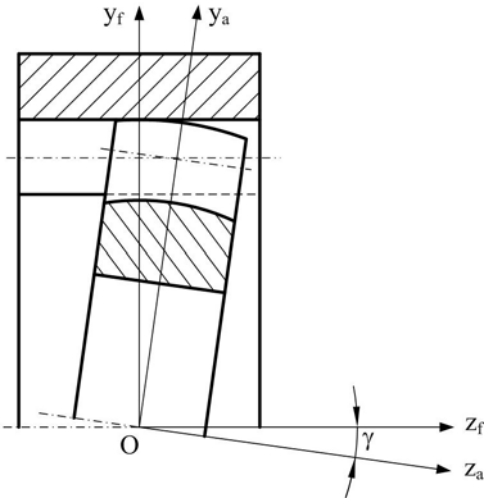


Figure 6. Gear coupling with angular misalignment

The shaft angle of the drive equals the angle of misalignment γ. The number of teeth on the hub equal to the number of teeth on the sleeve. The crowned tooth surface of the hub and the involute spur tooth surface of the sleeve are in point contact. The meshing analysis may be formulated as follows:

- Equations of gear-tooth surfaces and the angle of misalignment are given.
- It is necessary to determine the law of motion, which is the relation between the angles of rotation.

4.1. Coordinate systems

We set up four coordinate systems. S_1 (O, x_1, y_1, z_1) and S_2 (O, x_2, y_2, z_2) are moving coordinate systems and they are rigidly connected with the hub (gear 1) and the sleeve (gear 2) respectively. S_f (O, x_f, y_f, z_f) and S_a (O, x_a, y_a, z_a) are stationary coordinate systems fixed to the frame. S_f is the global system and S_a is an auxiliary ones. If there is no angular misalignment ($\gamma = 0$) S_a coincides with S_f (*Figure 6*).

All of the coordinate systems have a common origin O.

S_1 rotates in S_a around axis z_a which coincides with z_1. The turning angle φ_1 is measured between axes x_a and x_1 (*Figure 7*). When $\varphi_1 = 0$ S_1 coincides with S_a.

Similarly, S_2 rotates in S_f around axis z_f which coincides with z_2. The turning angle φ_2 is measured between axes x_f and x_2 (see *Figure 7*). When $\varphi_2 = 0$ S_2 coincides with S_f.
The relationships of transformation among coordinate systems are the following:

$$\mathbf{r}_a = \mathbf{M}_{a1}\,\mathbf{r}_1, \tag{14}$$

$$\mathbf{r}_f = \mathbf{M}_{f2}\,\mathbf{r}_2, \tag{15}$$

$$\mathbf{r}_f = \mathbf{M}_{fa}\,\mathbf{r}_a, \tag{16}$$

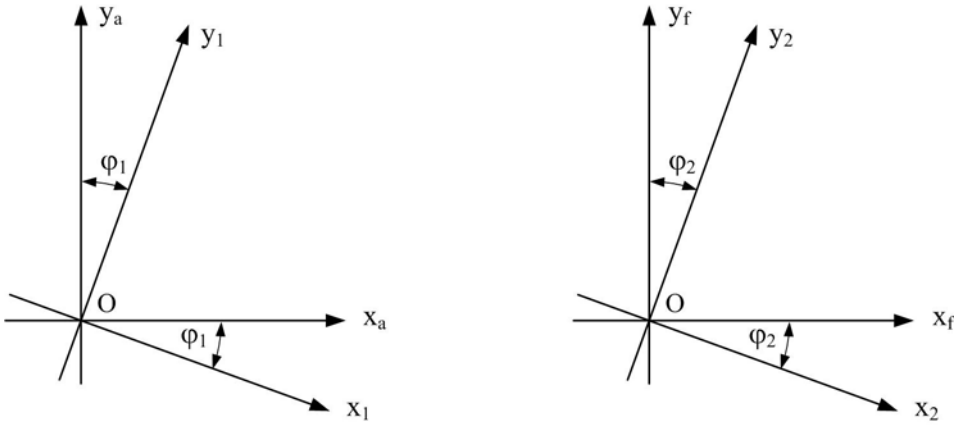


Figure 7. Relation between coordinate systems

where the position vectors in coordinate systems S_1, S_2, S_a and S_f are

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix},\ \mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix},\ \mathbf{r}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}, \text{ and } \mathbf{r}_f = \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} \text{ respectively.}$$

The transition matrices between coordinate systems are

$$\mathbf{M}_{a1} = \begin{bmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

$$\mathbf{M}_{fa} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}, \tag{18}$$

$$\mathbf{M}_{f2} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{19}$$

$\mathbf{M}_{nm}$ means the transformation from S_m system to S_n system.

4.2. Contact points on the tooth surfaces

The contact point between the tooth surfaces of the hub and the sleeve is a point in the coordinate system S_f, at which the position vectors and the surface unit normals coincide with another. Thus

$$\mathbf{r}_f^{(1)}(r_{y1}, t_1, \varphi_1) = \mathbf{r}_f^{(2)}(r_{y2}, t_2, \varphi_2), \tag{20}$$

$$\mathbf{n}_f^{(1)}(r_{y1}, t_1, \varphi_1) = \mathbf{n}_f^{(2)}(r_{y2}, t_2, \varphi_2). \tag{21}$$

Vector equation (20) yields three scalar equations, but equation (21) yields only two independent scalar equations because of the both surface normals are unit vector.

$$\left|\mathbf{n}_f^{(1)}\right| = \left|\mathbf{n}_f^{(2)}\right| = 1. \tag{22}$$

The tooth surfaces of the hub and the sleeve are represented in coordinate systems S_1 and S_2 respectively by the following equations

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} r_{y1}\sin\theta_1 \\ r_{y1}\cos\theta_1 \\ t_1 \end{bmatrix} \tag{23}$$

and

$$\mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} r_{y2}\sin\theta_2 \\ r_{y2}\cos\theta_2 \\ t_2 \end{bmatrix}. \tag{24}$$

The coordinates are known from equations (4) and (10).
Both tooth surfaces rotate about own axis located in the frame. The rotating tooth surfaces are generated in the coordinate system S_f. The locus of these surfaces are determined by the following equations

$$\mathbf{r}_f^{(1)} = \mathbf{M}_{fa}\mathbf{M}_{a1}\mathbf{r}_1 \tag{25}$$

and

$$\mathbf{r}_f^{(2)} = \mathbf{M}_{f2}\mathbf{r}_2. \tag{26}$$

The tooth surface normals in the coordinate system S_f are:

$$\mathbf{n}_f^{(1)} = \mathbf{M}_{fa}\,\mathbf{M}_{a1}\,\mathbf{n}_1\,, \tag{27}$$

$$\mathbf{n}_f^{(2)} = \mathbf{M}_{f2}\,\mathbf{n}_2\,, \tag{28}$$

where the surface unit normals are represented as follows:

$$\mathbf{n}_1 = \frac{\dfrac{\partial \mathbf{r}_1}{\partial r_{y1}} \times \dfrac{\partial \mathbf{r}_1}{\partial t_1}}{\left| \dfrac{\partial \mathbf{r}_1}{\partial r_{y1}} \times \dfrac{\partial \mathbf{r}_1}{\partial t_1} \right|}, \tag{29}$$

$$\mathbf{n}_2 = \frac{\dfrac{\partial \mathbf{r}_2}{\partial r_{y2}} \times \dfrac{\partial \mathbf{r}_2}{\partial t_2}}{\left| \dfrac{\partial \mathbf{r}_2}{\partial r_{y2}} \times \dfrac{\partial \mathbf{r}_2}{\partial t_2} \right|}. \tag{30}$$

To solve equation system (20, 21) contained five nonlinear scalar equations requires any iterative numerical procedure. It should be based on computer aided solution. The solving is very complex and difficult, so that we present a simplified computing method to determine the locus of contact points and the law of motion.

4.3. An approximate calculation for tooth contact analysis

To simplify the procedure of gear meshing analysis the tooth surface on the hub is divided into several nodes by a grid, which contains curves in radial direction and parallel profiles along the tooth face (*Figure 8*). The curves along the tooth profile are obtained as the intersecting curves between some cylinders having different radii and the crowned surface. The curves parallel to the transverse plane are involute profiles.

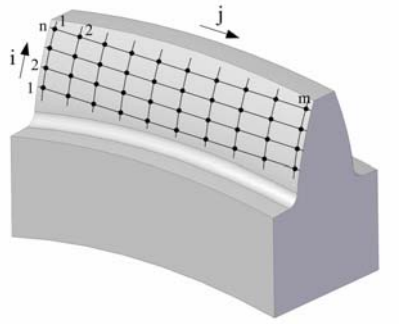


Figure 8. Nodes on the tooth surface of the hub

The number of nodes is n in i direction and m in j direction (see *Figure 8*).

The coordinates of the nodes in the coordinate system S_1 are the following:

$$\left.\begin{aligned} x_{1i,j} &= r_{y1i}\sin\theta_{1i,j}, \\ y_{1i,j} &= r_{y1i}\cos\theta_{1i,j}, \\ z_{1j} &= z_{\min} + j\Delta z, \end{aligned}\right\} \tag{31}$$

where
$$r_{y1i} = r_{\min} + i\Delta r. \tag{32}$$

The nodes on the tooth surface of turning hub in the stationary coordinate system S_a are:

$$\left.\begin{aligned} x_{ai,j} &= r_{y1i}\sin\left(\theta_{1i,j} + \varphi_1\right), \\ y_{ai,j} &= r_{y1i}\cos\left(\theta_{1i,j} + \varphi_1\right), \\ z_{aj} &= z_{1j}. \end{aligned}\right\} \tag{33}$$

When the gear coupling has an angular misalignment γ, the coordinates of nodes in the coordinate system S_f are:

$$\left.\begin{aligned} x_{fi,j} &= x_{ai,j}, \\ y_{fi,j} &= y_{ai,j}\cos\gamma - z_{aj}\sin\gamma, \\ z_{fi,j} &= y_{ai,j}\sin\gamma + z_{aj}\cos\gamma. \end{aligned}\right\} \tag{34}$$

The position of nodes is characterized by the radius r_{y2} and the angle β in global stationary system S_f as follows:

$$r_{y2i,j} = \sqrt{x_{fi,j}^2 + y_{fi,j}^2}\,, \tag{35}$$

$$\beta_{i,j} = \arcsin\frac{x_{fi,j}}{r_{y2i,j}}\,. \tag{36}$$

Contact points on the tooth surface of sleeve are common points with the nodes. These contact points are represented by radii $r_{y2i,j}$, profile angles $\theta_{2i,j}$ and rotation angles $\varphi_{2i,j}$. $\theta_{2i,j}$ may be calculated based on formula (11). Rotation angle is determined by the following expression:

$$\varphi_{2i,j} = \beta_{i,j} - \theta_{2i,j}\,. \tag{37}$$

In this computing algorithm rotation angle φ_1 is the parameter. If $\varphi_1 = \varphi_0$ is given as a starting value the result to φ_2 gives different values at the different nodes. The contact point between tooth surfaces is that node which gives the maximum of φ_2 from the all $(n \cdot m)$ solutions. Smaller value of φ_2 means clearance at the given nodes.

Maximum value of φ_2 and φ_1 give one point for the law of motion. After that φ_1 is stepped by $\Delta\varphi$ and the new maximum of φ_2 is determined. If φ_1 is stepped from 0 to 2π and

maximum of φ_2 is obtained for all φ_1 the law of motion for gear coupling is generated for one tooth-pair (*Figure 9*).

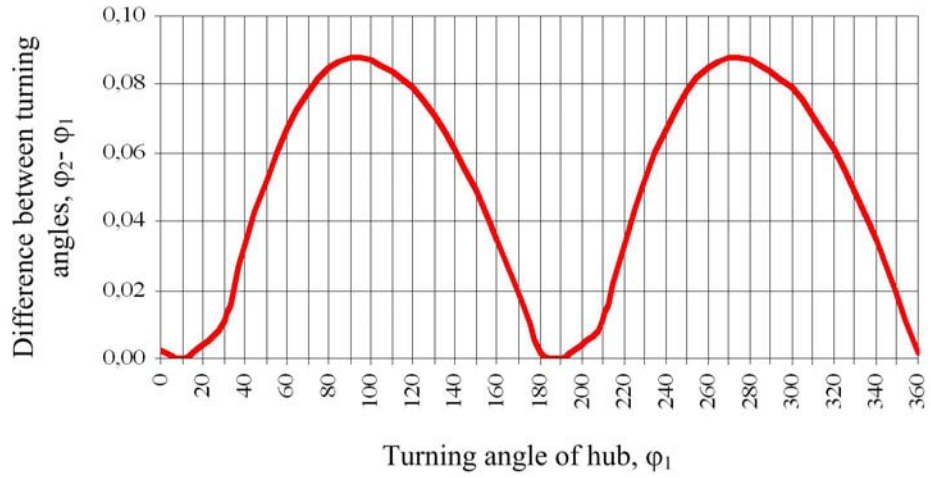


Figure 9. Law of motion in case of one tooth-pair connection

The following figure shows curves from several tooth-pair engagement (Figure 10.).

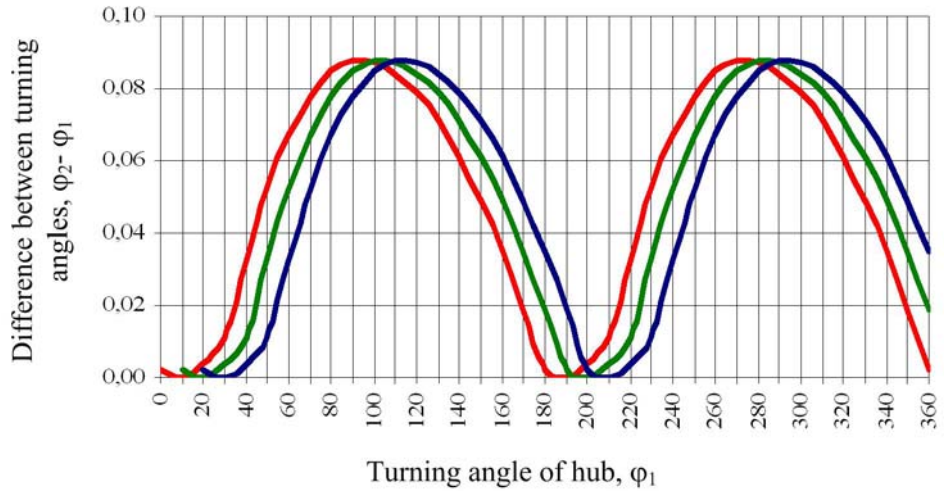


Figure 10. Law of motion in case of more tooth-pair connection

The law of motion for gear coupling contains the upper tips of the curves only, since at intersection points the next tooth-pair receives the driving (*Figure 11*).

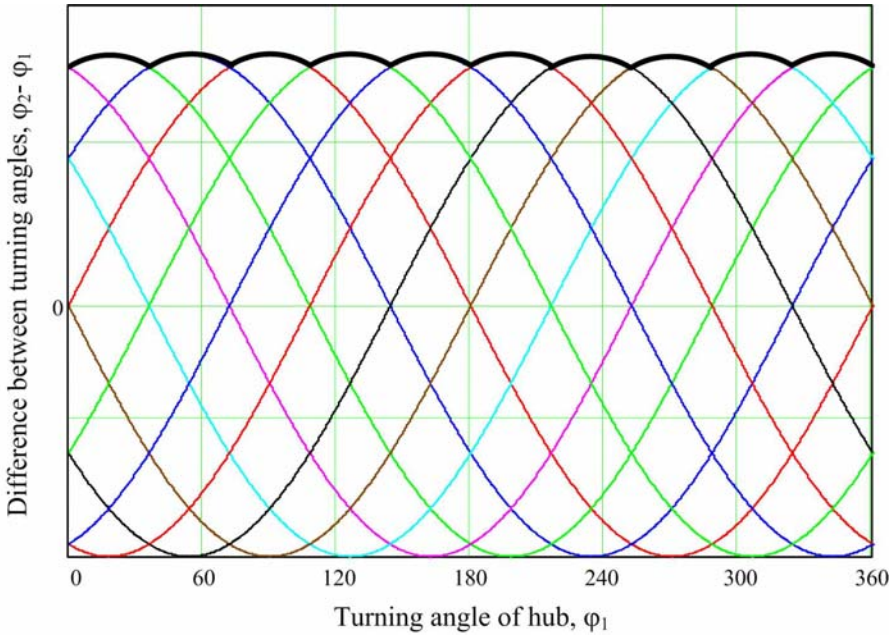


Figure 11. Law of motion for the gear coupling

This calculation method has any approximation, since the common unit normals at contact points of the tooth surfaces are not considered. The real solution is in the neighbourhood of the approximate solution. A consistent grid on the tooth surface of the hub causes smaller difference between the correct and the approximate solution and it gives more precise result.

Acknowledgements

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