Design of Machines and Structures, Vol. 2, No. 2 (2012), pp. 81–92.
https://doi.org/10.32972/dms.2012.016

HELICAL SPRINGS IN EPICYCLIC TRACTION DRIVES

GÉZA NÉMETH–JÓZSEF PÉTER–ÁDÁM DÖBRÖCZÖNI
Department of Machine and Product Design, University of Miskolc
H-3515 Miskolc-Egyetemváros
machng@uni-miskolc.hu, machpj@uni-miskolc.hu, machda@uni-miskolc.hu

Abstract. The paper tries to introduce an epicyclic traction drive containing one or more helical springs as basic element. Our goal is to assure the necessary normal forces between the contacting surfaces, proportionally to the external loads. Some suitable mechanical models were used to obtain the solution of the elasticity problem with the necessary accuracy.

Keywords: helical torsion spring, epicyclic traction drive, strain energy, pressure device, deflection

1. INTRODUCTION

At the Department of Machine and Product Design in the University of Miskolc the epicyclic gear drive took up a great deal of room ever since its foundation. By the leading of Professor Zénó Terplán the researches generated a scientific school in which numerous theses were born. Beside the dimensioning and strength calculations of the epicyclic drives, the selection of drives on the basis of high efficiency, the analysis of load distribution among the planet gears and along the face width the research field was widened by new domains. Practical results were arisen in the scopes of research, design, production and measurement of harmonic gear drives, and also in the design area of coupled epicyclic drives and the continuously variable drives. Both the changed needs and possibilities influenced the newly coming into view of the epicyclic traction drives. The new needs are the high speed, the noiseless operation, the variable transmission ratio, the clamping force proportional to the load, low production costs and the simple design. The new possibilities are the high surface hardness, the precise machining accuracy, the traction lubricants and the high power density. This paper tries to join to the research tradition of our department with an originally developed epicyclic traction drive instead of gear drive.

After collecting some example for the flexible elements on the area of planetary drives and the antifriction bearings, the common operation states of the rolling bearing and the planetary drives is detected. The helical springs should be classified to select the proper type for our purpose. It is installed into an io type (containing one inner and one outer connection) frictional planetary drive. It is investigated how can be assured the necessary clamping force and a mechanical model for the sun gear should be suggested. The creation of the model is followed by the dimensioning and strength calculation of a kinematic drive (low power drive). At last the results are summarized and the direction for further research is charted.

2. FLEXIBLE MACHINE PARTS IN EPICYCLIC DRIVES

Flexible basic parts in epicyclic drives are not unprecedented at all. Let us just think about the harmonic gear drive. The operational interference in inner mesh having small difference in number of teeth of the pair of gears, can be avoided only with flexible, high elastic deformation elements [1]. The originally cylindrical, cup-shaped flexspline and the originally circular rings of flexible wave generator bearing are capable of large elastic deformation, as shown in *Figure 1*. The principle of harmonic drives is applicable for frictional drives, too. However the flexibility of the flexspline does not contribute to the clamping force of the force-locking connection. In the epicyclic traction drive there is an effort made to assure a clamping force proportional to the external load of the drive. Both axial and radial force-closing clutches, self tightening wedge roller are applicable. One of their common disadvantages is the danger of sliding of the rollers when the operating torque is changing, that needs a special care to prevent them. The well known solutions were reviewed in one of our previous papers [2]. In one of the planetary drive patents of the Timken Bearing Company the only planetary wheel is a sufficiently flexible ring [3]. Two functions in one machine element are united in this solution – one of the elements of the frictional pair and the initial tensioning element.

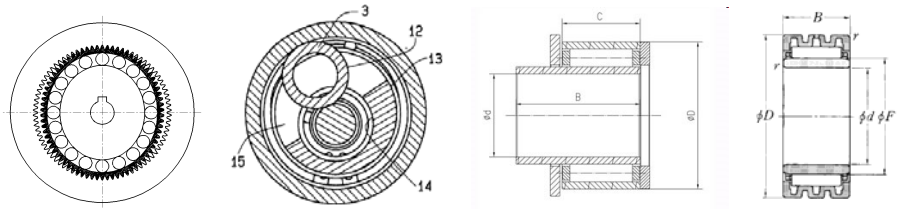


Figure 1. Harmonic gear drive, planetary traction drive [3], spring roller bearing and needle roller bearing with clearance adjustment

The application of flexible elements at the area of the rolling bearings is not unprecedented either. The rollers and/or rings of the spring roller bearing, are helical springs made of steel strip. The radial clearance of some needle roller bearings is reduced to the proper value by compressing axially the outer ring, similar to the Spieth-type clamping sleeve. Hydrodynamic radial slide bearing sleeves inserted in Spieth-type ring are also made by the radial clearance adjustment [4].

The rollers of a roller bearing make planetary motion. When the cage of this bearing is jointed to an output shaft, an io type epicyclic drive is obtained. The bearing also may be a spring roller bearing. *Figure 2* shows the mentioned roller bearing, spring roller bearing and epicyclic drive having the same radial dimensions. The main goal of the application of spring elements in roller bearings is to prevent the bearing against getting stuck, caused by unequal thermal expansion. That of in epicyclic traction drive is to assure the necessary clamping force. By the latter sample we would like to introduce an epicyclic traction drive having high efficiency, using the well known design principle and using basic elements of helical springs.

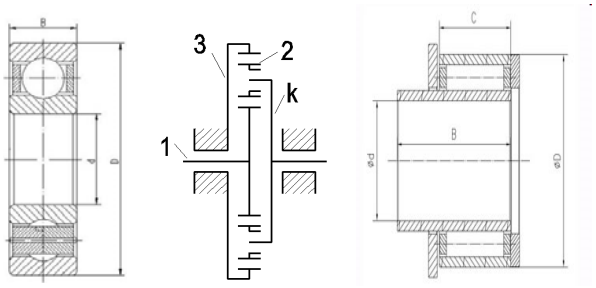


Figure 2. Epicyclic traction drive (io type) as a possible model of roller bearings

3. HELICAL SPRINGS

3.1. Characteristics, loads, production technologies

One type of the mechanically operating springs, made of steel is the cylindrical helical spring, which characteristic is approximately linear. The main load of each cross-section of the working threads of helical compression and helical tension springs, acting the external loads only at their ends, is an approximately constant torque, and that of the helical torsion spring is an approximately constant bending moment. Due to the loads there is a shear stress in the compression and tension springs and a normal stress in the torsion spring. The location of maximum stress is in the sections at the inner diameter. The slightly increment of the diameter of helical compression springs due to load should be mentioned. It requires attention when the compression spring is lead in hollow cylinder to avoid buckling. The helical torsion springs are designed for torque that increases their curvature and decreases their mean diameter. When the torsion spring is lead by a pin to decrease the danger of buckling or because of insufficient end clamping, the decrement in diameter and the friction on the pin should be also considered. The effect of frictional torque to the stiffness, relatively to the frictionless condition is an increase when loading the spring and decrease when unloading the spring. Their manufacturing technologies are forming (winding), machining (turning, milling, wire EDM, laser cutting) or both of them. The residual stresses after machining are much less than after forming, so the ideal characteristics of springs is assured by machined springs. There is a chance to make multiple coil spring which compensate the additional radial loads. The end forms are more versatile, and the load can transfer not only at the ends but anywhere along its length.

3.2. Additional deformations

There are some areas of application where instead of the large deformation caused by the load, the additional deformation is used for our purpose. This principle is used in certain force-locking connections, where an axial load produces the necessary radial increment of the elastic body diameter (ring spring, Belleville spring, or other springs). Helical springs are also applied in non-conventional areas. In the wrap spring clutch the friction torque is

produced by a similar manner than in the rope drive. The decrease in diameter of a helical torsion spring provides the additional deformation.

3.3. The shapes of helical springs

The shape of cross-sections of the springs may be circle, square, rectangle, or other, and the situation of rectangle may be standing or lying. The cross section of spring wire is constant (prismatic bar) or continuously variable. The spring forms a cylinder, cone or other solid of revolution. When one of the rolling elements of an epicyclic traction drive should be substituted by helical torsion spring, the rectangular cross section spring is suggested for our purpose. Not only is the size of cross section variable but the magnitude and sense of lead along the length of spring, and also the size of gap between the coils. The type of spring and production method is chosen to a current task.

4. THE EPICYCLIC TRACTION DRIVE DESIGN

4.1. Locations of helical springs

An io type epicyclic drive contain one inner and one outer connection, the load is distributed between more planet wheel. Any wheel (sun, planet or ring) can be rigid or highly elastic spring. All of the rolling elements of the drive may be rigid wheels and all of them may be helical springs. The number of all the variants is eight considering the two types of all the three rolling elements. *Figure 3* shows the kinematic and velocity diagram of the mentioned epicyclic drive containing sun- (1), planet- (2) and annular wheel (3) and a planet carrier (k). The small lead angle, α and the clockwise angular velocity vector, ω are the two important characteristics of the helical torsion spring having right hand helix, which can substitute the sun wheel. The relative relationship of the sense of rotation and direction of helix is important, because the spring can transmit torque only in one way. The last diagram of *Figure 3* shows a variant where both the rigid sun wheel and rigid annular wheel were substituted by right-hand helix helical torsion springs. The annular wheel is fixed to the frame, the sun wheel joined to the input shaft and the planet carrier to the output shaft.

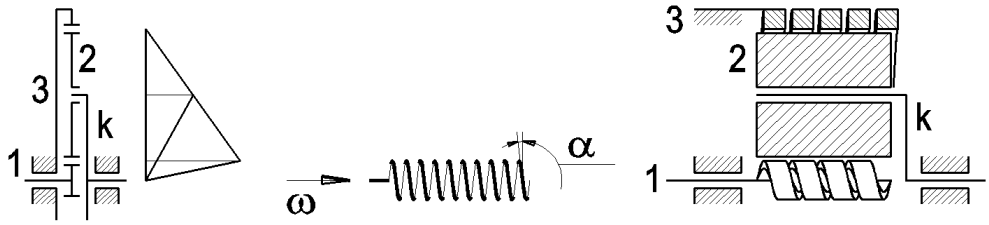


Figure 3. An io type epicyclic drive containing two helical torsion springs as basic elements

4.2. Operation of pressure device

The usual method, to assure the proper clamping force at the friction pairs, is the installation of one or more pressure devices. At the analysed drives where some of the basic elements are substituted by helical torsion spring elements, the detached pressure device is unnecessary. The springs perform the functions of both one element of the friction pair and the pressure device. As it visible in *Figure 3* the wires of springs have rectangular cross-sections and are prismatic, with small gap between the coils. The number of planet wheels is $Q = 3$, e.g. The planet wheels are fixed to the carrier with clearance, the outer diameter of the unloaded sun spring is greater, and that of the annular spring is less, than the current circle diameter determined by the rigid planet wheels. The condition of operation is the initial tensioning of the springs. The greater the external load, the greater the twist angle and the greater the clamping force acting between the mated rolling elements. This is the basis of the proper service of the pressure device. The clamping force is proportional to the external load (to the torque acting at the output shaft).The drive is unidirectional, but the direction of power flow is changeable.

5. THE MECHANICAL MODEL OF THE SUN WHEEL

5.1. Assumptions and simplifications

The previous train of thought and the study of the suggested traction drive design lead to a hopefully sufficiently accurate, usable mechanical model of a helical torsion spring with special loading. The common helical springs are loaded only at their ends, but the load of spring installed into the epicyclic traction drive as a sun gear, is more complicated. The input torque is supplied through one end of the spring, which has a relatively small lead angle, and when this helical spring having the number of coils, n, is in contact with Q piece of planet wheels, the torque is forwarded to the planet wheels through $n \cdot Q$ points. Instead of the other end, the input torque is forwarded along the spring through $n \cdot Q$ points. Another assumption is, that all the contact points are equally loaded, i.e. the radial and tangential reactions are equal to each other. Due to the small lead angle the spring can be modeled by a circular, prismatic beam with rectangular cross-section. The original shape of the spring that can produce an equally loaded circular beam at all the contact point, which are located on the surface of a perfect cylinder, yet is not interesting for us. Due to the small value of traction coefficient, μ_t the tangential components of the contact forces (reactions) are much less than that of the radials.

5.2. Arc-shaped flat beam model with radial and tangential loads

The arc coordinate, s is started at the free end of the beam. The mean radius of the beam is ρ_0. The angular coordinate, φ at point *j* of the beam, is

$$\varphi_j = \frac{s_j}{\rho_0}, \tag{1}$$

and $$(j-1)\frac{2\pi}{Q}+2k\pi \le \varphi_j \le j\frac{2\pi}{Q}+2k\pi\,, \tag{2}$$

where $$k=0,1,2,\ldots,ent(n) \text{ and } j=1,2,\ldots,Q\,. \tag{3}$$

The radial forces and consequently the tangential forces, due to the Coulomb friction law, are supposed to be equal, so

$$F_1=F_2=\ldots=F_j=F \text{ and } F_{t1}=F_{t2}=\ldots=F_{tj}=F_t\,. \tag{4}$$

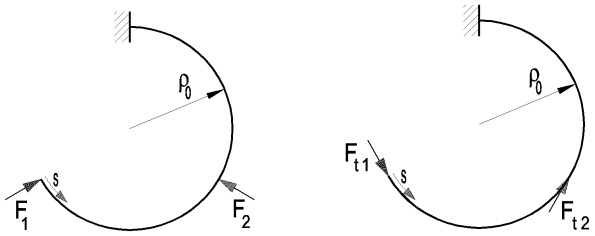


Figure 4. Arc-shaped flat beam model with radial and tangential loads

The normal load *N*, shear load *S* and the bending moment M_b along the beam, when radial loads F_j are acting, is changing as

$$N_j=-F\sum_{i=1}^{j}\sin\left[\varphi-(i-1)\frac{2\pi}{Q}\right], \tag{5}$$

$$S_j=F\sum_{i=1}^{j}\cos\left[\varphi-(i-1)\frac{2\pi}{Q}\right], \tag{6}$$

and $$M_{bj}=\rho_0 F\sum_{i=1}^{j}\sin\left[\varphi-(i-1)\frac{2\pi}{Q}\right], \tag{7}$$

respectively. The normal load, *N*, shear load, *S* and the bending moment M_b along the beam, when tangential loads, F_{tj} are acting, is changing as

$$N_j=-F_t\sum_{i=1}^{j}\cos\left[\varphi-(i-1)\frac{2\pi}{Q}\right], \tag{8}$$

$$S_j = -F_t \sum_{i=1}^{j} \sin\left[\varphi - (i-1)\frac{2\pi}{Q}\right], \tag{9}$$

and

$$M_{bj} = -\rho_0 F_t \left\{ kQ + \sum_{i=1}^{j} \left[i - \cos\left(\varphi - (i-1)\frac{2\pi}{Q} \right) \right] \right\}, \tag{10}$$

respectively. Due to the radial loads, along the neutral axes of the beam the loads are periodically changing, and due to the tangential loads only the normal and shear loads are periodical, the bending moment is monotonically increasing function, as *Figure 5* shows.

The diagrams consider a helical spring with n = 2.75 working coils contacting with Q = 3 piece of planet wheels.

5.3. Slope of the circular beam

The radially acting forces along the circular beam increase the curvature. At the free end of the beam ($\varphi = 0°$), when the number of planet wheel is Q = 3, the slope is the maximum, i.e.

$$\delta'_{M_0 1} = \frac{\rho_0^2 F}{I_{red} E} 3n\,, \tag{11}$$

that was obtained by Castigliano's theorem. The area moment of inertia, I_{red}, the Young module, *E* and the number of working threads of spring, *n* is known. The increment of slope is approximately linear along the beam. At any discrete point it is

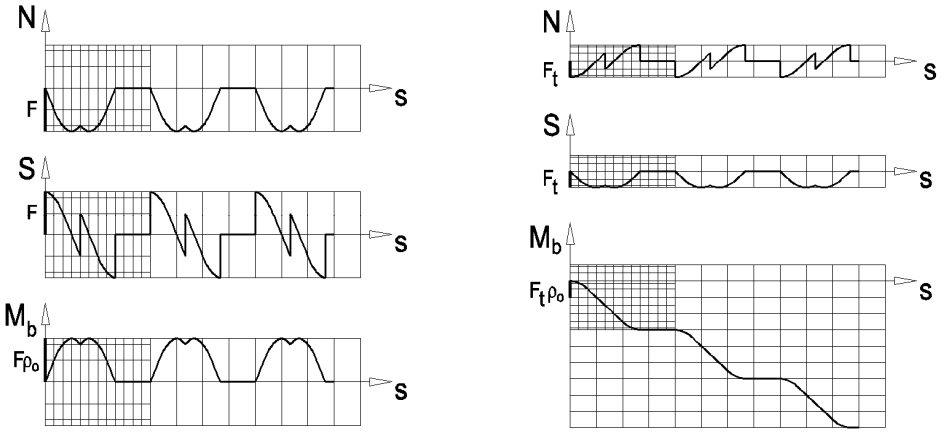


Figure 5. The N(s), S(s) and $M_b(s)$ functions, due to the radial and tangential loads, respectively (Q = 3, n = 2.75)

$$\delta'_{M_0}(\varphi) = \delta'_{M_0 1}\left(1 - \frac{1}{2\pi n}\varphi\right), \tag{12}$$

where φ is in radian. In an instant the discrete points are expressed by formula (2). The tangential forces are the function of the radial ones. When traction lubricant is used in the drive, a possible value of the traction coefficient, $\mu_t = 0.06$. The tangential forces along the circular beam decrease the curvature. The maximum slope is observed at the free end of the beam, that is

$$\delta''_{M_0 1} = -\frac{\rho_0^2 F_t}{I_{red} E} n(3n+1)\pi \,. \tag{13}$$

The slope function along the beam is approximately parabolic. Its value at any discrete point is

$$\delta''_{M_0}(\varphi) = \delta''_{M_0 1}\left(1 - \frac{1}{(2\pi n)^2}\varphi^2\right). \tag{14}$$

The resulting slope, using the superposition principle, is obtained of the sum of the previous slopes, i.e.

$$\delta_{M_0}(\varphi) = \delta'_{M_0}(\varphi) + \delta''_{M_0}(\varphi),$$

$$\delta_{M_0}(\varphi) = \frac{\rho_0^2 F}{I_{red} E}\left\{\mu_t \frac{3n+1}{4\pi n}\varphi^2 - \frac{3}{2\pi}\varphi + n\left[3 - \mu_t \pi(3n+1)\right]\right\}. \tag{15}$$

This resulting slope function has an extreme value at

$$\varphi = \frac{1}{\mu_t}\frac{3n}{3n+1}. \tag{16}$$

The slope per each spring threads are the difference of the slopes at the ends of the stages, so

$$\Delta\delta_{M_0}(k2\pi) = \delta_{M_0}(k2\pi) - \delta_{M_0}((k-1)2\pi), \quad where \;\; k = 1, 2, \ldots, n$$

$$\Delta\delta_{M_0}(k2\pi) = \frac{\rho_0^2 F}{I_{red} E}\left[\mu_t \frac{3n+1}{n}\pi(2k-1) - 3\right]. \tag{17}$$

The resulting slope among the threads is changing linearly. This function is negative everywhere. It means that all of the cross-section of the beam turns outwards. However in the case of two adjacent threads, between the points, are at $\Delta s = 2\pi\rho_0$ distance, may occur increase in curvature. This takes place just at the free end of the beam. The change in

the radius of the neutral layer of the circular beam influenced by the loads (F_j , F_{tj}) is negative only until the location of the extreme value (16), i.e.

$$\Delta\rho_{0k} = \rho_0\left(1 - \frac{2\pi}{2\pi + \Delta\delta_{M_0 k}}\right) \quad , \tag{18}$$

where $k = 1, 2, \ldots, n$ and $\Delta\delta_{M_0 k} = \Delta\delta_{M_0}(k2\pi)$.

The constraints of $F_j = const.$ and $F_{tj} = const.$ loads can be fulfilled if only the outer diameter of the helical spring is changed. The radius of the unloaded circular beam should be increased at the free end, and at the other end, where the input torque is supplied, the diameter should be decreased as shown in the *Figure 6*. In the case of Q = 3 and n = 10, the extreme value is at $\varphi = 16.13\ rad \approx 2.5 \cdot (2\pi)$, due to the formula (16).

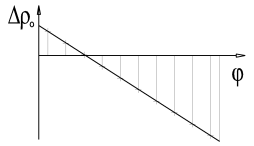


Figure 6. The change of radius of the neutral layer along the whole circular beam

6. LOAD CARRYING CAPACITY OF A TINY DRIVE

6.1. The allowable radial and tangential loads

Simple structure low noise traction drives are frequently used at the area of low power electric motor to modify their speed greatly. Some extremity prosthetics contain io type epicyclic traction drives which both diameter and length are less than 10 mm. Considering the increasing need of the mechatronics, the sample problem introduces a small power drive. Current paper deals with the design questions of sun wheel. The outer diameter of the wheel, that is now a helical torsion spring, is 5 mm. The cross section of the spring steel wire is rectangular, $a \times b$, as *Figure 7* shows. Considering the cross section and the material, the allowable strength, σ_{all} of the spring is selected. The dimensions and material properties are summarized in *Table 1*. The number of working threads, n and each thread is in contact with Q number of planet wheel. The number of contact points is $Q \cdot n$, and at each contact points there are radial and tangential forces acting. Both the radial and tangential forces are supposed to be equal to each other. The location of the maximum stress is at the vicinity of the torque supply (at the end of the n-th thread), at the inner layer of the beam, at distance η of the neutral layer. The beam is considered to be strongly curved, because the quotient of the mean circle radius, ρ_0 and the distance of extreme layer, $e_{\max}$ is equal to 4.

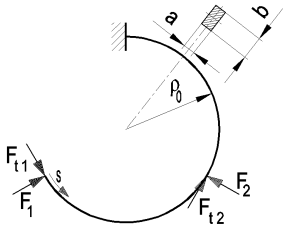


Figure 7. Geometry of the loaded circular beam

The influence of the shear compared to the bending is negligible. Knowing the allowable normal stress, the maximum allowable bending moment,

$$M_{b\max} \approx \frac{\sigma_{all}}{\frac{1}{\rho_0 ab} + \frac{1}{I_{red}} \frac{\rho_0 \eta}{\rho_0 + \eta}} = \ldots = -76 Nmm \,, \tag{19}$$

computed by the well known Grashof formula. Knowing the bending moment, the radial forces may be estimated as

$$F_{\max} = \frac{-M_{h\max}}{\mu_t \rho_0 n Q} = \ldots = 21N \,. \tag{20}$$

Table 1

Geometry and material properties of the sum wheel

$Q = 3 \;\; n = 10 \;\; \mu_t = 0{,}06 \;\; \sigma_{all} = 1100 MPa \;\; \eta = -\frac{b}{2}$	
$F_1, F_{2,}, \ldots, F_{(Qn)}$	$E = 2{,}1 \cdot 10^5 \, MPa$
$F_{t1}, F_{t2,}, \ldots, F_{t(Qn)}$	$\nu = 0{,}3$
$\rho_0 = 2 \, mm$ $\quad a = 0{,}5 \, mm$	$b = 1 \, mm$
$e_{\max} = \frac{b}{2}$	$\frac{\rho_0}{e_{\max}} = \ldots = 4$
$I_{red} = a\rho_0^3 \left(\ln \frac{2\rho_0 + b}{2\rho_0 - b} - \frac{b}{\rho_0} \right) = \ldots = 0{,}0433 mm^4$	

Let the values of radial forces, $F = 20N$ and that of the tangential forces, $F_t = \mu_t F = 1{,}2N$. The stress distribution in the beam is very unequal.

6.2. The constraints of the proper operation

According to the theoretical model, when constant values of F and F_t forces are acting along the beam, the shape (the meridian curve) of the helical spring at the free end is less than the original, and at the input shaft it is greater than the original, as it derived from the formula (18). Vice versa, the outside diameter of the spring should be modified to obtain the constraints of the original problem. The radial deflection of beam is very small, it is changing between -6.8 μm and +22.4 μm. Let suppose, that in case of so small deflection, relatively to the mean radius, ρ_0 of the beam, the forces generate approximately the same value of deflections, when the unloaded condition deviates of the perfect cylinder, as *Figure 6* shows, and the loaded shape will be a perfect cylinder. The original cylindrical shape of the spring should be modified by grinding, e.g., to get an unloaded spring having 0.0068mm greater radius at the free end and 0.0224mm less at the torque input. The proper operation requires some initial tensioning, that is why the theoretical generatrix of the cone should be shifted farther of the axis of the cone. To develop the accurate deflection in the sun spring, the bearings of planet wheels should be installed into the planet carrier with high accuracy.

6.3. Spring characteristics, assembly of the sun wheel

The maximum input torque can be estimated by

$$T_{\max} = \frac{K\sigma_{all}}{k_0} \approx M_{b\max} = 75Nmm \tag{21}$$

The maximum distorsion of the helical spring, at the maximum torque is

$$\varphi_{\max} = \frac{2\pi\rho_0 n}{I_{red}E} T_{\max} = \ldots = 1{,}07rad = 61{,}3° . \tag{22}$$

The decrement of the diameter according to the previous distorsion is

$$\Delta d = 2\Delta\rho_0 = 2\left(\rho_0 - \rho_0^{'}\right) = 2\rho_0 \left[1 - \frac{n}{\left(n + \frac{\varphi_{\max}}{2\pi} \right)} \right] = \ldots = 0{,}067mm , \tag{23}$$

and this provide the chance of a convenient assembly process.

6.4. The contact stresses

Knowing the radius and the material properties of the contacting sun wheel (spring) and the planet wheel, and using the symbols of *Figure 3*, the Hertzian contact stress can be computer by the following process.

$$E^* = \frac{E}{2\left(1-\nu^2\right)} = \ldots = 1{,}132 \cdot 10^5 MPa\,,$$

$$r_1 = 2{,}5mm \qquad r_2 = 8{,}5mm \qquad R = \frac{1}{r_1} + \frac{1}{r_2} = \ldots = 1{,}93mm\,,$$

$$p_0 = \sqrt{\frac{FE^*}{a\pi R}} = \sqrt{\frac{20N \cdot 1{,}132 \cdot 10^5 MPa}{0{,}5mm \cdot \pi \cdot 1{,}93mm}} = 864MPa\,. \tag{24}$$

The obtained Hertzian stress is allowable in case of usual spring steels.

7. SUMMARY

In the area of epicyclic traction drives an innovative approach was introduced. The clamping force required for the operation was ensured by the flexible design of the traction wheels. In this solution detached pressure device is no longer required. The static model of the sun wheel formed of helical torsion spring was created. The unloaded shape of helical spring, capable for assuring the uniform load distribution along the spring, was calculated. The maximum allowable torque and the maximum angle of twist during the assembly process were calculated. The checking of Hertzian contact stress was also made. Substitution of the annular wheel by helical spring, too, helps us to assure the clamping force proportional to the external load. The possibility of the self aligning of planet wheels hopefully helps the more uniform load distribution. The most sensitive element of the introduced tracton drive is the sun wheel that can transmit approximately 75 Nmm torque which is the input torque of the drive.

Acknowledgement

"This research was carried out as part of the TAMOP-4.2.1.B-10/2/KONV-2010-0001 project with support by the European Union, co-financed by the European Social Fund."

References

[1] J. Péter: *Fogazott hullámhajtómű kapcsolódásának vizsgálata*. Egyetemi doktori értekezés, Miskolci Egyetem, 1981, 174 p.
[2] G. Németh– Á. Döbröczöni: *Dörzsbolygóhajtások áttekintése*. GÉP, LXII. évf., 11. szám, 2011. pp. 18–21.
[3] Ai Xiaolan: *Eccentric planetary traction drive transmission with a single planetary roller*. Panent no.: US 6702704 B2, Assignee: The Timken Company, Canton, OH (US), Date of Patent: 9th March, 2004.
[4] SPIETH Series GLM Radial Slide Bearing, Catalogue no. SN 00.31 e 0409/0000/0409, SPIETH-Maschinenelemente GmbH & Co KG Alleenstrasse 41 73730 Esslingen, p. 20.
[5] Z. Terplán– G. Nagy– I. Herczeg: *Különleges tengelykapcsolók.* Műszaki Könyvkiadó, Budapest, 1971. p. 680, 769.