

## **DYNAMICAL INVESTIGATION OF A SUPERFINISHING DEVICE**

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**Abstract.** This article concerns some dynamical aspects of the design process of a vibrational superfinishing device. As the first approximation, a linear dynamical model is applied, which is then replaced by an improved nonlinear one. The Coulomb-type damping, which is supposed to be the model of the superfinishing process, is considered to be the nonlinearity. The approximation solutions of the nonlinear governing equations are established by the method of linearization above the phase curve, thus the amplitude-frequency expressions and the expressions of the power characteristics in function of the system parameters can be set up. The optimal operation conditions have been established by revealing the extremums of these expressions. The results obtained analytically, have been proven and supported numerically.

**Keywords:** superfinishing, dynamical, nonlinear vibration, power characteristics

### **1. INTRODUCTION**

Superfinishing is applied for fine manufacturing of cylindrical surfaces such as pistons, valves, raceways of rolling bearings etc. Such manufacturing process is generally performed by accessory devices adapted either by lathes or, in some cases, grinding machines as base machines. The smoothness of the manufactured surface is achieved as a superposition of a turning and an alternating motion produced by the basic machine and the superfinishing device. Since the superfinishing process requires vibration- and thermal-proof manufacturing environment, several dynamical problems, those are emerged during even the development process of the device, should be considered. Due to the capability of adaptation of such devices, their overall dimensions and weight should also be considered carefully. These features are determined by the power characteristics of the device, which can be evaluated also by the means of dynamic analysis. The analysis can be performed on the basis of different types of models. Usually the linear model, which is the simplest, can already lead us to recognize some important phenomenon, whilst phenomenon of much wider range can be revealed by the application of nonlinear models.

### **2. INDUSTRIAL BACKGROUNDS**

The Department of Machine Tools, University of Miskolc, as one of the members of a consortium established within an EU6 frame project, has participated in a project during the recent years to develop a superfinishing device which is capable of being mounted on a CNC turning machine of high precision for performing a combined superfinishing proce-

dure. To be able to perform such a combined superfinishing procedure (turning and finishing in one clamping), a device has been needed to construct, that can be integrated on a certain CNC turning machine. The task of the University of Miskolc was to develop the prototype device mentioned above, and its realization. Besides the University of Miskolc, the other members of the consortium were the Hembrug Machine Tool Company, Haarlem, The Netherlands, who was providing the CNC turning machine, the IPT, Fraunhofer Institute, Aachen, Germany, who was performing the manufacturing test series, the Cerobear and HWG bearing manufacturing companies from Germany, and the Diasfin Company from Romania providing the superfinishing stones. On the closing conference of the project, the industrial participants of the consortium raised the feasibility of a superfinishing device operated by a different principle, where the overall dimensions and weight of the new device is determined in such a way, that it could be adopted by a base machine without its reconstruction, and without blocking the displacement of the turret along the strokes. Due to some preconceptions, we think that if the power characteristics were to be increased and the revealed optimal superfinishing parameters were to be considered, a new device meeting the market requirements could be constructed, that works among significantly more efficient conditions, and thus it is much smaller and weighs less than the prototype device.

### **3. LITERATURE REVIEW**

The power characteristics are revealed by the application of different mechanical models by the means of dynamic analysis, where the methods of the nonlinear vibrations and the mathematical methods concerning the approximation solutions of nonlinear motion-equations are applied [1, 3, 5, 6]. The approximation solutions of the nonlinear governing equations are achieved by the method of linearization above the phase-curve [7–8]. The results of the analytical solutions are compared to those of some numerical methods provided by the MAPLE12 and MATLAB/SIMULINK 6.5 softwares [2, 4].

### **4. THEORETICAL PRINCIPLES**

Let the prototype superfinishing device, constructed and manufactured by the University of Miskolc, be assumed as a damped oscillating system, which is stimulated by a linear motor unit. The stimulation can be set up within a wide range of frequency and amplitude, so that the optimal superfinishing parameters could be established. The different combinations of superfinishing amplitude-frequency values will require different exciting force amplitudes and, of course, power. The higher the applied frequency and amplitude, the higher force amplitude and more power are required. However, if an appropriately tuned spring was connected to the oscillating parts of the device, thereby the device is tuned the appropriate manufacturing frequency, increased power characteristics could be obtained. The establishment of the optimum superfinishing parameters reduced the applied frequency and amplitude range to a significant extent, enabling the application of such a “well-tuned” spring mentioned above. Linear and non-linear models are assigned to the device, which enable us to explore the power characteristics of the device, and to establish the optimum superfinishing frequencies.

## 5. THE LINEAR DYNAMIC MODEL

The model of an electro-dynamical vibrator serves as the mechanical model of the devices under investigation *Figure 1*, where  $m$ ,  $k$  and  $\kappa$  represent the mass of the oscillating parts, the spring stiffness and the coefficient of the linear damping which comes from the superfinishing process. As regards the electric parts,  $R_1$ ,  $R_2$  and  $L_1$ ,  $L_2$  represent the ohmic resistances and the self-inductances of the circuits 1 and 2, whilst  $M_{12}$  the flux-linkage. Symbols of  $U_1$  and  $u_2$  refer to the stimulating voltages of the circuits, and  $x_2$  represents the displacement of the moving circuit 2.

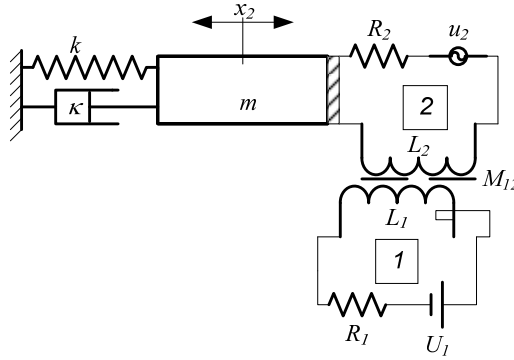


Figure 1. The linear electro-dynamical model

As usual in mechanical engineering practice, first the linear model with the linear governing equations evolved from the Lagrangian-equations of the second kind was established (1), where it was assumed that the maximum displacement of the moving body is considerably less than the longitudinal dimensions of the coils and the current state of the fixed coil as a separate exciter is not influenced by the moving part. The governing equation system takes the form of

$$m\ddot{x} + \kappa\dot{x} + kx - \alpha i = 0$$

$$L \frac{di}{dt} + Ri + \Gamma \dot{x} = U_0 \cos \Omega t \quad (1)$$

where now  $U_0$  and  $R$  refer only to the circuit 2. The expressions of the power characteristics, which come from the solutions of (1), and are established in function of the stimulating frequencies, display clearly that the optimum power characteristics are realized when the stimulating frequency is equal to the natural frequency of the device. The following diagrams display the extremums of some of the power characteristics ( $\cos \psi_i$  the power-factor, and  $E_{be}$  the electric energy consumption), which refer to the optimum power condi-

tions (Figure 2), where  $f$  is the stimulating frequency,  $\lambda$  is the dimensionless frequency and, when  $\lambda = 1$ , the system is stimulated at its natural frequency.

It is pointed out numerically, that – whilst the same manufacturing conditions ( $A$  amplitude of the finishing motion,  $\Omega$  angular frequency of finishing; Table 1) and system parameters are assumed, and then the power characteristics ( $i_0$  current amplitude,  $F_0$  amplitude of the pulling force,  $P_m$ ,  $P_{eff}$ ,  $P_s$  voltless, effective and apparent power etc) of both devices are compared to each other – the new device, which is stimulated at its natural frequency, operates among far more efficient power conditions than the prototype (Table 1).

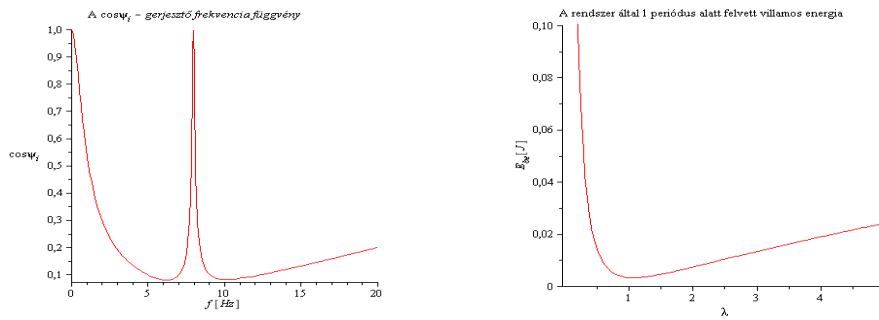


Figure 2. The extremums referring to the optimum power conditions

Table 1

The power characteristics of the prototype and the tuned device

| Power characteristics |      | The values of the power characteristics                                  |                                                                                 |
|-----------------------|------|--------------------------------------------------------------------------|---------------------------------------------------------------------------------|
|                       |      | Prototype device ( $k = 0$ ,<br>$\Omega = \Omega_0 = 50 \frac{rad}{s}$ ) | New device ( $k \neq 0$ ,<br>tuned to its natural<br>frequency: $\lambda = 1$ ) |
| $\sin \varphi_i$      |      | 0,994                                                                    | 0,000366                                                                        |
| $\cos \varphi_i$      |      | 0,104                                                                    | 0,999999                                                                        |
| $A$                   | [mm] | 5                                                                        | 5                                                                               |
| $i_0$                 | [A]  | 0,253                                                                    | 0,005                                                                           |
| $F_0$                 | [N]  | 6,200                                                                    | 0,122                                                                           |
| $P_m$                 | [VA] | 0,755                                                                    | 0,000056                                                                        |
| $P_h$                 | [W]  | 0,079                                                                    | 0,015                                                                           |
| $P_s$                 | [VA] | 0,759                                                                    | 0,015                                                                           |
| $\Upsilon_R$          |      | 0,806                                                                    | 0,001663                                                                        |
| $\Upsilon_{Rh}$       |      | 4,166                                                                    | 0,001665                                                                        |
| $E_R$                 | [J]  | 0,008                                                                    | 0,000003254                                                                     |
| $E_{be}$              | [J]  | 0,010                                                                    | 0,001957                                                                        |

A similar dynamical analysis has been performed on the basis of a possible nonlinear model of the new, tuned superfinishing device, which is going to be discussed below.

## 6. THE NON-LINEAR DYNAMIC MODEL

A better model of the superfinishing process can be established with the application of a nonlinear damping characteristic such as, for example, the Coulomb-type dry friction, whose expression can be considered with the form of  $S = -F_s \operatorname{sgn}(\dot{x})$ , where  $F_s$  is the absolute value of the  $S$  kinetic friction force, and  $\operatorname{sgn}$  is for the *signum* function. It is noted, that there is no difference distinguished between the static and kinetic friction forces during the analysis.

The nonlinear governing equations read

$$\begin{aligned} m\ddot{x} + F_s \operatorname{sgn}(\dot{x}) + kx - \alpha i &= 0 \\ L \frac{di}{dt} + Ri + \Gamma \dot{x} &= U_0 \cos \Omega t \end{aligned} \quad (2)$$

where the other symbols are already defined. In order to see the influence of the system parameters on the power conditions of the model, the solutions as the continuous functions of the parameters would be of interest. Thus the approximation solutions of the differential-equations (2) are established by the method of linearization above the phase curve [7][8]. Periodic solutions of (2) should be assumed, so that the linearization method could be applied. As a result of the linearization, the linear governing equation system reads

$$\begin{aligned} m\ddot{x} + \frac{4F_s}{\pi a_1 \Omega} \operatorname{sgn}(\dot{x}) + kx - \alpha i &= 0 \\ L \frac{di}{dt} + Ri + \Gamma \dot{x} &= U_0 \cos \Omega t \end{aligned} \quad (3)$$

where  $a_1$  is the unknown displacement amplitude. When the method of linearization above the phase-curve is applied, the solutions are assumed to take the forms of  $x(t) = a_1 \cos(\Omega t + \varphi_A)$  and  $i(t) = i_0 \cos(\Omega t + \varphi_i)$ , where  $i(t)$  is the expression of the current-function along circuit 2 and  $i_0$  is the unknown current-amplitude.  $\varphi_A$  and  $\varphi_i$  are the unknown angels of phases to the stimulating voltage. If  $x(t)$  and  $i(t)$  is replaced in (3) and the algebraic transformation of the expressions is carried out, then the analytical expressions of  $a_1$ ,  $i_0$ ,  $\varphi_A$  and  $\varphi_i$  as the functions of the stimulating angular frequency and the system parameters are obtained.

Introducing the  $\frac{k}{m} = \nu^2$ ,  $\frac{\Omega}{\nu} = \eta$ ,  $\eta^2 = \xi$  new variables, the expression of

$$a_1^\pm(\eta) = -\frac{\frac{8F_s}{\pi} \alpha R \nu \eta}{2\left[\left[mR\nu^2(1-\eta^2)\right]^2 + \nu^2\eta^2\left[mL\nu^2(1-\eta^2) + \alpha\right]^2\right]} \pm \sqrt{\frac{\left(\frac{8F_s}{\pi} \alpha R \nu \eta\right)^2 - 4\left[\left[mR\nu^2(1-\eta^2)\right]^2 + \nu^2\eta^2\left[mL\nu^2(1-\eta^2) + \alpha\right]^2\right]\left\{\left(\frac{4F_s}{\pi}\right)^2(R^2 + L^2\nu^2\eta^2) - (\alpha U_o)^2\right\}}{2\left[\left[mR\nu^2(1-\eta^2)\right]^2 + \nu^2\eta^2\left[mL\nu^2(1-\eta^2) + \alpha\right]^2\right}}} \quad (4)$$

is obtained, where  $a_1^+$  and  $a_1^-$  refer to the curves identified by the positive or negative sign in front of the expression of square root, and  $\nu$  is the natural frequency of the system, and  $\eta$  is a dimensionless angular frequency. Since the exact solutions of (3) are obtained as the continuous functions of the system parameters, so their influence on the power conditions can be tracked down clearly and more or less exactly.

As the second result of the approximation method, two curves have been obtained for the displacement amplitude, so stability analysis had to be performed to decide whether which curve real numerical values belong to. When the real amplitude-curve is known, it is pointed out, that stimulating the nonlinear model at its natural frequency, whilst some combinations of parameters are fulfilled, it operates among optimum power conditions. The results obtained analytically, have been checked numerically.

## 7. THE POWER CONDITIONS

After the stable one of the curves (4) had been concluded, the expressions of the power characteristics can be established as the functions as the stimulating dimensionless frequencies. It was pointed out, that if the relations of

$$(\rho^2 - 2\gamma)^2 - 3(\gamma^2 - 2\rho^2) > 0, \quad \rho = \frac{R}{L\nu} \gg 1, \quad r = \frac{F_{sH}}{F_s} \gg 1, \quad \rho = \frac{R}{L\nu} \gg 1$$

were fulfilled at one time and the system is stimulated at its natural frequency, then the device would operate among optimal power conditions. The analytical results have been checked by numerical investigations. The following diagram displays the extremums of some of the power characteristics ( $E_{be}$  the total electric energy consumption of the system,  $E_R$  the electric energy consumption of the ohmic resistance,  $E_h$  the electric energy consumption of the Coulomb-damping, which is considered to be equivalent to the finishing process), which refer to the optimum power conditions (Figure 3).  $\eta$  is the dimensionless frequency.

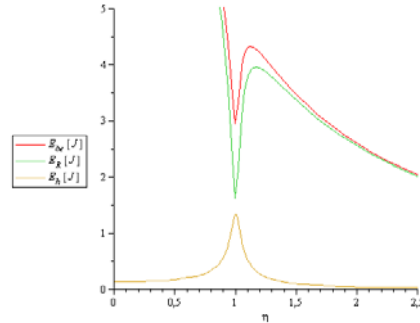


Figure 3. The extremums of some power characteristics at resonance frequency

The following table contains the numerical values of each power characteristic by which the power conditions of the nonlinear models of the prototype and the new devices are compared to each other, whilst the manufacturing parameters are assumed to be the same for both models (Table 2). It can be seen clearly, that adding an appropriate flexible element to the prototype device, the optimum manufacturing parameters can be achieved by less pulling force ( $F_0$ ) and non-significant wattles power ( $P_m$ ), whilst the electric energy ( $E_{be}$ ) and the current consumption ( $i_0$ ) is significantly reduced comparing to those of the prototype device.

Table 2

The power conditions of the prototype and the properly tuned devices based on nonlinear models

| Power characteristics | Prototype device ( $k = 0$ ) | New device ( $k \neq 0$ , tuned to its natural frequency: $\eta = 1$ ) |
|-----------------------|------------------------------|------------------------------------------------------------------------|
| $\sin \varphi_i$      | 0,63                         | 0,083                                                                  |
| $\cos \varphi_i$      | 0,77                         | 0,996                                                                  |
| $A$ [mm]              | 1                            | 1                                                                      |
| $i_0$ [A]             | 23,19                        | 3,74                                                                   |
| $F_0$ [N]             | 728,16                       | 76,40                                                                  |
| $P_m$ [VA]            | 605,04                       | 4,62                                                                   |
| $P_h$ [W]             | 733,93                       | 54,95                                                                  |
| $P$ [VA]              | 951,17                       | 55,15                                                                  |
| $Y_R$                 | 0,99                         | 0,86                                                                   |
| $Y_{Rh}$              | 98,98                        | 6,57                                                                   |
| $E_R$ [J]             | 24,21                        | 1,59                                                                   |
| $E_{be}$ [J]          | 24,46                        | 1,83                                                                   |

These optimum power conditions make it possible for us to construct such a linear motor driven device, whose overall dimensions and weigh is much more favourable than those of

the prototype device, and thus can be adopted on a certain ultraprecise hard-turning machine without significant reconstructions and producing thermal and vibrational disturbances which would have some effects on the work-room of the base-machine.

## 8. SUMMARY

This article is a brief summary of the results of some dynamic and power calculations emerging in the development process of a superfinishing prototype-device. The subject of the investigation is a 1-phase electromechanical superfinishing device operated by electricity and performing an oscillating movement of short-stroke. Linear and non-linear models are assigned to the device, which enables us to explore the power characteristics of the device and to establish the optimum superfinishing frequencies. The positive effect of the appropriate flexible element was demonstrated, i.e. considering constant manufacturing parameters, and exciting the non-linear system at its natural frequency, it becomes possible to construct an improved device of a significantly smaller size and of increased power consumption efficiency than the prototype.

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