Design of Machines and Structures, Vol.2 2, No. 2 (2012), pp. 137–147.
https://doi.org/10.32972/dms.2012.021

DETERMINATION OF BACKLASH FOR GEAR DYNAMIC ANALYSIS

RENÁTA SZŰCS–LÁSZLÓ KAMONDI
Department of Machine and Product Design, University of Miskolc
H-3515, Miskolc-Egyetemváros
szucs.renata@citromail.hu; machkl@uni-miskolc.hu

Abstract. In this paper our aim is to determine the backlash introduced to the mechanical model of gears. On one hand an overview of the relevant standards is presented and on the other hand definition of backlash in the motion of equation is also demonstrated as well as its relationship with other important gear characteristics namely with meshing stiffness and tooth thickness. Backlash is a very significant parameter with great influence on the dynamic behaviour of gears.

1. INTRODUCTION

Gears are one of the most frequently used power transmission machine elements. As a consequence of combination of the increased demands claimed against gears and economical production nonlinear dynamic analysis has a great importance still nowadays. During this dynamic analysis the correct chose of backlash is pivotal.

On one hand backlash is necessary because of wear, thermal expansion, manufacturing errors, lubrication etc. but on the other hand due to backlash dynamic behaviour of gears are worse than backlash-free gears. Practically, backlash-free gears exist only theoretically on account of the fore-mentioned reasons. We can say that backlash is the necessary evil. Since manufacturing costs are greatly increased by reduction of backlash designers have great responsibility for choosing the right value of backlash. In case of gears of lower accuracy wider tolerances can be allowed but for precise machines backlash should be minimized. From the point of the application the maximum allowable backlash should be determined, this backlash is that by which the required performance can be achieved by the minimum manufacturing costs i.e. between the maximum allowable tolerances. In case of gears smaller in size determination of backlash are also very important. The aim is to use tolerances not tighter than necessary to obtain the required performance.

In this study we would like to show the correlation between meshing stiffness and backlash as well as the recommendations for backlash according to ISO standards.

2. EQUATION OF MOTION

For dynamic analysis of gear drives a model shown on *Figure 1* has been developed [4–6]. The driver and the driven gear are modeled by masses rotating around the central points of the gears, while meshing is modeled by an equivalent spring and a damping proportional with the velocity in the direction of line of action. This model contains backlash but neglects friction and installation inaccuracies.

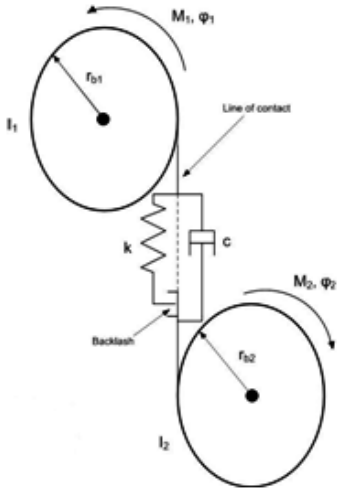


Figure 1. Dynamic model

Apart from development for the equation of motion the following relation can be written:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k(t) \cdot h(x) = f_T(t) + f_M(t) \tag{1}$$

where:

$$m = \frac{I_1 \cdot I_2}{I_1 \cdot r_{b2}^2 + I_2 \cdot r_{b1}^1} \tag{2}$$

$$f_T(t) = \frac{r_{b1} \cdot I_2 \cdot M_1 - r_{b2} \cdot I_1 \cdot M_2}{I_1 \cdot r_{b2}^2 + I_2 \cdot r_{b1}^2} \tag{3}$$

$$f_M(t) = -m \cdot \ddot{e}(t) \tag{4}$$

$$h(x) = \begin{cases} x - b, & x \geq b \\ 0, & |x| > b \\ x + b, & x \leq -b \end{cases} \tag{5}$$

Table 1

List of symbols and parameters

I_1, I_2	moments of inertia
φ_1, φ_2	rotational angles
$x = r_{b1}\varphi_1 - r_{b2}\varphi_2 - \mathrm{e(t)}$	relative displacement
r_{b1}, r_{b2}	base circles
M_1, M_2	external torques
2b	whole backlash
c	damping
k(t)	meshing stiffness
e(t)	static transmission error

3. DEFINITION OF BACKLASH

Backlash is the amount by which a tooth space exceeds the thickness of a gear tooth engaged in mesh. The general purpose of backlash is to prevent gears from jamming by making contact on both sides of their teeth simultaneously [3]. Usually the backlash under stabilized working conditions (working backlash) is different from (smaller than) the backlash which is measured when the gears are mounted in the housing under static conditions (assembly backlash) [2].

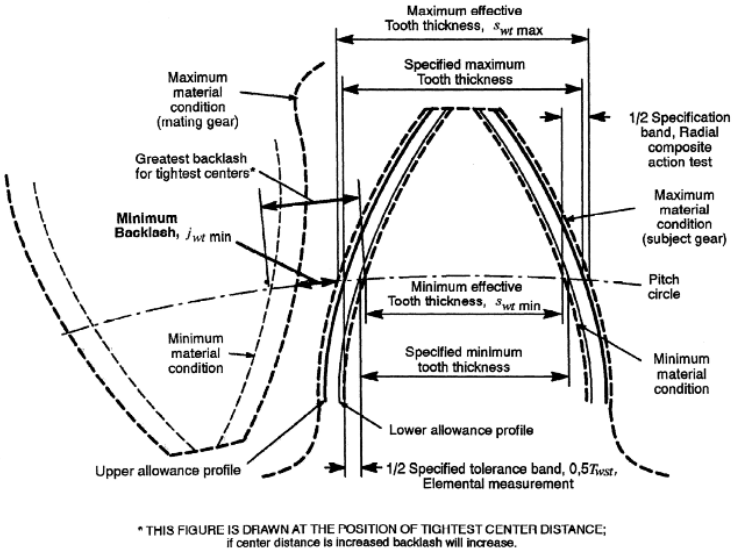


Figure 2. Tooth thickness, transverse plane [2]

According to ISO 1122-1 and ISO TR 10064-2 the following backlashes are determined [1, 2]:

- *Circumferential backlash (j_{wt}):* length of arc of the pitch circle through which a gear can be turned when the mating gear is fixed.

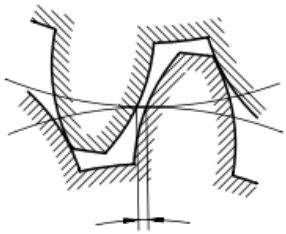


Figure 3. Circumferential backlash [1]

- *Normal backlash (j_{bn}):* is the shortest distance between non-operating flanks when the operating flanks are in contact.

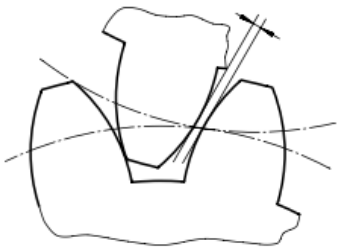


Figure 4. Normal backlash [1]

- *Reference backlash:* length of arc of the reference circle, equal to the product of the reference diameter and the circumferential backlash, divided by the pitch diameter.
- *Angular backlash (j_θ):* maximum value of the angle through which a gear can be turned when the mating gear is fixed and the centre distance has the specified value.
- *Radial backlash (j_r):* is the amount by which the center distance has to be diminished till the position in which left and right flanks of mating gears are in contact.
- *Minimum backlash ($j_{t,min}$):* is the minimum circumferential backlash on the pitch circle when the gear tooth with the greatest allowable effective tooth thickness is in mesh with the mating gear tooth having its greatest allowable effective tooth thickness, at the tightest allowable center distance, under static conditions. The tightest center distance is the minimum working center distance for external gears.
- *Maximum backlash ($j_{t,max}$):* is the maximum circumferential backlash on the pitch circle when the gear tooth with the smallest allowable effective tooth thickness is in mesh with the mating gear tooth having its smallest allowable effective tooth thickness, at the largest allowable center distance, under static conditions.

In case of spur gears the following relationships can be written between the different types of backlashes [3]:

$$j_{bn} = j_{wt} \cdot cos\alpha_{wt} \tag{6}$$

$$j_r = \frac{j_{wt}}{2 \cdot tan\alpha_{wt}} \tag{7}$$

$$j_\theta = j_{wt} \cdot \frac{360}{\pi d} \tag{8}$$

4. BACKLASH ACCORDING TO ISO STANDARDS

For selecting and measuring of backlash recommendations can be found in ISO TR 10064-2.
Backlash, *j,* in an assembled gear set is the clearance between the teeth of the meshing gears. Backlash may be measured in the normal plane or along the line of action, but it is calculated and specified in the transverse plane or in the plane of action (base tangent plane) [2].

An individual gear does not have backlash, it has only a tooth thickness. Backlash in a mesh is governed by the center distance at which a pair of gears is operated and the effective tooth thickness of each of the gears [2].

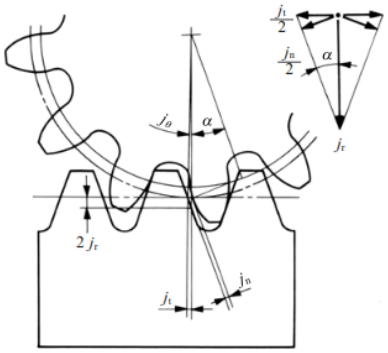


Figure 5. Kinds of backlash and their direction [3]

Backlash in a given mesh varies during operation as a result of changes in speed, temperature, load, etc. Adequate backlash should be present during static conditions, when it can be measured, to assure sufficient backlash under load at the most adverse working condition [2].

4.1. Maximum Tooth Thickness [2]

Maximum tooth thickness of a gear is determined as if the gear were in mesh with a perfect mating gear at the minimum center distance, allowing for the desired minimum backlash. Tooth thickness deviations reduce the maximum tooth thickness from the maximum value, and increase backlash.

For x = 0 gears, the theoretical or nominal tooth thickness is customarily equal to one half the circular pitch on the reference circle. Unless otherwise specified, the actual maximum tooth thickness on an unassembled gear will usually be less than the theoretical value, since the manufacturer usually makes a reduction in tooth thickness to allow for backlash.

4.2. Minimum Backlash [2]

Minimum backlash $j_{bn,min}$, is the minimum backlash allowable when the gear tooth with the greatest allowable effective tooth thickness is in mesh with a mating tooth having its greatest allowable effective tooth thickness at the tightest allowable center distance, under static conditions. This is the traditional “backlash allowance” provided by the designer to provide for:

a) deflections of housings, shafts and bearings
b) misalignments of gear axes due to housing deviations and bearing clearances
c) skew of gear axes due to housing deviations and bearing clearances
d) mounting errors such as shaft eccentricity
e) bearing runouts

f) temperature effects (a function of temperature differences between housing and gear elements, center distance and material difference)
g) centrifugal growth of rotating elements
h) other factors, such as allowance for contamination of lubricant and swelling of non-metallic gear materials

The value of minimum backlash can be small if the factors listed above are controlled. Each factor can be evaluated by analyzing the tolerances, and then a minimum requirement can be calculated. Judgment and experience are required to assess the minimum expected requirement, since the worst case tolerances are not likely to coincide.

Figure 6 shows values of minimum backlash recommended for industrial drives with ferrous gears in ferrous housings, working at pitchline speeds less than 15 m/s, with typical commercial manufacturing tolerances for housings, shafts and bearings.

The values found in *Figure 6* may be calculated from $j_{bn\,min} = \left(\frac{2}{3}\right)[0{,}06 + 0{,}0005a_i + 0{,}03m_n]$.

m_n in mm	Minimum center distance a_i in mm					
	50	100	200	400	800	1600
1,5	0,09	0,11	---	---	---	---
2	0,10	0,12	0,15	---	---	---
3	0,12	0,14	0,17	0,24	---	---
5	---	0,18	0,21	0,28	---	---
8	---	0,24	0,27	0,34	0,47	---
12	---	---	0,35	0,42	0,55	---
18	---	---	---	0,54	0,67	0,94

Figure 6. Recommended values minimum backlash $j_{bn\,min}$ for coarse pitch gears [2]

4.3. Maximum Backlash [2]

The maximum backlash in a gear set, $j_{bn,max}$, is the sum of tooth thickness tolerance, the effect of center distance variation, and the effects of gear tooth geometry variation. The theoretical maximum backlash occurs when two perfect gears, made to the minimum tooth thickness specification, are meshed at the loosest allowable center distance. The loosest center distance is the maximum for external gears or the minimum for internals.

The maximum theoretical backlash will also occur when two teeth, made to the minimum effective tooth thickness, $s_{wt\,min}$, coincide while operating at the loosest center distance. Neither occurrence is likely in practice.

The maximum expected backlash is a function of $j_{bn,max}$ and the statistical distribution of the individual elements of tooth and center distance variation. Any tooth deviations due to manufacturing will decrease the maximum expected backlash. Experience and judgment are required to estimate reasonable values.

If maximum backlash must be controlled, a careful study of each element of maximum backlash must be made and an accuracy grade selected which will limit tooth deviations as necessary.

When maximum backlash of an assembled gear drive, particularly a gear drive with multiple stages, is used as an acceptance criterion, the maximum acceptable value must be

carefully chosen to allow reasonable manufacturing tolerances for each part in the assembly.

4.4. Limits [2]

The (actual) sizes of the tooth thicknesses of pinion and gear and of the shaft center distance, together with the respective gear element deviations, determine the backlash j of the gear teeth; i.e. the clearance between the non-working flanks at the working diameter.

Usually, maximum backlash does not affect the function or smoothness of transmission motion, and effective tooth thickness deviation is not the main consideration in the selection of gear accuracy. In many applications, allowing a larger range of tooth thickness tolerance or working backlash will not affect the performance or load capacity of gears and may allow more economical manufacturing. A tight tooth thickness tolerance should not be used unless absolutely necessary, since it has a strong influence on manufacturing cost. In those cases where maximum backlash must be closely controlled, a careful study of the influence factors must be made and the gear accuracy grade, center distance tolerance and measurement methods must be carefully specified.

It may be necessary to specify a more precise accuracy grade to hold maximum backlash within the desired limits.

Minimum working backlash may not be allowed to become zero or negative. Because working backlash is determined by the assembled backlash and the working conditions; i.e., by the influences of deflections, misalignment, bearing runouts, temperature effects, and any unknowns, one has to distinguish: assembled backlash and working backlash.

Backlash does not have a fixed value, but may vary at different tooth positions due to manufacturing tolerances and working conditions.

The values given for tooth thickness and backlash are to be selected by the designer to suit the application.

4.5. Backlash from our point of view

Several kind of backlash are determined by standards but as it can seen from section 4.1–4.4 standards does not give strict values for backlash only recommendations and for coarse pitch gears some minimum values can be found. Backlash is influenced by several parameters which have to be taken into account during design. Our aim is to determine the minimum and maximum backlash by which the desired working conditions can be achieved. It includes the optimization of the tooth profile and of the tolerances. Solution of Eq. 1 should be done for the working backlash, but dynamic behavior, i.e. determination of vibration, of the gear transmission should also be determined in the case of the minimum and maximum backlashes in order to provide the desired dynamic parameters under these conditions. While a few parameters such as lubrication, housing etc. are not included in our dynamic model we have to take them into account at the stage of determination of the minimum and maximum backlashes.

In our studies we deal with not commonly used gears but with gears for special applications where parameters such as tooth deflection can't be neglected. For these kind of gears factual values can't be found even in standards. Recommendation included in standard mostly relates to gears not for special application. The goal of our research is to give not

only recommendation but factual values for backlash in case of special application with regarding the optimum gear profile under specified load conditions.

5. RELATIONSHIP BETWEEN MESHING STIFFNESS, BACKLASH AND TOOTH THICKNESS

The amount of backlash required depends on the size of the gears, their accuracy, mounting and the application. Backlash can be produce by enlarging the center distance and by reducing the tooth thickness [3]. Reducing the tooth thickness is the more popular method. By reducing the tooth thickness meshing stiffness is varied as it can be seen from the following equations obtained from [6]. In [6] the meshing stiffness is discussed from derivating it deflections resulted from the external load. In this study the tooth was considered elastic and modeled as a non-uniform cantilever embedded through its width b (See *Figure 7*).

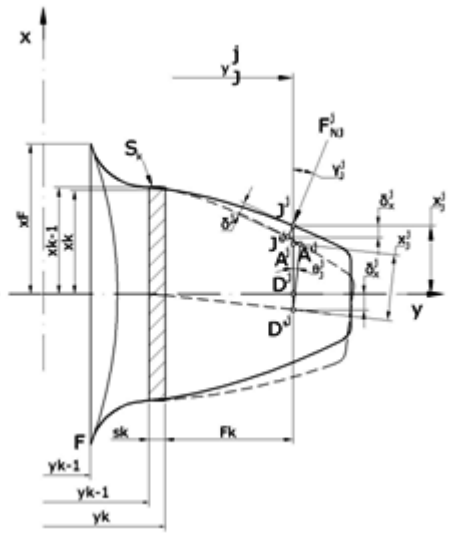


Figure 7. Model of the tooth

Bending deflection and meshing stiffness arising from it:

$$\delta_{Ja}^{j} = \frac{F_{NJ}^{j}}{E_v}\left[\frac{cos^2\gamma_J^j}{6}\sum_{k=1}^{N}\frac{s_k}{I_K}(2s_k^2 + 5F_k s_k + 3F_k^2) - cos\,\gamma_J^j sin\gamma_J^j \cdot x_J^j \sum_{k=1}^{N}\frac{s_k}{I_k}(s_k + 2F_k) + sin^2\gamma_J^j \cdot (x_J^j)^2 \sum_{k=1}^{N}\frac{s_k}{I_k}\right],\ k_{Ja}^{j} = \frac{F_{NJ}^{j}}{\delta_{Ja}^{j}} \tag{9}$$

Shearing deflection and meshing stiffness arising from it:

$$\delta_{Jb}^{j} = \delta_{xb}^{j} \cdot cos\gamma_{J}^{j} = \frac{1{,}2F_{NJ}^{j} \cdot cos^{2}\gamma_{J}^{j}}{G} \sum_{k=1}^{N} \frac{s_k}{\overline{A}_k}, k_{Jb}^{j} = \frac{F_{NJ}^{j}}{\delta_{Jb}^{j}} \tag{10}$$

Fillet-foundation deflection and meshing stiffness arising from it:

$$\delta_{Jc}^{j} = \frac{1{,}327F_{NJ}^{j} \cdot \left[cos\gamma_{J}^{j} \cdot \left(y_{J}^{j} - y_{F}\right) - sin\gamma_{J}^{j} \cdot x_{J}^{j}\right]^{2}}{E_{v} \cdot x_{F}^{2} \cdot b}, k_{Jc}^{j} = \frac{F_{NJ}^{j}}{\delta_{Jc}^{j}} \tag{11}$$

Contact deflection and meshing stiffness arising from it:

$$\delta_{Jd1,2}^{j} = \frac{5 \cdot 10^{-3} \cdot {F_{NJ}^{j}}^{0,9}}{E_{v1,2}^{0,9} \cdot b_{1,2}^{0,8}}, k_{Jd1,2}^{j} = \frac{F_{NJ}^{j}}{\delta_{Jd1,2}^{j}} \tag{12}$$

For *k(t),* when one pair of teeth in contact the following relationship has been obtained:

$$k_{1,2}^{j} = k_{J1,abc}^{j} + k_{J2,abc}^{j} + k_{Jd1,2}^{j}.$$

As it can be seen from (2–5), actual tooth thickness (tolerance of the nominal tooth thickness) has a significant influence on meshing stiffness while directly relates backlash. So tolerances on the thickness are very important factors.

5.1. Backlash in the motion of equation

According to Eq. 5 backlash can be illustrated by *Figure 8* considering teeth configuration in pursuance of *Figure 9*.

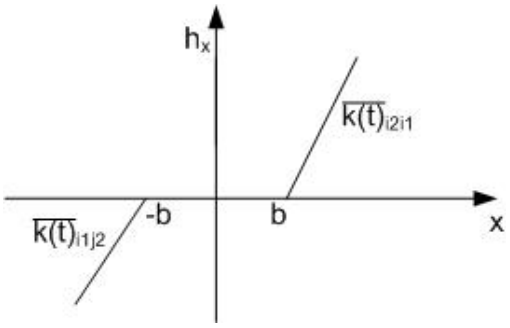


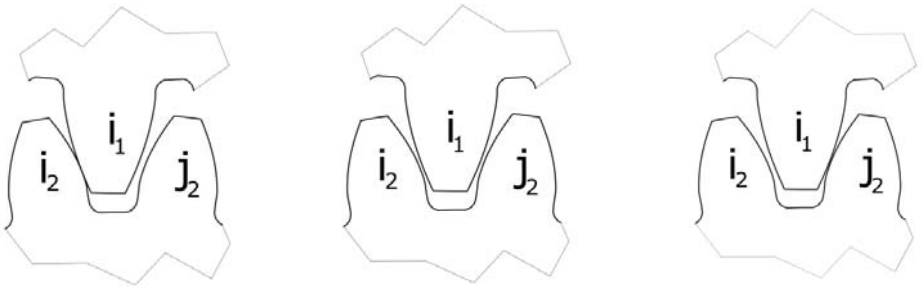
Figure 8. Backlash

Key for *Figure 8*:

$\overline{k(t)}_{i1j2}$: linearized meshing stiffness between tooth *i* of gear 1 and tooth *j* of gear 2;

$\overline{k(t)}_{i2i1}$: linearized meshing stiffness between tooth *i* of gear 2 and tooth *i* of gear 1;

Figure 9 shows the teeth configuration.



a) *b)* *c)*

Figure 9. Teeth confoguration: a) – first case, $x \geq b$ b) – second case, $|x| > b$; c) – third case, $x \leq -b$

In the second case backlash causes contact loss which results bad dynamic behaviour (noise, vibration) mainly in case of low loaded gears.

According to the above-mentioned Eq. 1 can be divided into three sub-equation:

a) if $x \geq b$:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k(t) \cdot (\mathrm{x} - \mathrm{b}) = f_T(t) + f_M(t)$$

so *k(t)* and *c* should be determined between tooth *i* of gear 2 and tooth *i* of gear 1;

b) if $|x| > b$:

$$m \cdot \ddot{x} = f_T(t) + f_M(t)$$

so there is no contact;

c) if $x \leq b$:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k(t) \cdot (\mathrm{x} - \mathrm{b}) = f_T(t) + f_M(t)$$

so *k(t)* and *c* should be determined between tooth *j* of gear 2 and tooth *i* of gear 1.

5.2. Methods of Controlling Backlash

For reducing backlash two methods are available: a static and a dynamic method. The static method concerns means of assembling gears and the making proper adjustments to achieve the desired low backlash. The dynamic method introduces an external force which continually eliminated all backlash regardless of rotational position. [3]

6. SUMMARY

Since backlash has great influence on the dynamic behaviour of gears determination of its value a very important task. The minimum and maximum value also should be determine in such a way that provides the desired performance. As backlash is in close relation with tooth thickness and meshing stiffness it can't be investigated as an individual parameter it should be determined as part of the optimizing process in which parameters influencing Eq. 1 are handled together. Our aim is to give factual data for the optimum gear profile under

the specified load that contains the tolerance of the tooth profile determining the minimum and maximum value of backlash.

Acknowledgement

"This research was carried out as part of the TAMOP-4.2.1.B-10/2/KONV-2010-0001 project with support by the European Union, co-financed by the European Social Fund."

References

[1] ISO 1122-1:1998(E/F): Vocabulary of gear terms – Part 1: Definitions related to geometry, Second edition, 1998-08-01
[2] ISO TR 10064-2: *Cylindrical gears – Code of inspection practice.* Part 2: Inspection related to radial composite deviations, runout, tooth thickness and backlash, First edition, 1996.
[3] *Practical Information on Gears.* Kohara Gear Industry Co., Ltd.: http://www.khkgears.co.jp/de/gear_technology/pdf/gear_guide2.pdf
[4] R. Szűcs–L. Kamondi: *Fogaskerekek dinamikai vizsgálatának egy lehetősége.* Kolozsvár: Erdélyi Magyar Műszaki Tudományos Társaság, Műszaki Szemle (2010), pp. 416–419.
[5] R. Szűcs–L. Kamondi: *Bevezetés a geometria dinamikai vizsgálatokra gyakorolt hatásába*. GÉP LXI. (2010/9-10.), Gépipari Tudományos Egyesület Műszaki Folyóirata, pp. 102–104.
[6] R. Szűcs–L. Kamondi: *Analytical model for determine meshing stiffness of spur gears*. Advanced Engineering 6 (2012)