

FLOW AND HEAT FLUX BEHAVIOUR IN THIN 2D CHANNEL WITH ROUGH MOVING WALLS

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Abstract: Paper describes the investigation of flow and heat flux in thin channel with rough moving walls in the 2D approach. The effect of the real measured roughness profiles upon Nusselt-number in comparison with smooth wall channel is shown. Both static and dynamic flow components were investigated.

Keywords: *thin channel, surface roughness, fluid flow, heat flux, moving wall*

1. Introduction

Study of the fluid flow and heat flux in thin channels of technological devices is one of the main problems of modern applied mechanics. Practical aspects of the problem are referred to the heat flux intensification in compact heat exchangers, heat flux study in moving parts of pumps and engines and so on. Behaviour of the flow and heat flux in the thin channel differs from the same processes in large size channels. Due to the small gap size, surface roughness becomes one of the main factors, having great impact on the resulted leakage and heat-exchange rate in the channel. One of the common ways to stimulate heat-exchange in large channels and pipes (especially for turbulent flow) is to produce some regular surface texture on its' walls. In the small channel the surface texture influence should be even more clear.

In authors' earlier work [1] the mathematical model of heat flux in the thin channels with rough immovable walls was proposed. Present work is an attempt for extending our researches over the heat flux in channels with rough moving walls. Such model allows to analyse the roughness effect upon the static and dynamic components of flow and heat flux in the thin channel.

2. Calculation model

The geometry model of the thin channel with moving walls formed by two rough surfaces h_1 , h_2 is presented in Figure 1. The gap between two rough surfaces (H) is taken as the distance between their mean lines. Upper and lower walls has coordinates $H_1(x) = H + h_1(x)$ and $H_2(x) = H + h_2(x)$. The current gap in the channel is calculated as $h_r(x) = H + h_1(x) + h_2(x)$. The upper wall slides in parallel direction to the lower with constant U speed.

Measured profiles of the real industrial surfaces or simulated roughness profiles with desirable features can be used as a surface geometry. Both profiles are always specified on regular grid with Δx step. Calculation step in x direction is considered as some fraction of the profile grid step $\Delta x = k\delta x$ (where k is an integer value). Calculation step in y direction δy is also considered regular comparable to δx .

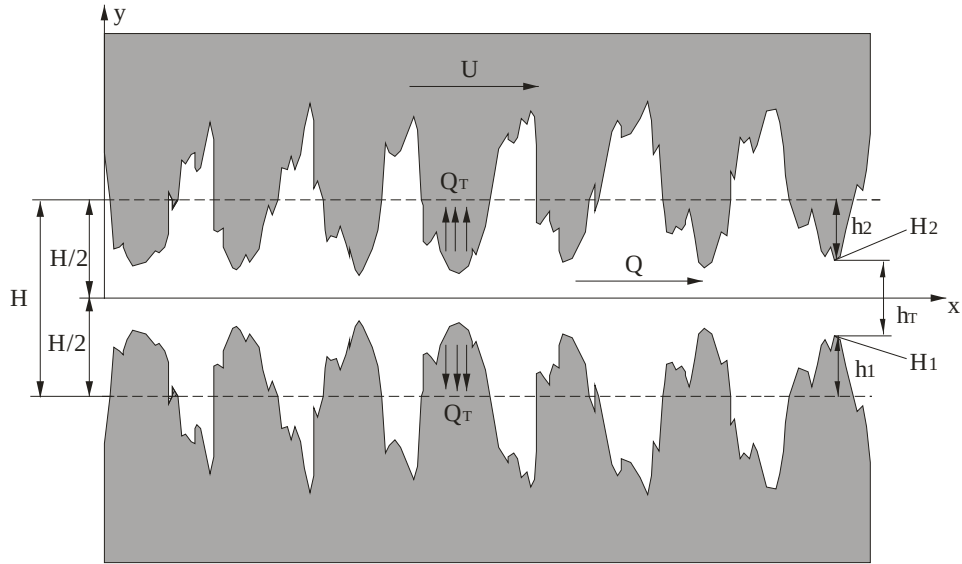


Figure 1. Geometry model of the thin channel with rough moving walls

To model the flow in the described thin channel the lubrication flow model was proposed. It is based on the common Reynolds equation (3D case):

$$\frac{\partial}{\partial x} \left(h_T^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(h_T^3 \frac{\partial p}{\partial y} \right) = 6\mu U \frac{\partial h_T}{\partial x}, \quad v_z = 0, \quad (1)$$

$$v_x = \frac{1}{2\mu} \frac{\partial p}{\partial x} z(z-h_T) + U \left(1 - \frac{z}{h_T}\right), \quad v_y = \frac{1}{2\mu} \frac{\partial p}{\partial y} z(z-h_T),$$

where p is the flow pressure, v is the local flow velocity, μ is the dynamic viscosity. For the 2D case Reynolds equation is significantly simplified and can be solved analytically, as shown in [2].

$$p(x) = p_{in} + 6\mu U \int_0^x \frac{d\xi}{h_T^2(\xi)} + \frac{p_{out} - p_{in} - 6\mu U \int_0^L \frac{d\xi}{h_T^2(\xi)}}{\int_0^L \frac{d\xi}{h_T^3(\xi)}} \int_0^x \frac{d\xi}{h_T^3(\xi)},$$

$$\frac{\partial p}{\partial x} = \frac{6\mu U}{h_T^2(x)} + \frac{p_{out} - p_{in} - 6\mu U \int_0^L \frac{d\xi}{h_T^2(\xi)}}{h_T^3(x) \int_0^L \frac{d\xi}{h_T^3(\xi)}}, \quad (2)$$

where p_{in} is the pressure at the channel inlet, p_{out} is the pressure at the channel outlet (boundary conditions).

To model the conjugated to the flow heat flux the thermal balance equation is commonly used:

$$\rho C_p \frac{dT}{dt} = \lambda \Delta(T) + \mu \Phi, \quad (3)$$

where T is the local flow temperature, ρ is the media density, C_p is the media thermal capacity, λ is the media thermal conductivity, t is the time, Φ is the dissipative function. The dissipation process is not natural for the described problem, so Φ was neglected in the proposed model.

The flow in the 2D thin channel is in general ordered in x direction, from inlet to the outlet. So $v_y = 0$ is considered from the flow equation. Supposing the process as stationary it gives:

$$\frac{dT}{dt} = v_x \frac{dT}{dx} + v_y \frac{dT}{dy} = v_x \frac{dT}{dx}. \quad (4)$$

The heat transfer in the 2D thin channel is generally ordered in y direction, from channel walls to the flow media. So d^2T/dx^2 in the Laplace operator was also neglected. The resulting heat transfer model is:

$$\rho C_p v_x \frac{dT}{dx} = \lambda \frac{d^2T}{dy^2}. \quad (5)$$

Equation (5), being the simplified approach to (3), still takes into account main process features.

Difference problem (5) was solved with known temperature T_w at the channel walls and known temperature of the flow T_{in} at the channel inlet. Under such boundary conditions the problem was solved by using the implicit finite-difference method. The stencil for that method is shown in Figure 2.

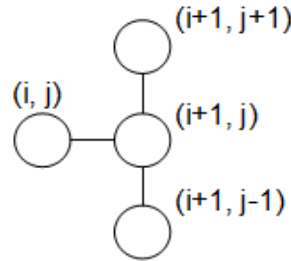


Figure 2. Implicit finite difference stencil for the thermal balance equation

According to the implicit finite difference method the temperature derivatives are substituted by the following finite differences:

$$\frac{dT}{dx} = \frac{T_{i+1,j} - T_{i,j}}{\delta x}, \quad \frac{d^2T}{dy^2} = \frac{T_{i+1,j-1} - 2T_{i+1,j} + T_{i+1,j+1}}{\delta y^2}. \quad (6)$$

So, the (5) equation is substituted by the following finite difference equation:

$$\rho C_p [v_x]_{i,j} \frac{T_{i+1,j} - T_{i,j}}{\delta x} = \lambda \frac{T_{i+1,j-1} - 2T_{i+1,j} + T_{i+1,j+1}}{\delta y^2}. \quad (7)$$

The sweep method (tridiagonal matrix algorithm) was used for the solution of (7). The scheme is always numerically stable and convergent. The scheme errors are linear in x direction and quadratic in y direction.

3. Heat flux characterisation

The operational capacity of the thin channel with rough walls was assessed by means of flow factors, and Nusselt-number. The dimensionless flow factors φ_x and Θ_x shows how decreases the leakage through the channel Q in comparison to the leakage through the equal channel with smooth walls Q^* :

$$Q_x^* = \frac{BH^3(p_{out} - p_{in})}{12\mu L} + \frac{BUH}{2}. \quad (8)$$

where B is the channel thickness.

It is clear from (8) that the flow is the algebraic sum of two flow components. The first component is the static flow. It is caused by the pressure gradient at the channel inlet and outlet. The second component is the dynamic flow. It is caused by the channel wall sliding. So, different flow factors for dynamical and static flow components should be calculated.

The static flow factor φ_x should be calculated taking into account only static part of leakage and not taking into account the wall movement (U=0):

$$Q_{xs} = \frac{Bh_T(L)^3}{12\mu} \frac{\partial p}{\partial x} \Big|_{x=L} = \frac{B(p_{out} - p_{in})}{12\mu \int_0^L \frac{d\xi}{h_T^3(\xi)}},$$

$$Q_{xs}^* = -\frac{BH^3(p_{out} - p_{in})}{12\mu L}, \quad \varphi_x = Q_{xs} / Q_{xs}^*. \quad (9)$$

The dynamical flow factor Θ_x is calculated taking into account only dynamic part of leakage and not taking into account the pressure gradient on the channel inlet and outlet:

$$Q_{xd} = BU \int_0^L \frac{d\xi}{h_T^2(\xi)} \Big/ 2 \int_0^L \frac{d\xi}{h_T^3(\xi)},$$

$$Q_{xd}^* = \frac{BUH}{2}, \quad \theta_x = Q_{xd} / Q_{xd}^*. \quad (10)$$

Such method of flow factors calculation provides that both of them do not depend on particular values of wall sliding velocity U and pressure gradient, viscosity and so on. They are mostly determined by the roughness form and roughness-to-gap proportion.

The Nusselt-number is the ratio of convective and conductive heat transfer normal to the boundary. The conductive component is measured under the same conditions as the heat convection but with a stagnant flow. In our case this dimensionless shows the efficiency of the reviewed channel for managing the heat transfer by manipulating the flow pressure and wall velocity.

For the fully developed laminar flow the Nusselt-number, calculated for some cross section ($x = \text{const}$) is [3]:

$$Nu = \frac{\alpha D_h}{\lambda}, \quad (11)$$

where α is the convective heat transfer coefficient, D_h is the hydraulic diameter. The hydraulic diameter of the channel in section is:

$$D_h = \frac{4 A_c}{P} = \frac{4BH}{2B + 2H} = \frac{2BH}{B + H}, \quad (12)$$

where A_c is the cross sectional area, P is the wetted perimeter of the cross section.

The convective heat transfer coefficient of the channel is calculated as:

$$\alpha = \frac{q}{T_w - T_m}, \quad (13)$$

where q is the normal heat flux near the walls, T_m is the representative flow temperature in the section.

The normal heat flux near the walls was calculated neglecting the roughness slopes as:

$$q = \lambda \frac{\partial T}{\partial y}. \quad (14)$$

The representative flow temperature was calculated by averaging the flow temperatures concerning the flow velocity:

$$T_m(x) = \frac{\int_{H_1(x)}^{H_2(x)} v_x(y) T(y) dy}{\int_{H_1(x)}^{H_2(x)} v_x(y) dy}. \quad (15)$$

The resulting Nusselt-number of the flow can be calculated by averaging the Nusselt-numbers in all the cross-sections of the channel except the developing zone near the channel inlet. The distinct static and dynamic Nusselt-numbers can be calculated for both static and dynamic case of the boundary condition.

4. Analysis

To verify the developed calculation model and computation software the channel with smooth walls was examined, which has already been studied by many authors [4]. In the stated assumptions the flow process is hydraulically developed from the beginning and flow velocity in any cross section has parabolic form. For the heat flux there is some zone near the channel inlet in which the process is developing. For the developed process the cross section of the temperature field has a clear parabolic shape. For the developing process it has a shape of flattened parabola. The results of the numerical experiments are agreed to the expected process behaviour. The resulting evolution of the heat flux in the channel with smooth walls is shown in Figure 3.

The quasy-parabolic distribution of the temperature was achieved for the inner sections of the channel (Figure 3b). The centre of the temperature distribution is shifted towards the moving wall direction. This is due to the higher velocity of the flow near the moving wall. Near the channel inlet the cross-section temperature gradually evolves from the constant boundary condition to the quasy-parabolic form (Figure 3a).

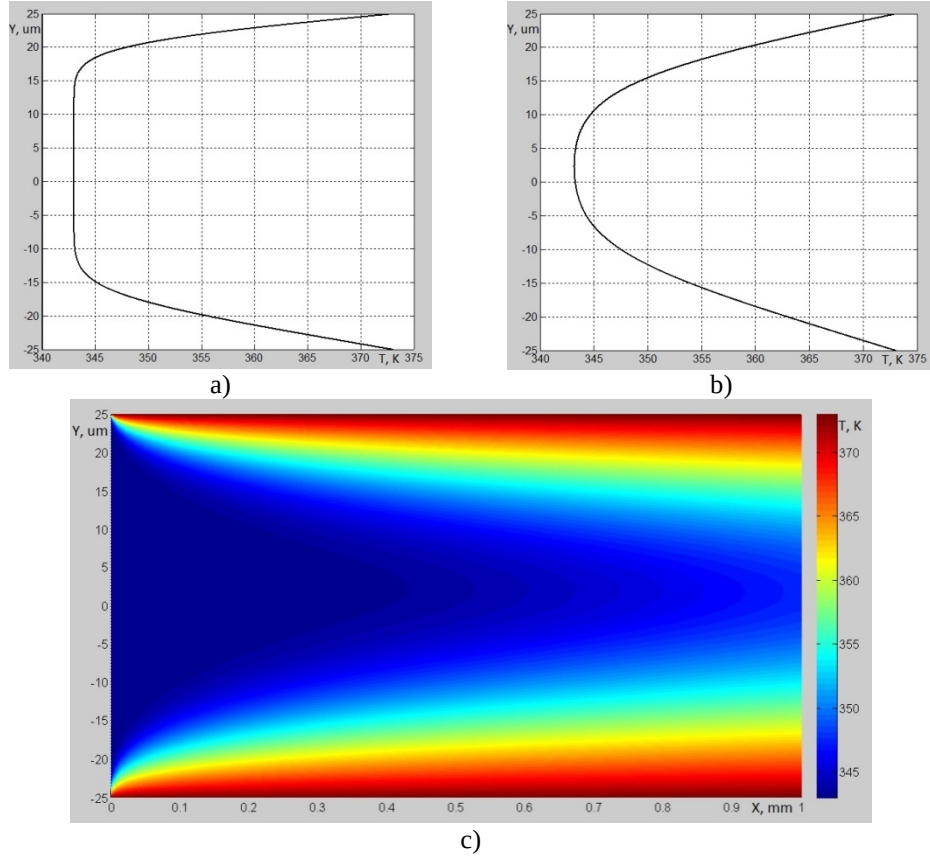


Figure 3. Flow temperature in the channel with smooth walls

a) – cross-section of flow temperature near the channel inlet, b) – cross-section of flow temperature in the middle zone, c) – flow temperature field

In complex the results achieved for the smooth wall channels allow to validate the correctness of the proposed numerical model.

Other numerical experiments were carried out for the rough wall channels. The surfaces after polishing, grinding, milling and turning were used (Figure 4). The length of the analysed channel was 0,8mm.

The experiments were carried out for $H = 40\mu\text{m}$, $p_A = 10\text{kPa}$, $p_B = 0$, $U = 200\text{m/s}$, $T_{\text{in}} = 343\text{K}$, $T_w = 373\text{K}$. Typical flow temperature fields for the static boundary condition is shown in Figure 5. Typical flow temperature fields for the dynamic boundary condition is shown in Figure 6.

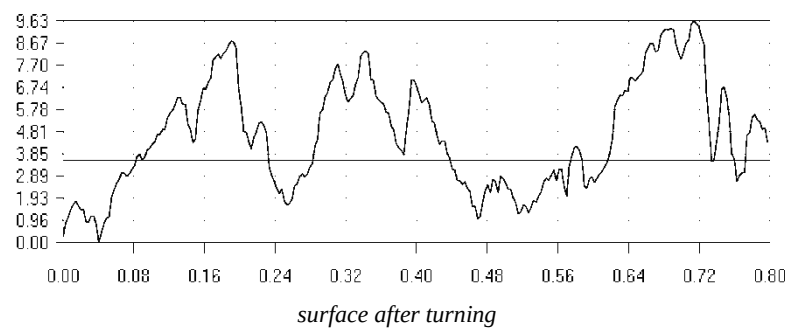
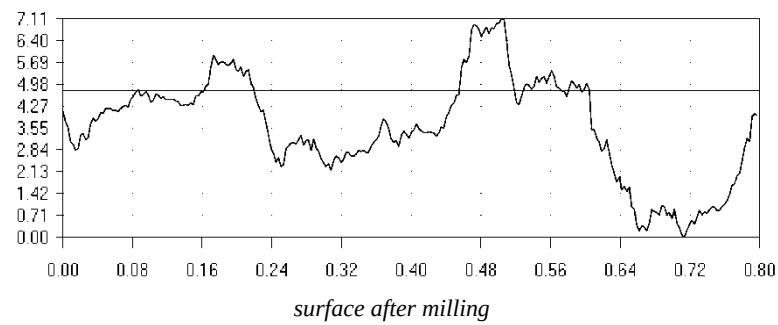
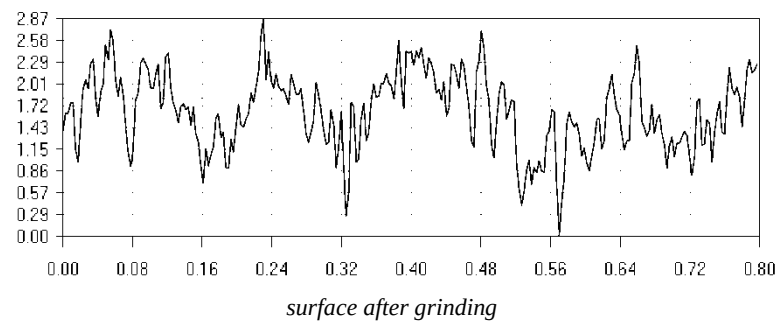
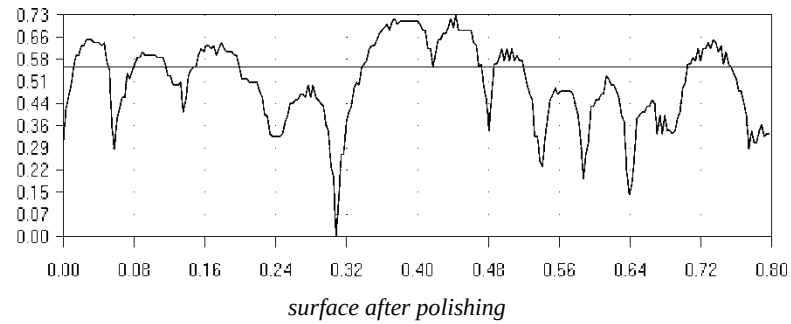


Figure 4. Surface roughness profiles

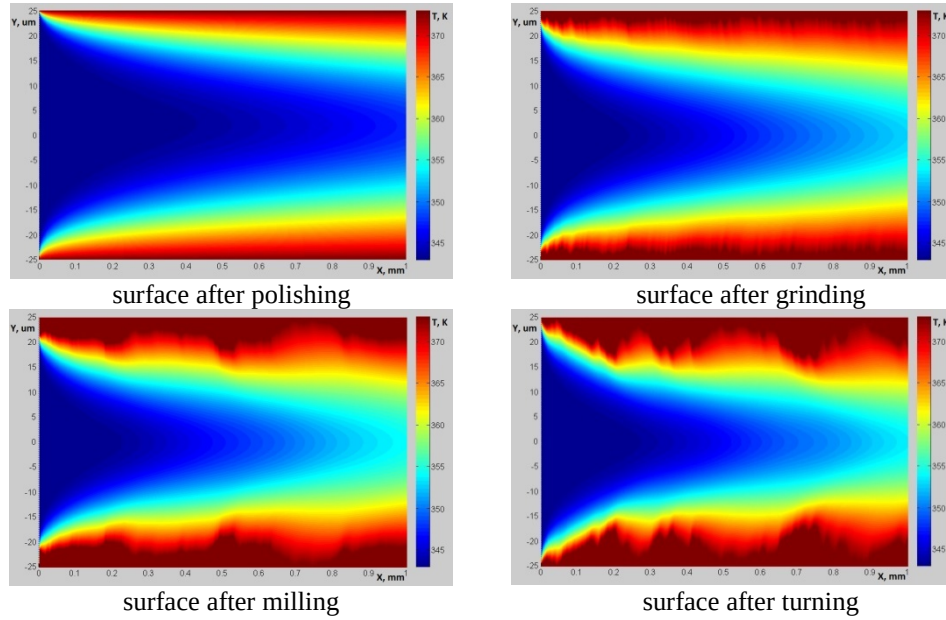


Figure 5. Flow temperature fields in channels with rough walls and static flow boundary

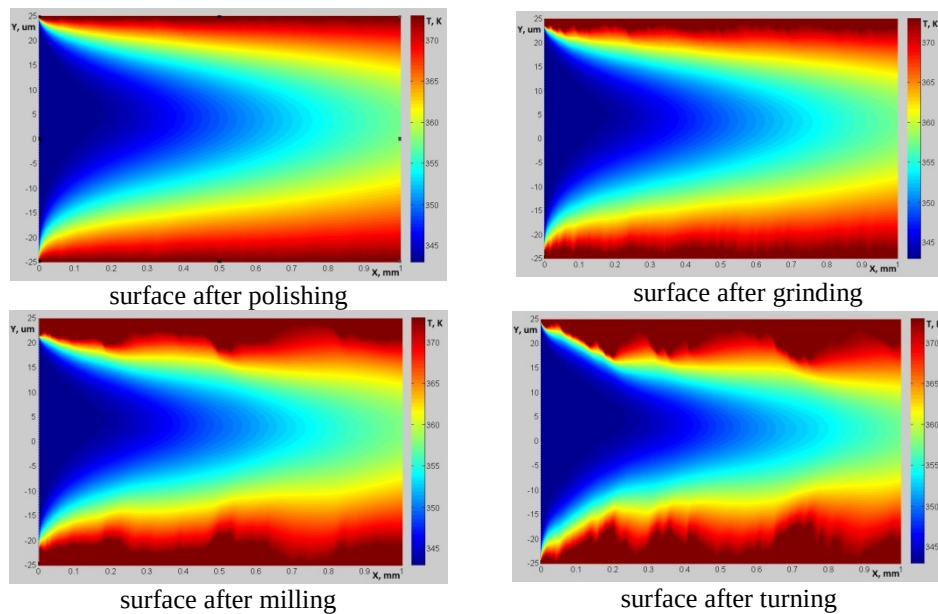


Figure 6. Flow temperature fields in channels with rough walls and dynamic flow boundary

The cross sections of the flow temperature fields are also has a quasy parabolic shape as for the smooth channel. There is also zone of the developing flow near the channel inlet. The heat flux is more intensive for the greater roughness heights.

The resulting Nusselt-number evolution for flows in channels with different surface roughness for static and dynamic boundary conditions are shown in Figure 7, 8. The results are shown as a graph of the Nusselt-number evolution while growing of the average gap from 40 to 60 μm .

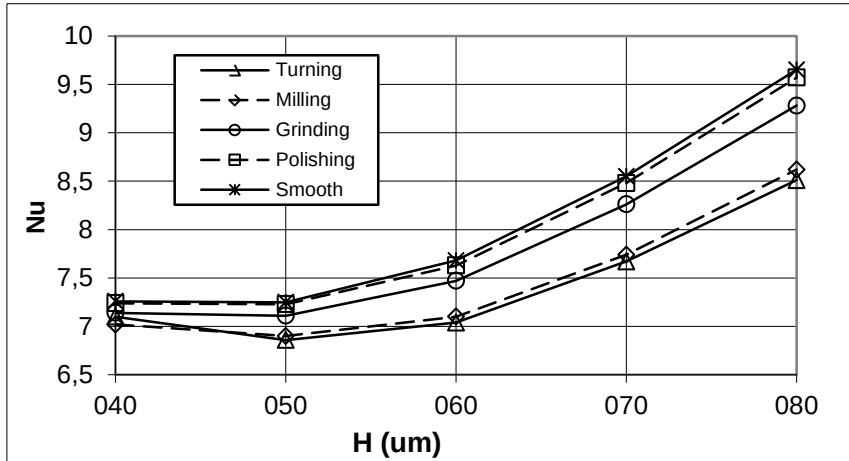


Figure 7. Evolution of Nusselt-number for static flow component

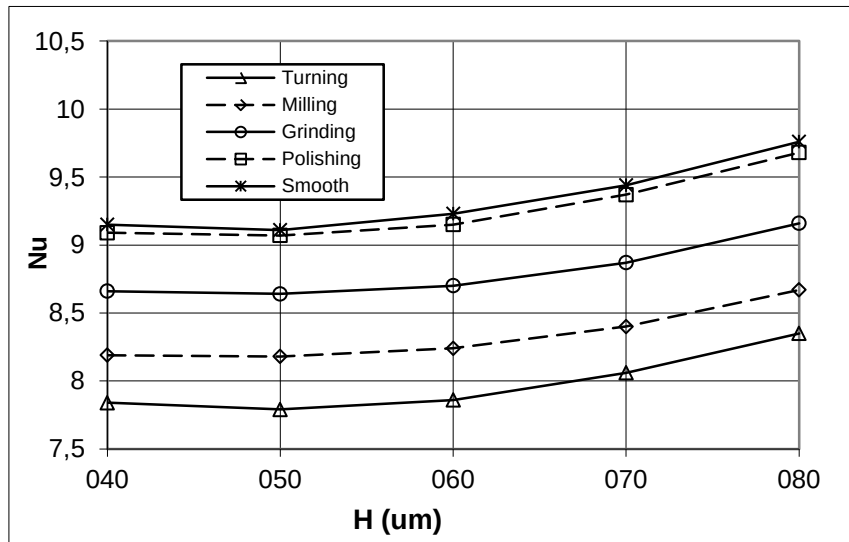


Figure 8. Evolution of Nusselt-number for dynamic flow component

Nusselt-number grows with growing average gap, because Reynolds-number of the flow also grows. It is consistent to the known theoretical assumptions and experimental data for macro-size channels.

For both static and dynamic flow components the Nusselt-numbers for the rough surfaces becomes more distinct from the smooth channel ones with growing of the average gap. But the difference in Nusselt-numbers is much lesser for the more thin channels.

For the static case the mean flow temperature is located approximately in the middle of the channel vertical section. The central flow layers are mostly affect the resulting Nusselt-number. For the dynamic case the mean flow temperature is biased in the direction of moving wall. The layers adjacent to the moving wall are much more involved into the heat flux. The Nu curves for the dynamic case are quite parallel to each other. That is an unsuspected fact that requires further detailed investigation.

5. Conclusion

Proposed mathematical model allows to forecast flow and heat flux in 2D channels with rough moving walls. It can be effective for the optimal selection of the processing methods of surfaces at the projecting stage.

Numerical experiments show that the surface roughness have a great influence upon the heat flux. For both static and dynamic flow components the Nusselt-numbers for the rough surfaces becomes more distinct from the smooth channel ones with growing of the average gap.

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