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MODELLING OF MOTORCYCLES AND THEIR COMPONENTS

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Abstract: One of the aims of the paper is to develop a mathematical model of a motorcycle using Lagrange equations; the model is verified thanks to ADAMS. Results of multibody simulations are used as inputs for boundary conditions of frame FEM simulation and following topology optimization which results in modifications of the frame design.

Keywords: *Multibody systems, motorcycle dynamics, FEM, topology optimization*

1. Introduction

Design of a motorcycle frame is a complicated task. The frame has to withstand significant dynamic forces without failure. The most reliable method to acquire necessary data is an experiment. However, in case there is no experimental data multibody modelling is arguably the best way to obtain the needed information. At the point when forces and loads are know they can be applied in FEM simulations. Topology optimization is the last step in the process that helps to find the stiffest structure possible taking into account a constraint of limited volume of material that can be used.

2. Mathematical model

2.1. Nonlinear model of a motorcycle

First step of the solution process is to develop a nonlinear mathematical model of a motorcycle. A planar 5-degre-of-freedom model was proposed with following generalized coordinates: x_2 – frame horizontal translation; y_2 – frame translation; φ_2 – frame pitch angle; y_3 – front assembly vertical translation; y_4 – rear assembly vertical translation. A scheme of the model containing generalized coordinates as well as other parameters is shown in Figure 1.
The generalized coordinates are included in vector q as shown in (1).

$$q = [x_2, y_2, \varphi_2, y_3, y_4]^T \tag{1}$$

Lagrange equations (2) were used to develop equations of motion of the system.

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} + \frac{\partial \mathcal{R}}{\partial \dot{q}_i} = Q_i \tag{2}$$

Where E_k, E_p and R represent kinetic energy function, potential energy function and Rayleigh dissipation function of the system. Each one of them is listed as the equations below.

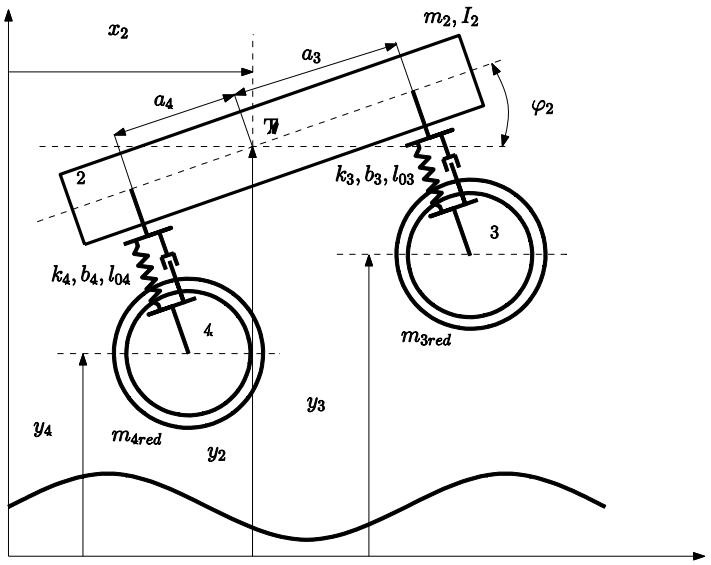


Figure 1. Scheme of the model

$$E_k = \frac{1}{2}\left\{ m\dot{x}_2^2 + I\dot{\varphi}_2^2 + m\dot{y}_2^2 + m_{3red}\left[\frac{d}{dt}\left(\frac{y_2 + a_3\sin\varphi_2 - y_3}{\cos\varphi_2}\right)\right]^2 + m_{4red}\left[\frac{d}{dt}\left(\frac{y_2 - a_4\sin\varphi_2 - y_4}{\cos\varphi_2}\right)\right]^2 \right\} \tag{3}$$

$$E_k = \frac{1}{2}\left[k_3\left(l_{03} - \frac{y_2 + a_3\sin\varphi_2 - y_3}{\cos\varphi_2}\right)^2 + k_4\left(l_{04} - \frac{y_2 - a_3\sin\varphi_{2-} y_4}{\cos\varphi_2}\right)^2 \right] \tag{4}$$

$$\mathcal{R} = \frac{1}{2}\left\{ b_3\left[\frac{d}{dt}\left(\frac{y_2 + a_3\sin\varphi_2 - y_3}{\cos\varphi_2}\right)\right]^2 + b_4\left[\frac{d}{dt}\left(\frac{y_2 - a_3\sin\varphi_2 - y_4}{\cos\varphi_2}\right)\right]^2 \right\} \tag{5}$$

After conducting partial derivatives according to (2) a mass matrix M, a stiffness matrix K and a damping matrix B can be developed as shown below.

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & m_{3red} & 0 \\ 0 & 0 & 0 & 0 & m_{4red} \end{bmatrix} \tag{6}$$

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_3 + k_4 & \dfrac{(k_3 a_3 - k_4 a_4)\sin\varphi_2}{\varphi_2} & -k_3 & -k_4 \\ 0 & (k_3 a_3 - k_4 a_4)\cos\varphi_2 & \dfrac{(k_3 {a_3}^2 - k_4 {a_4}^2)\sin 2\varphi_2}{2\varphi_2} & -k_3 a_3 \cos\varphi_2 & -k_4 a_4 \cos\varphi_2 \\ 0 & -k_3 & \dfrac{-k_3 a_3 \sin\varphi_2}{\varphi_2} & k_3 & 0 \\ 0 & -k_4 & \dfrac{k_4 a_4 \sin\varphi_2}{\varphi_2} & 0 & k_4 \end{bmatrix} \tag{7}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & b_3 + b_4 & (b_3 a_3 - b_4 a_4)\cos\varphi_2 & -b_3 & -b_4 \\ 0 & (b_3 a_3 - b_4 a_4)\cos\varphi_2 & (b_3 {a_3}^2 - b {a_4}^2)\cos^2\varphi_2 & b_3 a_3 \cos\varphi_2 & b_4 a_4 \cos\varphi_2 \\ 0 & -b_3 & -b_3 a_3 \cos\varphi_2 & b_3 & 0 \\ 0 & -b_4 & b_4 a_4 \sin\varphi_2 & 0 & b_4 \end{bmatrix} \tag{8}$$

The stiffness matrix and the damping matrix are both not constant and depend on the generalized coordinate φ_2 which means the system is nonlinear and provides more accurate results as displacements increase. However, this also places requirements on the integration method of the motion equations itself.

2.2. Tyre contact model

Interaction between tyres and road is a significant source of forces that act on the frame. In this case longitudinal driving and braking forces are neglected and the key task is to obtain normal contact forces. As stated previously the whole model is planar therefore lateral cornering forces are not taken into account either. The model of the tyre contact is shown in Figure 2.

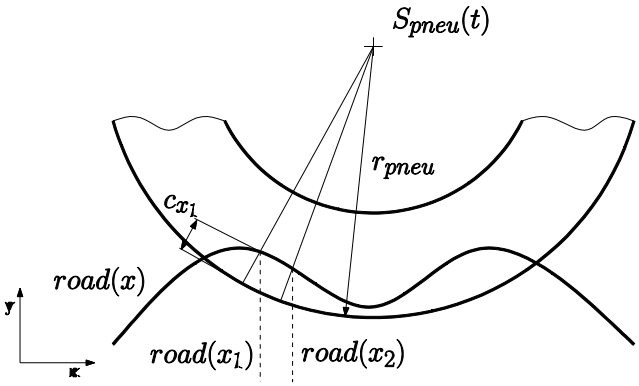


Figure 2. Tyre contact model

Let's assume that the location of tyre centre in known at any time. In case that any point of road surface is closer to the tyre centre than tyre radius a contact pressure exerting on the tyre can be calculated as

$$p_k = k_p c \tag{9}$$

Where $k_p[N/m^2]$ describes stiffness characteristics of the tyre and c is the difference between the tyre radius and the distance. Differential normal force can be obtained according to (10).

$$dN = k_p c ds \tag{10}$$

Differential ds relates to tyre arc. Differential normal force is always perpendicular to the road surface. For our problem it is crucial to determine its components in horizontal and vertical direction. This can be accomplished by using derivative of road function as described in the following equations.

$$dN_x = k_p c \sin\left(\arctan\frac{droad}{dx}\right) dx \tag{11}$$

$$dN_y = k_p c \cos\left(\arctan\frac{droad}{dx}\right) dx \tag{12}$$

In order to determine the components of the actual normal force, integration is conducted as the very last step.

$$N_x = \int_{x(S_{pneu})-r_{pneu}}^{x(S_{pneu})+r_{pneu}} dN_x \tag{13}$$

$$N_y = \int_{y(S_{pneu})-r_{pneu}}^{y(S_{pneu})+r_{pneu}} dN_x \tag{14}$$

The tyre contact model proposed above is based on elastic contact model. Tyre hysteresis effects can be taken into consideration as well assuming damping forces have similar character to normal contact forces as stated in the following equation.

$$N_b = \dot{c} N \frac{b_k}{k_k} \tag{15}$$

The variable $\dot{c}$ from previous equation relates to tyre deformation rate.

3. Numerical integration of the equations of motion

Once we acquired the forces acting on tyres we can proceed to solution process of the motion equations with MATLAB. Set of 5 nonlinear ordinary differential equations is summed up in the (16).

$$\boldsymbol{M}\ddot{\boldsymbol{q}}_{(t)} + \boldsymbol{B}\dot{\boldsymbol{q}}_{(t)} + \boldsymbol{K}\boldsymbol{q}_{(t)} = \boldsymbol{f}_{n(\dot{\boldsymbol{q}},\boldsymbol{q},t)} \tag{16}$$

Constant average acceleration integration method was used due to its relative simplicity and numerical stability. This method is predictor-corrector based as the system in question is nonlinear. A road function was designed using modified sin functions to represent an obstacle, which is supposed to cause a loss of contact between the tyres and the road. In this way a jump of a motorcycle can be simulated.

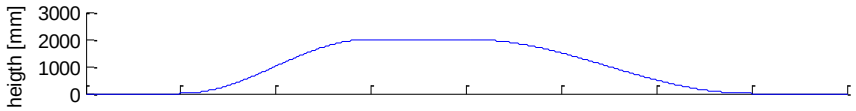


Figure 3. Road profile

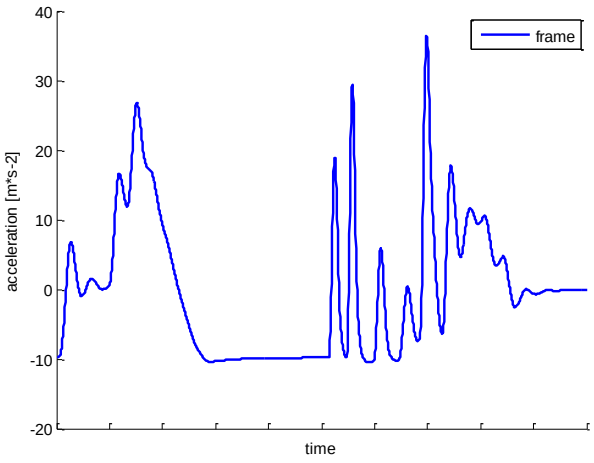


Figure 4. Frame acceleration results

Acceleration results are shown in the figures below. In Figure 4 there are acceleration results of the frame. The initial steep increase relates to the beginning of the obstacle. As the acceleration drops to approximately -10 $\mathbf{ms^{-2}}$ it means that the tyres lost contact with surface. The peak values occur after landing reaching 36 $\mathbf{ms^{-2}}$. Acceleration results of front and rear assembly in Figure 5 can be explained in a similar way.

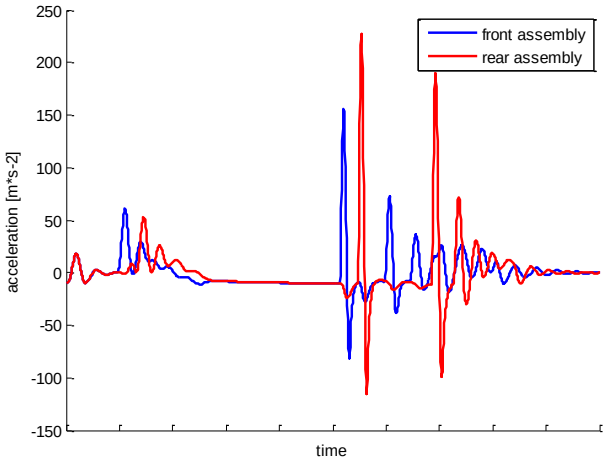


Figure 5. Front and rear assembly acceleration results

4. Model verification

The mathematical model that was developed provides reasonable results; however, further verification is necessary. In order to do so, ADAMS/VI-Motorcycle module was used to compare results. In Figure 6 there is a motorcycle model in VI-Motorcycle interface running over an obstacle defined in the same as in the MATLAB model.

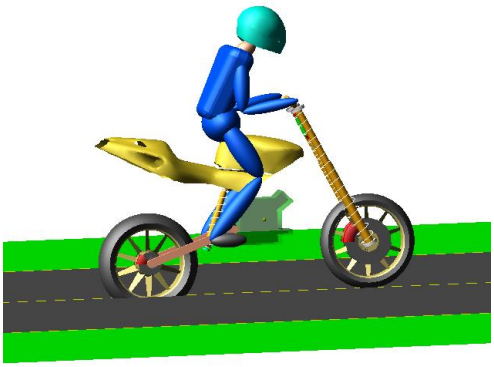


Figure 6. VI-Motorcycle model

In Figure 7 there are acceleration results of front assembly in ADAMS and MATLAB. In this simulation tyre damping properties were neglected in MATLAB, which explains the key difference between both curves.

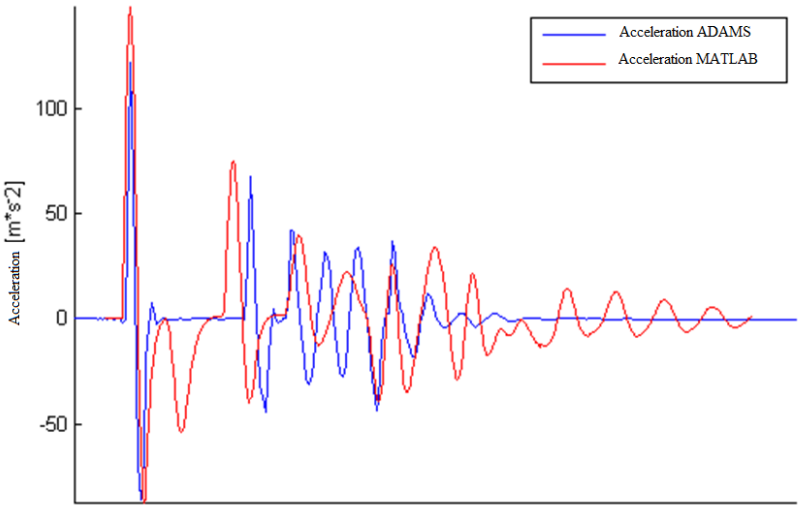


Figure 7. Front assembly acceleration results

5. FEM Simulation

The last step that concludes this paper is a FEM simulation followed by a topology optimization. Boundary conditions that are necessary for the simulation were obtained in the previous step. As it can be seen in Figure 8 the frame is an open tubular frame, which means an engine plays major role in overall stiffness. Engine stiffness properties were determined separately and these effects were expressed by 1D CBUSH elements in the whole model. Topology optimization enabled design modifications resulting in stiffness increase by 33% while saving 200g in weight. Lower overall values of stress were also achieved thanks to topology optimization.

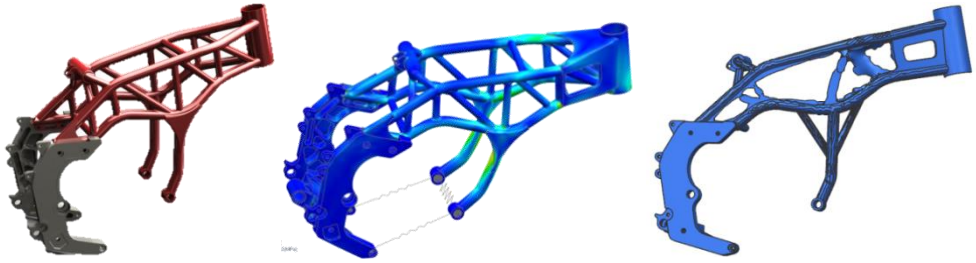


Figure 8. CAD, FEM and Topology optimization model

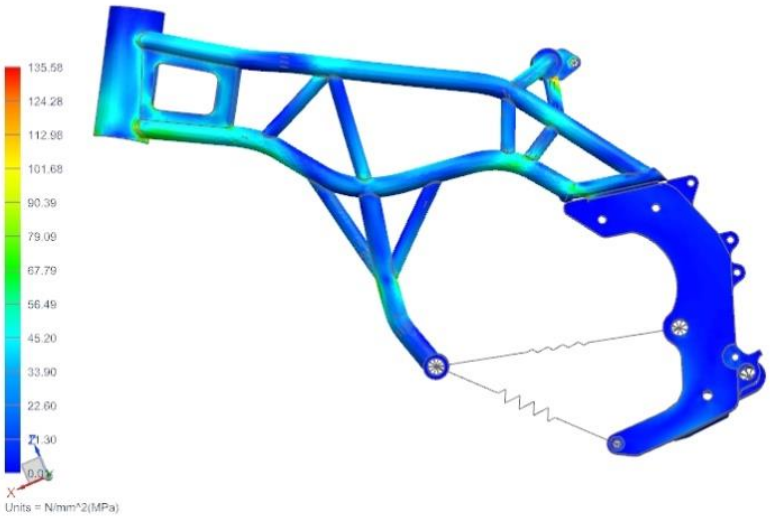


Figure 9. FEM results after topology optimization

6. Summary

Design of a racing motorcycle frame is a complicated task. Without previous experience is experimental data multibody modelling is arguably the only way to obtain boundary conditions for FEM simulations. Nevertheless, to describe motorcycle behaviour and exerting forces properly a large number of simulations would be necessary. Therefore the data acquired in this paper are more suited to compare design alternatives instead of exact FE modelling. Topology optimization is a powerful tool allowing shortening the whole design process in order to find optimal solution.

7. References

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