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FLOW IN CAPILLARY TUBE

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Abstract: In this study, I showed the equations that describe the steady state flow in the capillary in a way that no distinction should be made according to whether there is a one or a two-phase flow. This is due to the fact that the enthalpy partial derivatives in the equations are prepared by numerical calculation. In case of steady state flow I showed the throttling effect on capillary flow, as well as how to calculate (numerically) the size of the unassignable capillary flow, and the enthalpy and density of the fluid which comes into the evaporator

Keywords: *capillary, numerical simulation, steady state flow*

1. Introduction

In the cooling system of the household refrigerators the most common expansion device is the capillary tube. The capillary tube controls the amount of refrigerant flowing from the condenser to the evaporator. The capillary tube as an expansion device is used widely due to its low cost and reliable operation (no moving parts). The capillary tube as a structural component of the cooling system is one of the simplest components, but it is more difficult to understand its physical function and to describe it mathematically.

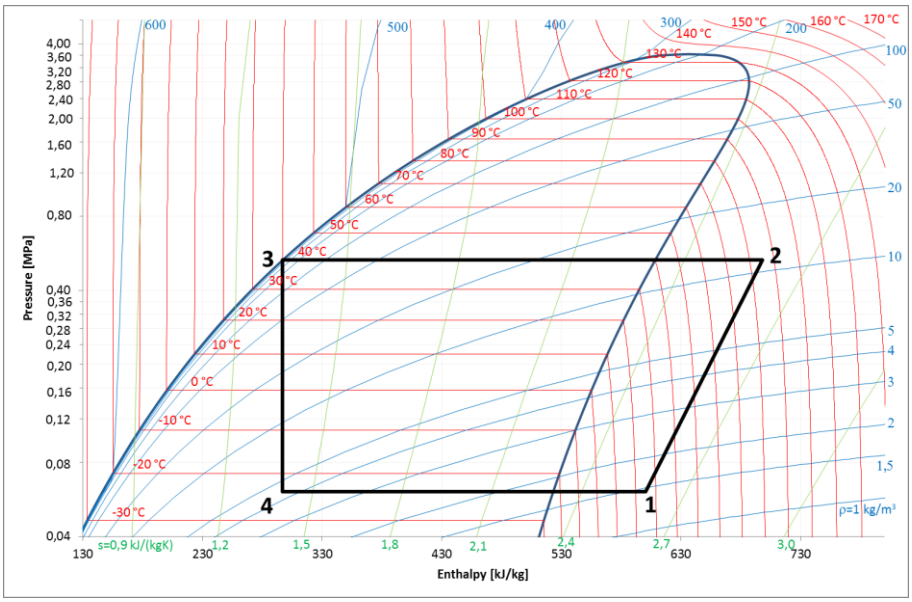


Figure 1. Theoretical cooling cycle in log(p)-h diagram

Cooling device developers determine the applicable capillary diameter and length through experiments and tests. In general, they specify the so-called capillary air flow rate. This means that they specify the amount of outgoing nitrogen volumetric flow in case of a 6 bar inlet and 1 bar outlet pressure ratio. . This value is usually given in litre/minute.

Capillary tubes are usually made of copper or aluminium. Their external diameter is 1,85 mm and the internal diameter range is 0,57; 0,63; 0,70; 0,76 mm (0,0225; 0,025; 0,0275; 0,030”). They select from this inner diameter range, and set the different air flow rate value by preparing the length of the capillary tube to the proper value.

Figure 1 shows a theoretical cooling cycle in a log(p)-h diagram. (Usually this kind of diagram is used to describe cooling cycles.) In this figure point 3 is the end of the condenser and point 4 is the front of the evaporator. These two points are connected by the capillary tube. In the theoretical cooling cycle it is assumed that the refrigerant flows from the condenser to the evaporator at a constant enthalpy (with a so called throttling state change).

In practice, the capillary tube and the suction tube, which latter connects the evaporator to the compressor, are counter flow heat exchangers. (It further complicates the description of the operation of the capillary.) Figure 2 shows this layout. The total length of the capillary is , the length of it before the heat exchanger is, the length of the heat exchanger is, the length of the capillary after the heat exchanger is which is in the insulating PUR foam. This section, therefore, we can take adiabatic.

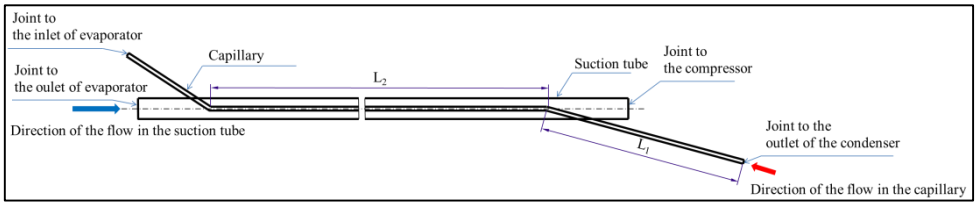


Figure 2. Structure of heat exchanger

We know what pressure (temperature) and density the medium has which enters the capillaries, but we do not know its quantity. (What is the mass flow rate?) We do not know the density (the amount of steam) and the enthalpy of the fluid, which exits the capillary and enters the evaporator. This is important to recognize so that we can tell what long evaporator is needed (or what is the size of its surface) for the entering amount of refrigerant to evaporate.

2. Equations describing the flow

Using the nominations of Figure 3 the following equation can be prescribed.

– Continuity equation

$$\frac{\partial \rho}{\partial t} + w\frac{\partial \rho}{\partial x} + \rho\frac{\partial w}{\partial x} = 0 \tag{1}$$

– Equation of motion (Euler Equation of motion)

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{\lambda}{2D}w|w| - g\sin\alpha \tag{2}$$

– Energy equation

$$\frac{\partial h}{\partial t} + w\frac{\partial h}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial t} + w\frac{\partial w}{\partial t} + w^2\frac{\partial w}{\partial x} = \frac{4k}{D\rho}\left(T_k - T\right) - wg\sin\alpha \tag{3}$$

– Equations of state

– Thermal equation of state

$$p = p(\rho, T) \tag{4}$$

– Caloric equation of state

$$h = h(\rho, T) \tag{5}$$

$$s = s(\rho, T) \tag{6}$$

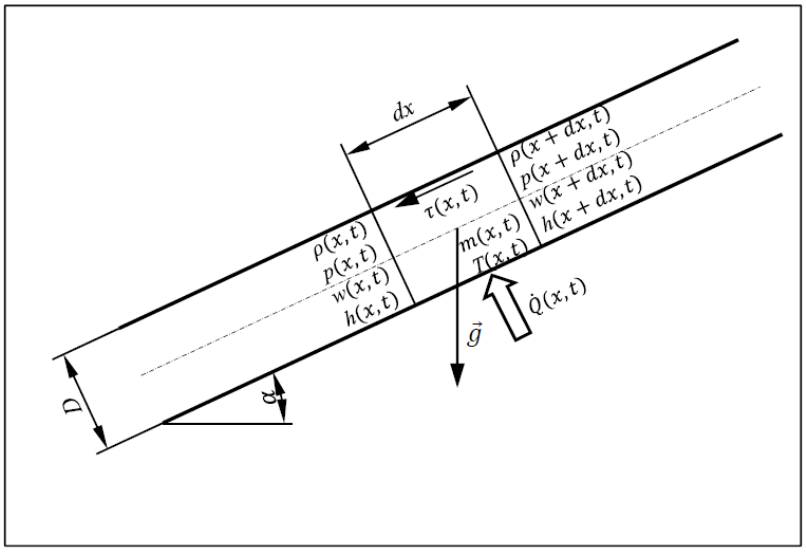


Figure 3. Elementary section of pipe with markings

– Equations for steady-state flow

In case of equations of the stationary (time constant) flow the partial derivatives of the time $\left(\frac{\partial}{\partial t}\right)$ disappear. The equations are as follows.

Continuity equation

$$\frac{\partial(\rho w)}{\partial x}=0 \quad , \tag{7}$$

this means that

$$\rho w = G = Cons \quad . \tag{8}$$

Equation of motion (Euler Equation of motion)

$$w\frac{\partial w}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=-\frac{\lambda}{2D}w|w|-g\sin\alpha \tag{9}$$

$$\underbrace{w\rho}_{G}\frac{\partial w}{\partial x}+\frac{\partial p}{\partial x}=-\frac{\lambda}{2D}\underbrace{\rho w}_{G}|w|-g\rho\sin\alpha \tag{10}$$

Energy equation

$$\frac{\partial h}{\partial x}+w\frac{\partial w}{\partial x}=\frac{4k}{D\rho w}(T_k-T)-g\sin\alpha \tag{11}$$

$$\frac{\partial h}{\partial x}+\frac{\partial\left(\frac{w^2}{2}\right)}{\partial x}=\frac{4k}{D\underbrace{\rho w}_{G}}(T_k-T)-g\sin\alpha \tag{12}$$

Equations of state are not changed.

Switching over from differentials to differences in equations (10) and (12) we get the following equations:

$$Gdw+dp=-\left(\frac{\lambda}{2D}\frac{G^2}{\rho}+g\rho\sin\alpha\right)dx \tag{13}$$

$$d\left(h+\frac{w^2}{2}\right)=\left(\frac{4k}{DG}(T_k-T)-g\sin\alpha\right)dx \quad . \tag{14}$$

$$G\Delta w+\Delta p=-\left(\frac{\lambda}{2D}\frac{G^2}{\rho}+g\rho\sin\alpha\right)\Delta x \tag{15}$$

$$\Delta\left(h+\frac{w^2}{2}\right)=\left(\frac{4k}{DG}(T_k-T)-g\sin\alpha\right)\Delta x \tag{16}$$

at the same time based on equation (8) we know that

$$w=\frac{G}{\rho} \quad . \tag{17}$$

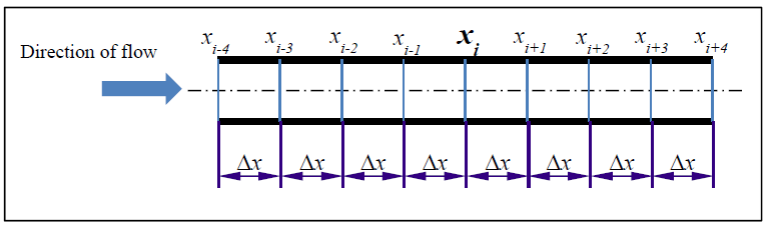


Figure 4. Net with equal scale interval

Thus, equations (15) and (16) can be written as follows (Figure 4):

$$G\left(w_{i+1}-w_i\right)+p_{i+1}-p_i=-\left(\frac{\lambda_i}{2D}\frac{G^2}{\rho_i}+g\rho_i\sin\alpha_i\right)\Delta x \tag{18}$$

$$h_{i+1}-h_i+\frac{w_{i+1}^2}{2}-\frac{w_i^2}{2}=\left(\frac{4k_i}{DG}\left(T_k-T_i\right)-g\sin\alpha_i\right)\Delta x \tag{19}$$

Taking relationship (17) into account and after rearranging (18) and (19) we get

$$p_{i+1}=-\left(\frac{\lambda_i}{2D}\frac{G^2}{\rho_i}+g\rho_i\sin\alpha_i\right)\Delta x+p_i+Gw_i-\frac{G^2}{\rho_{i+1}} \tag{20}$$

$$h_{i+1}=\left(\frac{4k_i}{DG}\left(T_k-T_i\right)-g\sin\alpha_i\right)\Delta x+h_i+\frac{w_i^2}{2}-\frac{G^2}{2\rho_{i+1}^2} \tag{21}$$

expressions. At the same time, also the caloric state equation must be satisfied i.e.

$$h_{i+1}=h\left(p_{i+1},\rho_{i+1}\right). \tag{22}$$

Equations (21) and (22) shall be equal, the pressure (p_{i+1}), calculated from relation (20), must be solved numerically concerning the density (ρ_{i+1}). It has to be solved in case of all the elementary section of pipe (Δx) under a given mass flow (G). This means that an implicit equation with high computational demands has to be solved in case of all the elementary section of pipe.

We present another option, which makes the calculations explicit (and faster). To do this refer to equations (13) and (14), and based on relation (17) express dw:

$$dw=d\left(\frac{G}{\rho}\right)=Gd\left(\frac{1}{\rho}\right)=-\frac{G}{\rho^2}d\rho \tag{1}$$

.

Substituting relation (23) into equations (13) and (14), we get the followings:

$$dp=-\left(\frac{\lambda}{2D}\frac{G^2}{\rho}+g\rho\sin\alpha\right)dx+\frac{G^2}{\rho^2}d\rho \tag{2}$$

$$dh=\left(\frac{4k}{DG}\left(T_k-T\right)-g\sin\alpha\right)dx+\frac{G^2}{\rho^3}d\rho \tag{3}$$

In addition to the independent variable dx the unknown variables are dh, dp and the dρ. We have three unknowns and two equations. The equation of state of fluid is missing. Let's make the equation of state of the fluid known numerically in h = h(p, ρ) form. From it dh can be expressed as follows:

$$dh=\left.\frac{\partial h}{\partial p}\right|_{\rho}dp+\left.\frac{\partial h}{\partial \rho}\right|_{p}d\rho \tag{4}$$

Substituting formula (26) into equation (25), we get the following relationship:

$$\left.\frac{\partial h}{\partial p}\right|_{\rho}dp=\left(\frac{4k}{DG}\left(T_k-T\right)-g\sin\alpha\right)dx+\left(\frac{G^2}{\rho^3}-\left.\frac{\partial h}{\partial \rho}\right|_{p}\right)d\rho \tag{5}$$

.

From equations (24) and (27) we get the elemental density change

$$d\rho = \frac{\left(\frac{4k}{DG}(T_k - T) - g\sin\alpha\right)dx + \left.\frac{\partial h}{\partial p}\right|_{\rho}\left(\frac{\lambda}{2D}\frac{G^2}{\rho} + g\rho\sin\alpha\right)dx}{\left.\frac{\partial h}{\partial p}\right|_{\rho}\frac{G^2}{\rho^2} + \left.\frac{\partial h}{\partial \rho}\right|_{p} - \frac{G^2}{\rho^3}} \tag{6}$$

.

Switching over to differences, the density change, the pressure change and the enthalpy change can be calculated as follows:

$$\Delta\rho_i = \frac{\left(\frac{4k_i}{DG}(T_k - T_i) - g\sin\alpha_i\right)\Delta x + \left.\frac{\partial h}{\partial p}\right|_{\rho i}\left(\frac{\lambda_i}{2D}\frac{G^2}{\rho_i} + g\rho_i\sin\alpha_i\right)\Delta x}{\left.\frac{\partial h}{\partial p}\right|_{\rho i}\frac{G^2}{\rho_i^2} + \left.\frac{\partial h}{\partial \rho}\right|_{pi} - \frac{G^2}{\rho_i^3}} \tag{29}$$

,

$$\Delta p_i = -\left(\frac{\lambda_i}{2D}\frac{G^2}{\rho_i} + g\rho_i\sin\alpha_i\right)\Delta x + \frac{G^2}{\rho_i^2}\Delta\rho_i \tag{30}$$

,

$$\Delta h_i = \left(\frac{4k_i}{DG}(T_k - T_i) - g\sin\alpha_i\right)\Delta x + \frac{G^2}{\rho_i^3}\Delta\rho_i \tag{31}$$

.

Knowing this the followings can be calculated:

$$\rho_{i+1} = \rho_i + \Delta\rho \tag{32}$$

,

$$p_{i+1} = p_i + \Delta p \tag{33}$$

,

$$h_{i+1} = h_i + \Delta h \tag{34}$$

.

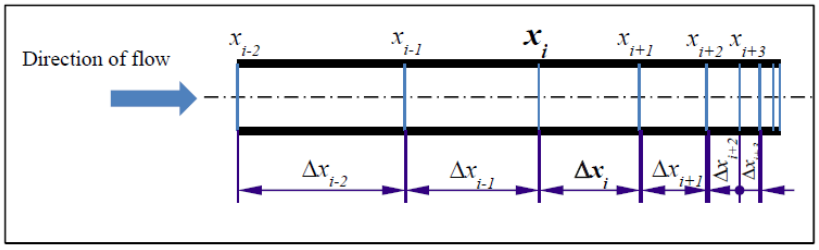


Figure 5. Net with variable scale interval

If we take Δp as independent variable, then we get the following relation for Δx_i (Figure 5) based on equations (29) and (30):

$$\Delta x_i = \frac{\left(\left.\frac{\partial h}{\partial p}\right|_{\rho i} + \frac{\rho_i^2}{G^2}\left.\frac{\partial h}{\partial \rho}\right|_{pi} - \frac{1}{\rho_i}\right)\Delta p}{\left(\frac{1}{\rho_i} - \frac{\rho_i^2}{G^2}\left.\frac{\partial h}{\partial \rho}\right|_{pi}\right)\left(\frac{\lambda_i}{2D}\frac{G^2}{\rho_i} + g\rho_i\sin\alpha_i\right) + \frac{4k_i}{DG}(T_k - T_i) - g\sin\alpha_i} \tag{35}$$

.

Δx calculated from expression (35) must be met the

$$\Delta x \geq 0 \tag{36}$$

condition. The borderline case $(\Delta x = 0)$ is met when

$$\left.\frac{\partial h}{\partial p}\right|_{\rho} + \frac{\rho^2}{G^2}\left.\frac{\partial h}{\partial \rho}\right|_{p} - \frac{1}{\rho} = 0 \tag{37}$$

equality is fulfilled. To analyse this relationship let's use the following based on expression (8)

$$\frac{\rho^2}{G^2} = \frac{1}{w^2}, \tag{38}$$

furthermore, we know the pressure, the enthalpy and the entropy as function of temperature and density based on equations (4), (5) and (6). Thus, we can write that

$$dp = \left.\frac{\partial p}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial p}{\partial T}\right|_{\rho} dT, \tag{39}$$

$$dh = \left.\frac{\partial h}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial h}{\partial T}\right|_{\rho} dT, \tag{40}$$

$$ds = \left.\frac{\partial s}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial s}{\partial T}\right|_{\rho} dT. \tag{41}$$

In equation (37) coefficients $\left.\frac{\partial h}{\partial p}\right|_{\rho}$ and $\left.\frac{\partial h}{\partial \rho}\right|_{p}$ can be expressed as follows, knowing that

$$\begin{aligned} dh &= \left.\frac{\partial h}{\partial p}\right|_{\rho} \underbrace{dp}_{\left.\frac{\partial p}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial p}{\partial T}\right|_{\rho} dT} + \left.\frac{\partial h}{\partial \rho}\right|_{p} d\rho = \\ &= \left(\left.\frac{\partial h}{\partial p}\right|_{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T} + \left.\frac{\partial h}{\partial \rho}\right|_{p}\right)d\rho + \left.\frac{\partial h}{\partial p}\right|_{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho} dT = \left.\frac{\partial h}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial h}{\partial T}\right|_{\rho} dT \end{aligned} \tag{42}$$

and relation (42) is true at any dρ and at any dT, so

$$\left.\frac{\partial h}{\partial p}\right|_{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T} + \left.\frac{\partial h}{\partial \rho}\right|_{p} = \left.\frac{\partial h}{\partial \rho}\right|_{T}, \tag{43}$$

$$\left.\frac{\partial h}{\partial p}\right|_{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho} = \left.\frac{\partial h}{\partial T}\right|_{\rho}. \tag{44}$$

Expressing coefficients $\left.\frac{\partial h}{\partial p}\right|_{\rho}$ and $\left.\frac{\partial h}{\partial \rho}\right|_{p}$ from relations (43) and (44)

$$\left.\frac{\partial h}{\partial p}\right|_{\rho} = \frac{\left.\frac{\partial h}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}}, \tag{45}$$

$$\left.\frac{\partial h}{\partial \rho}\right|_{p} = \left.\frac{\partial h}{\partial \rho}\right|_{T} - \frac{\left.\frac{\partial h}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}} \left.\frac{\partial p}{\partial \rho}\right|_{T}, \tag{46}$$

and substituting them into equation (37) we get

$$\frac{\left.\frac{\partial h}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}} + \frac{1}{w^2}\left(\left.\frac{\partial h}{\partial \rho}\right|_{T} - \frac{\left.\frac{\partial h}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}} \left.\frac{\partial p}{\partial \rho}\right|_{T}\right) - \frac{1}{\rho} = 0 \tag{47}$$

.

Write the first law of thermodynamics in the

$$dh - \frac{dp}{\rho} = Tds \tag{48}$$

form and using relations (39), (40) and (41), we can write

$$\left.\frac{\partial h}{\partial \rho}\right|_{T} d\rho + \left.\frac{\partial h}{\partial T}\right|_{\rho} dT - \frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T} d\rho - \frac{1}{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho} dT = T\left.\frac{\partial s}{\partial \rho}\right|_{T} d\rho + T\left.\frac{\partial s}{\partial T}\right|_{\rho} dT \tag{49}$$

.

Equation (49) must be fulfilled at each dρ and dT, therefore

$$\left.\frac{\partial h}{\partial \rho}\right|_{T} - \frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T} = T\left.\frac{\partial s}{\partial \rho}\right|_{T}, \tag{50}$$

$$\left.\frac{\partial h}{\partial T}\right|_{\rho} - \frac{1}{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho} = T\left.\frac{\partial s}{\partial T}\right|_{\rho}. \tag{51}$$

.

Expressing partial derivatives $\left.\frac{\partial h}{\partial \rho}\right|_{T}$ and $\left.\frac{\partial h}{\partial T}\right|_{\rho}$ from equations (50) and (51):

$$\left.\frac{\partial h}{\partial \rho}\right|_{T} = T\left.\frac{\partial s}{\partial \rho}\right|_{T} + \frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T}, \tag{52}$$

$$\left.\frac{\partial h}{\partial T}\right|_{\rho} = T\left.\frac{\partial s}{\partial T}\right|_{\rho} + \frac{1}{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho}, \tag{53}$$

and substituting them into equation (47)

$$\frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho} + \frac{1}{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}} + \frac{1}{w^2}\left(T\left.\frac{\partial s}{\partial \rho}\right|_{T} + \frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T} - \frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho} + \frac{1}{\rho}\left.\frac{\partial p}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}} \left.\frac{\partial p}{\partial \rho}\right|_{T}\right) - \frac{1}{\rho} = 0 \tag{54}$$

,

$$\frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}}+\frac{1}{\rho}+\frac{1}{w^2}\left(T\left.\frac{\partial s}{\partial \rho}\right|_{T}+\frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T}-\frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}}\left.\frac{\partial p}{\partial \rho}\right|_{T}-\frac{1}{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T}\right)-\frac{1}{\rho}=0 \tag{55}$$

,

$$\frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}}+\frac{1}{w^2}\left(T\left.\frac{\partial s}{\partial \rho}\right|_{T}-\frac{T\left.\frac{\partial s}{\partial T}\right|_{\rho}}{\left.\frac{\partial p}{\partial T}\right|_{\rho}}\left.\frac{\partial p}{\partial \rho}\right|_{T}\right)=0 \tag{56}$$

,

$$\frac{T}{w^2\left.\frac{\partial p}{\partial T}\right|_{\rho}}\underbrace{\left(w^2\left.\frac{\partial s}{\partial T}\right|_{\rho}+\left.\frac{\partial s}{\partial \rho}\right|_{T}\left.\frac{\partial p}{\partial T}\right|_{\rho}-\left.\frac{\partial s}{\partial T}\right|_{\rho}\left.\frac{\partial p}{\partial \rho}\right|_{T}\right)}_{=0}=0 \tag{57}$$

.

It follows that

$$w^2=\left.\frac{\partial p}{\partial \rho}\right|_{T}-\frac{\left.\frac{\partial p}{\partial T}\right|_{\rho}}{\left.\frac{\partial s}{\partial T}\right|_{\rho}}\left.\frac{\partial s}{\partial \rho}\right|_{T}=\left.\frac{\partial p}{\partial \rho}\right|_{s}=a^2 \tag{58}$$

,

i.e.

$$w=a \tag{59}$$

where the speed of sound is $a=\sqrt{\left.\frac{\partial p}{\partial \rho}\right|_{s}}$.

In relation (58) the second equality can be provided as follows. Write down the pressure as the function of entropy and density, i.e. p = p(s, ρ), so in this case

$$dp=\left.\frac{\partial p}{\partial s}\right|_{\rho}ds+\left.\frac{\partial p}{\partial \rho}\right|_{s}d\rho \tag{60}$$

.

Using relations (39) and (41)

$$dp=\left(\left.\frac{\partial p}{\partial s}\right|_{\rho}\left.\frac{\partial s}{\partial \rho}\right|_{T}+\left.\frac{\partial p}{\partial \rho}\right|_{s}\right)d\rho+\left.\frac{\partial p}{\partial s}\right|_{\rho}\left.\frac{\partial s}{\partial T}\right|_{\rho}dT=\left.\frac{\partial p}{\partial \rho}\right|_{T}d\rho+\left.\frac{\partial p}{\partial T}\right|_{\rho}dT \tag{61}$$

,

$$\left.\frac{\partial p}{\partial s}\right|_{\rho}\left.\frac{\partial s}{\partial T}\right|_{\rho}=\left.\frac{\partial p}{\partial T}\right|_{\rho}\Rightarrow\left.\frac{\partial p}{\partial s}\right|_{\rho}=\frac{\left.\frac{\partial p}{\partial T}\right|_{\rho}}{\left.\frac{\partial s}{\partial T}\right|_{\rho}} \tag{62}$$

,

$$\left.\frac{\partial p}{\partial s}\right|_{\rho}\left.\frac{\partial s}{\partial \rho}\right|_{T}+\left.\frac{\partial p}{\partial \rho}\right|_{s}=\left.\frac{\partial p}{\partial \rho}\right|_{T}\Rightarrow\left.\frac{\partial p}{\partial \rho}\right|_{s}=\left.\frac{\partial p}{\partial \rho}\right|_{T}-\left.\frac{\partial p}{\partial s}\right|_{\rho}\left.\frac{\partial s}{\partial \rho}\right|_{T}=\left.\frac{\partial p}{\partial \rho}\right|_{T}-\frac{\left.\frac{\partial p}{\partial T}\right|_{\rho}}{\left.\frac{\partial s}{\partial T}\right|_{\rho}}\left.\frac{\partial s}{\partial \rho}\right|_{T} \tag{63}$$

.

In summary the $\Delta x = 0$ is satisfied if the speed is equal to the speed of sound. In addition, the condition $\Delta x = 0$ at location $x \geq L$ must be met.

When $\Delta x = 0$ (but now let's turn back to differentials and using relation $dx = 0$), then equation (10) is as follows:

$$dw = -\frac{dp}{G}, \tag{64}$$

and relation (11) is

$$dh + wdw = 0. \tag{65}$$

Rearranging it and replacing dh by relation (48) and replacing dw by expression (64) we can write

$$Tds + \frac{dp}{\rho} - \underbrace{\frac{w}{G}}_{\frac{1}{\rho}} dp = 0. \tag{66}$$

It follows that, where $\Delta x = 0$, there the change in the entropy also becomes zero ($ds = 0$), i.e., the entropy reaches its maximum. Entropy is defined by:

$$ds = \frac{\delta q}{T} \tag{67}$$

if the $\delta q \geq 0$, then the $ds \geq 0$, i.e. the entropy cannot be reduced.

If at given mass flow the speed of flow reaches the speed of sound at location $x < L$ (it is not possible), then the capillary is not able to let that mass flow rate flow through, but only a smaller mass flow rate where the speed of flow reaches the speed of sound at location $x = L$. This phenomenon is called throttling.

At the given mass flow rate $\dot{m}$ the pressure p can be calculated where the velocity reaches the speed of sound namely by the iterative solution of relationship

$$\frac{\dot{m}}{A} = G = \rho w = \rho a(\rho, p). \tag{68}$$

If the resulting pressure p is less than the evaporation pressure (p_e), then no throttling occurs. However, if the pressure value is higher than the evaporation pressure, then throttling occurs, which means that the capillary is not capable of transmitting this flow rate. Thus, the flow rate has to be determined (also in an iterative way) so that equality $w = a$ can meet at place $x = L$. The resulting mass flow must redefine the pressure at which the velocity reaches the speed of sound. The iteration has to be carried out, as long as relation (68) and equality $w = a$ simultaneously satisfies at place $x = L$. So it can determine numerically the capillary mass flow rate.

3. Numerical methods

The first step is to determine Δp. To do this, we first need to calculate the pressure value p where at a given mass flow the rate of flow reaches the speed of sound. If

this pressure is lower than the evaporation pressure, then $\Delta p = \frac{p_c - p_e}{n}$, if it is greater, then $\Delta p = \frac{p_c - p}{n}$.

To determine pressure p we must know the speed of sound, as a function of the pressure and the density. The bases of the numerical determination of the speed of sound is given by equation (63). (Details below.)

Now let's suppose that we were able to determine Δp. Thus, the next step is the calculation of Δx based on relation (35).

The determinations of coefficients in equation (35) are the followings.

1. Numerical determination of $\left.\frac{\partial h}{\partial p}\right|_{\rho}$

The bases of the numerical determination of $\left.\frac{\partial h}{\partial p}\right|_{\rho}$ gives equation(45). So we trace back $\left.\frac{\partial h}{\partial p}\right|_{\rho}$ to the numerical determination of $\left.\frac{\partial h}{\partial T}\right|_{\rho}$ and $\left.\frac{\partial p}{\partial T}\right|_{\rho}$.

According to equations (4) and (5) we know the pressure and the enthalpy as a function of the density and temperature. Here we assume the numerical knowledge of these functions.

Based on Figure 6 the numerical partial derivatives can be prepared by the following formulas:

$$\left.\frac{\partial h}{\partial T}\right|_{\rho i} = \frac{h_{i-2} - 8h_{i-1} + 8h_{i+1} - h_{i+2}}{12\Delta T} + \mathrm{O}\left((\Delta T)^4\right) \tag{69}$$

$$\left.\frac{\partial p}{\partial T}\right|_{\rho i} = \frac{p_{i-2} - 8p_{i-1} + 8p_{i+1} - p_{i+2}}{12\Delta T} + \mathrm{O}\left((\Delta T)^4\right) \tag{70}$$

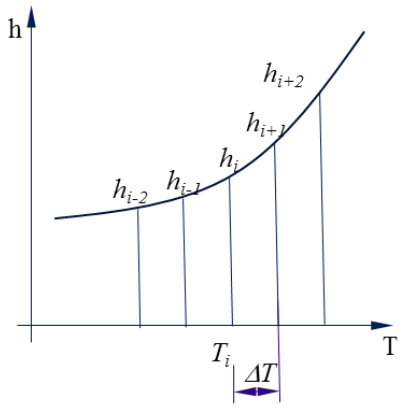


Figure 6. Numerical determination of partial derivatives

Similarly to relationships (69) and (70) we could also set $\left.\frac{\partial h}{\partial p}\right|_{\rho}$, but the pressure and the enthalpy are known as a function of temperature and density so it is appropriate to introduce partial derivatives of these variables.

Figure 7 and Figure 8 show functions $p(T)|_{\rho}$ and $h(T)|_{\rho}$ in case of isobutene (R600a).

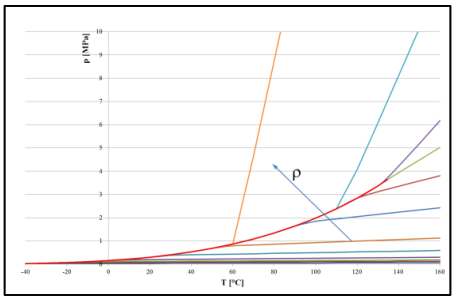


Figure 7. p(T) ρ = Constant

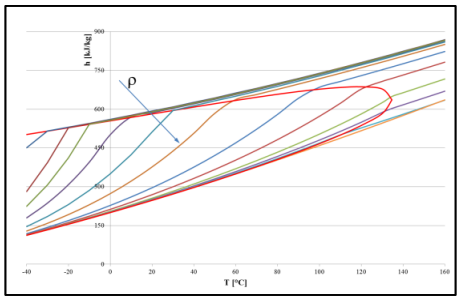


Figure 8. h(T) ρ = Constant

2. Numerical determination of $\left.\frac{\partial h}{\partial \rho}\right|_{p}$

Like in the previous point, here we also trace it back to the partial derivatives of the temperature and density. This tracing back is possible according to relation (46). In addition to the foregoing, here are also the enthalpy and pressure partial derivatives of density at a constant temperature. Numerical determination of them is possible according to the approximate relationships (71, 72). Figure 7 and Figure 8 show the functions $p(\rho)|_{T}$ and $h(\rho)|_{T}$ in case of the isobutene (R600a).

$$\left.\frac{\partial h}{\partial \rho}\right|_{T\,i} = \frac{h_{i-2} - 8h_{i-1} + 8h_{i+1} - h_{i+2}}{12\Delta\rho} + \mathrm{O}\left((\Delta\rho)^4\right) \quad (71)$$

,

$$\left.\frac{\partial p}{\partial \rho}\right|_{T\,i} = \frac{p_{i-2} - 8p_{i-1} + 8p_{i+1} - p_{i+2}}{12\Delta\rho} + \mathrm{O}\left((\Delta\rho)^4\right) \quad (72)$$

.

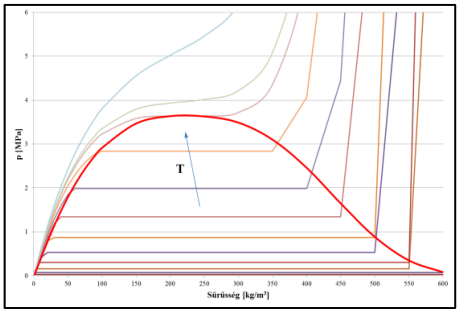


Figure 9. p(ρ) T = Constant

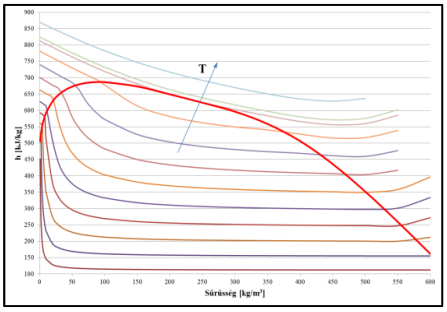


Figure 10. h(ρ) T = Constant

3. λ pipe friction factor is defined in the function of the Reynolds-number.

For two-phase in the calculation of Reynolds-number $\mathrm{Re} = \frac{Dw}{\upsilon} = \frac{DG}{\eta}$ the dynamic viscosity is considered below according to relation [13]

$$\eta = \frac{1}{\frac{x_g}{\eta_g} + \frac{1 - x_g}{\eta_f}} \quad . \tag{73}$$

Depending on the value of the Reynolds number pipe friction factor can be calculated as follows according to [17]:

if $\mathrm{Re} \leq 2320$, then in the context of laminar flow

$$\lambda = \frac{64}{\mathrm{Re}} \quad , \tag{74}$$

if $2320 < \mathrm{Re} \leq 10^5$, then

$$\lambda = 0{,}3164 \cdot \mathrm{Re}^{-0{,}25} \tag{75}$$

according to the Blasius equation,

if $10^5 < \mathrm{Re} < 10^8$, then

$$\lambda = 0{,}0032 + 0{,}221 \cdot \mathrm{Re}^{-0{,}237} \tag{76}$$

according to Nikuradse.

4. Definition of the heat transfer coefficient k

The heat transfer coefficient of the inner cylindrical wall surface has to be calculated according to the following equation [5]:

$$k_b = \frac{1}{\frac{1}{\alpha_b} + \frac{D \cdot \ln\left(\frac{D_k}{D}\right)}{2\lambda_{cső}} + \frac{D}{D_k \alpha_k}} \quad , \tag{77}$$

where $\alpha_k = 1{,}32\left(\frac{\Delta T}{D_k}\right)^{1/4}$, $\alpha_b = \frac{\lambda Nu}{D}$ can be calculated.

For a phase the Nusselt-number is calculated according to the Dittus–Boelter-equation

$$Nu = 0{,}023 \cdot \mathrm{Re}^{0{,}8}\, \mathrm{Pr}^{0{,}3} \quad , \tag{78}$$

$$\mathrm{Pr} = \frac{c_p \eta}{\lambda_h} \quad . \tag{79}$$

For two-phases [7]:

$$Nu = \frac{\alpha_b D}{\lambda_{hf}} = 0{,}026 \cdot \mathrm{Re}_m^{0{,}8}\, \mathrm{Pr}_f^{1/3} \quad , \tag{80}$$

where (m: mixture)

$$\mathrm{Re}_m = \frac{Dw}{\eta_f}\left[\rho_f + \rho_g\left(\frac{\rho_f}{\rho_g}\right)^{1/2}\right]. \tag{81}$$

5. Determination of the partial derivative $\left.\frac{\partial p}{\partial \rho}\right|_s$ for the calculation of the speed of sound

We also trace it back to the partial derivatives of the temperature and density, according to relation

$$\left.\frac{\partial p}{\partial \rho}\right|_s = \left.\frac{\partial p}{\partial \rho}\right|_T - \frac{\left.\frac{\partial p}{\partial T}\right|_\rho}{\left.\frac{\partial s}{\partial T}\right|_\rho}\left.\frac{\partial s}{\partial \rho}\right|_T. \tag{82}$$

$\left.\frac{\partial p}{\partial \rho}\right|_T$ can be calculated based on the (72), and $\left.\frac{\partial p}{\partial T}\right|_\rho$ based on the approximation (70).

Similarly, the missing partial derivatives can be determined using expressions (83, 84) and with the help of Figure 11 and Figure 12.

$$\left.\frac{\partial s}{\partial T}\right|_{\rho i} = \frac{s_{i-2} - 8s_{i-1} + 8s_{i+1} - s_{i+2}}{12\Delta T} + \mathrm{O}\left((\Delta T)^4\right), \tag{83}$$

$$\left.\frac{\partial s}{\partial \rho}\right|_{T i} = \frac{s_{i-2} - 8s_{i-1} + 8s_{i+1} - s_{i+2}}{12\Delta \rho} + \mathrm{O}\left((\Delta \rho)^4\right). \tag{84}$$

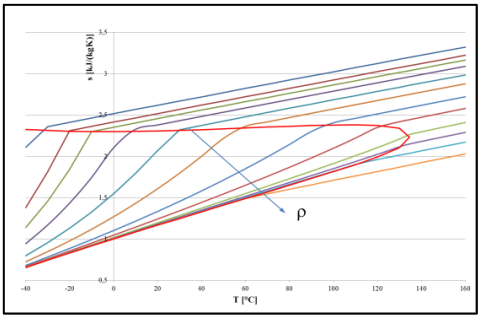


Figure 11. s(T) ρ = Constant

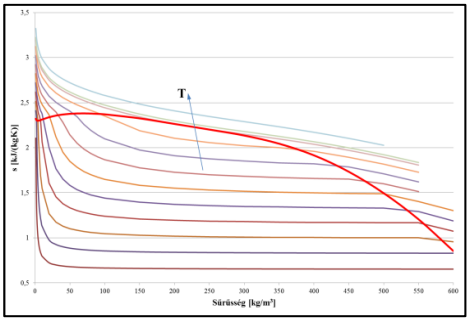


Figure 12. s(ρ) T = Constant

4. Results

Knowing the state surfaces of the fluid according to relation (4), (5) and (6) we can calculate the pressure, the density, the enthalpy and the velocity as a function of location. In addition, we can calculate how much mass flow the given geometry

capillary is capable to transmit through knowing the pressure, density and enthalpy of the entering fluid.

If the fluid is nitrogen, then the capillary airflow rate can be determined by calculation, as well as the length of the capillary needed to achieve the desired airflow rate at a selected inside diameter of the pipe.

Figure 1 shows a theoretical cooling cycle in log(p)-h chart. It is theoretical, because the capillary flow assumes a constant enthalpy (between points 3 and 4) and the compressor presses the refrigerant into the condenser at constant entropy (between points 1 and 2). This study does not deal with the compressor. This study deals with the flow between points 3 and 4, and we are particularly interested in the flow around point 4.

Let's take a look at how to develop the flow in the capillary tube under the following assumptions:

The inner diameter of the capillary is 0,7 mm, its total length (L) is 2800 mm, the length before the heat exchanger (L_1) is 380 mm, the length of the heat exchanger (L_2) is 1330 mm.

The temperature of the refrigerant (R600a) in the condenser is 40°C, and it is a saturated liquid.

The temperature of the fluid in the evaporator is –24°C.

On the length before the heat exchanger (L_1) the capillary tube is in contact with the static ambient air, having a temperature of 25°C.

On the heat exchanger section the air temperature varies linearly between –20°C and +10°C around the capillary.

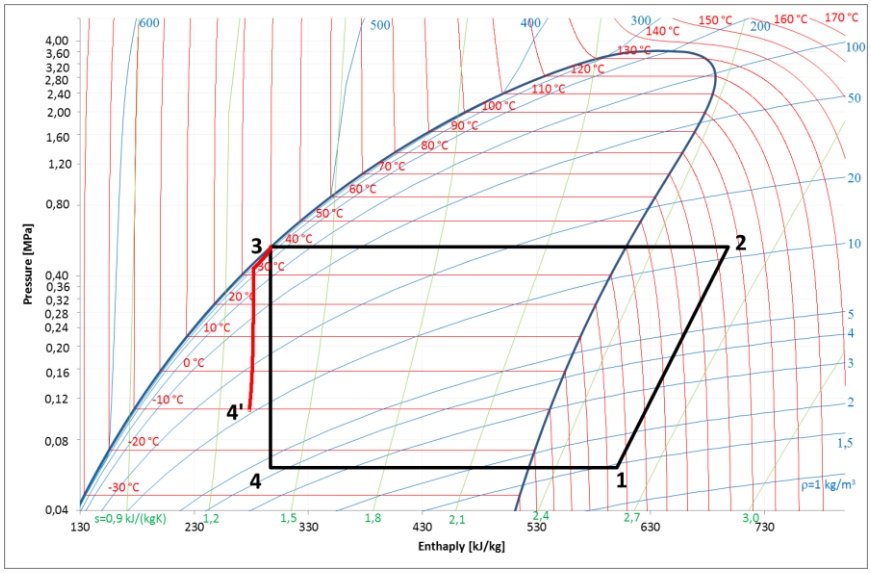


Figure 13. The pressure in capillary as a function of the enthalpy

After the heat exchanger, the capillary is in the insulating foam (PU foam). This section is considered to be adiabatic.

The resulting pressure in the capillary is marked on Figure 13 with a thick red curve (between points 3 and 4) in a log(p)-h chart. It can be seen that the capillary pressure is higher at end than the pressure in the evaporator (point 4). This also means that throttling occurs. The capillary is capable of transmitting a 1,278593 kg/h mass flow under the above conditions. The pressure is 0,106859 MPa at the end of the capillary, which corresponds to a 10,3048°C fluid.

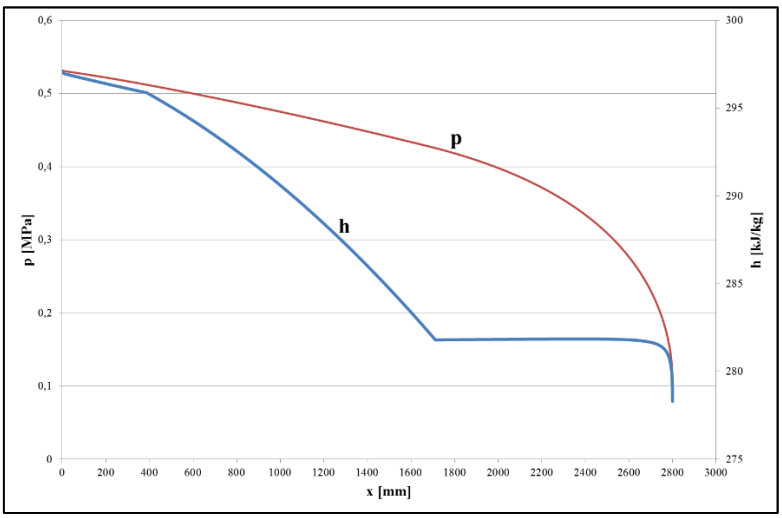


Figure 14. h(x) and p(x) in the capillary

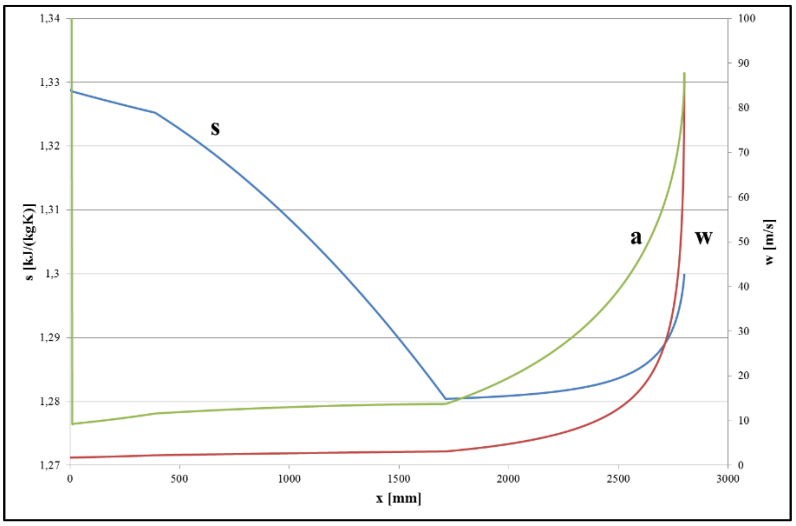


Figure 15. s(x), w(x) and a(x) in the capillary

5. Summary

In this study, I showed the equations that describe the steady state flow in the capillary in a way that no distinction should be made according to whether there is a one or a two-phase flow. This is due to the fact that the enthalpy partial derivatives in the equations are prepared by numerical calculation.

In case of steady state flow I showed the throttling effect on capillary flow, as well as how to calculate (numerically) the size of the unassignable capillary flow, and the enthalpy and density of the fluid which comes into the evaporator. With that I have answered the questions asked in the first part.

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