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THEORETICAL VIBRATION ANALYSIS OF A MANUFACTURING DEVICE

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Abstract: This article explores a certain theoretical method for investigating a large-sized manufacturing device for face-milling rectangular aluminium ingots. First a single DOF (degree of freedom) dynamic model which contains a piecewise linear spring force and damping is created, then the governing equation system is established which is solved by the means of numerical analysis. As a consequence, the theoretically obtained results are compared to the ones gained by the experimental investigations.

Keywords: *analytical dynamics, nonlinear vibration, numerical methods*

1. INTRODUCTION

A theoretical investigation of a large-sized face-milling device is detailed in this article. This machine is applied for the improvement of the surface quality of aluminium ingots manufactured by face-milling. This operation is required by the high demands made on the multi-layered rolled aluminium products from the side of the automotive industry. The remanent surface roughness however, as the result of the face-milling process, often brings forth problems in the forms of bubbles, inclusions during the subsequent multilayered rolling procedure. To eliminate this problem, several technological experiments, investigations and modifications had been carried out and implemented, the surface roughness of the ingots, however, still have not been approved enough, and not always met the requirements. In summary, some 50% improvement of the surface quality had been achieved as the result of the technological investigations.

Besides these improvements, which still have turned out to be unsatisfactory in several cases, investigations have revealed additional concerns. The theoretical surface roughness for example, which comes purely from the geometry of the cutting inserts, the feedrate and the depth of cut, still has not been able to be achieved, hence some other sources of the surface errors need to be searched for, which might come from the dynamic properties of the manufacturing device.

The face-milling device and the aluminium ingot are depicted in *Figure 1*, and clearly seen that a portal-type milling machine is investigated. Face-milling of the aluminium ingot *(1)* is performed by the milling disk *(2)*, and the feeding motion is perpendicular to the drawing plane and takes place between the columns.

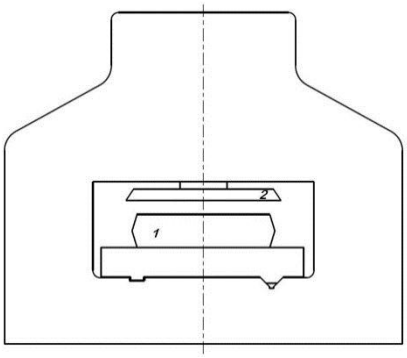


Figure 1. The portal-type face-milling machine

2. THE OBJECTIVE AND THE METHOD OF THE THEORETICAL INVESTIGATIONS

The main objective of the investigations on one hand is to reveal the dynamic properties of the milling machine, focusing on the possible sources of vibration which might affect the surface quality and, on the other hand, the validation of the experimental results obtained by the instrumental vibration analysis.

First of all the *1*-DOF vibroimpact-type dynamic model of the device is created, then the piecewise linear governing equations are established which then are solved with the means of numerical analysis (R-K 4th-order) and, finally, the analytical results are compared to the profilograms displaying the surface roughness of the ingot and had been recorded after the face-milling process.

3. THEORETICAL ANALYSIS

A single-DOF, periodically excited vibro-impact model is considered, which contains some additional features, such as a gap of y_0, a mass point of m, and a piecewise linear spring [1] and dampings. This model is based on some results of a former experimental investigation where the gap of $y_0 = 0{,}4\ mm$ had been detected within the ball bearings and which is assumed to be the source of the axial vibration of the spindle-milling disk assembly. Due to our assumption, this gap might play an important role in the inappropriate surface roughness of the ingot. This article is supposed to demonstrate our "a priori" assumption.

3.1. The dynamic model

During the face-milling procedure, the axis of the spindle – milling-disk unit is bevelled to the face of the ingot to be manufactured in order to avoid the

manufactured surface from the returning cutting edges. As a result of this bevelled position is seen in *Figure 2*, and, since the stiffness of the milling-disk is much less than that of the spindle and acts like a spring, the cutting edges, due to the stimulation of the cutting procedure, will perform vibrations which influences the expected theoretical surface roughness by making it more irregular and, in some cases, unacceptably wavy. This suggests that the insufficient stiffness of the milling-disk and the axial gap within the rolling bearings which induce the axial vibration of the spindle unit, are of great importance in respect with the irregular surface quality of the ingot. This theoretical assumption is going to be demonstrated in the followings. The dynamic model applied for the investigations is depicted in *Figure 2*.

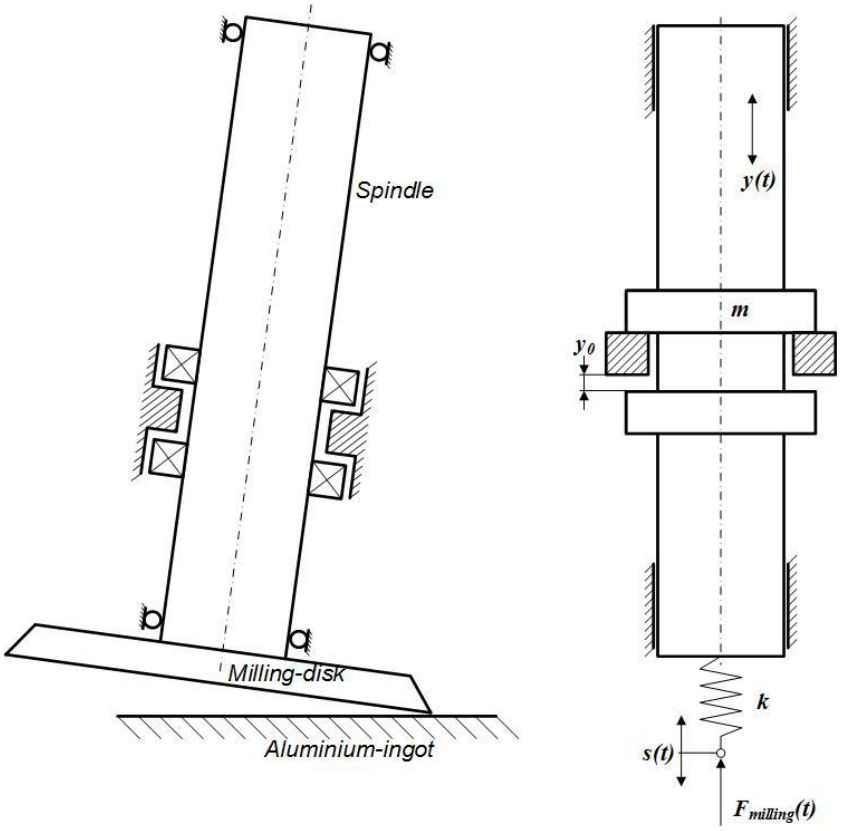


Figure 2. The dynamic model of the spindle – milling-disk assembly

During the analysis the spindle is assumed to be a rigid body with the mass of m, while the milling-disk is considered to be a massless spring having the stiffness of k and which is connected to the spindle at one end, and periodically excited by the cutting procedure $\left(F_{milling}(t)\right)$ at the other end. The vibration is assumed to be

uniaxial, and performed along only the Y-axis, and the maximal displacement of the mass is y_0.

Two different types of damping are considered during the analysis: the material damping on one hand, which is acting when the spindle impacts into the walls of the gap and, on the other hand, the Coulomb-type damping, which comes from the friction between the sliding parts.

3.2. The mass and the spring elements of the model

The main conclusion of this investigation is the displacement of the point of the cutting edge in cut vs. time, which function enables us to draw another conclusion on the surface quality of the ingot. The considered point is the acting point of the stimulating force coming from the cutting procedure. The numerical values of the above mentioned parameters had been established by the former technological experiments, the drawing documents from the side of the company, and by estimations based on the literature. First of all the 3D assembly model of the spindle – milling-disk unit *(Figure 3)* was created by which the mass of the entire unit could be calculated yielding $m = 6000\ kg$.



Figure 3. The 3D CAD model of the spindle – milling-disk assembly

The total weigh of the mass point of the dynamic model *(Figure 2, m)* comes from the sum of the masses of the spindle and the milling-disk, hence the mass of the „spring" in the model shall be neglected in the followings. By the 3D model seen in *Figure 3* the stiffness k of the milling-disk, which is the spring constant of the model *(Figure 2, k),* has also been calculated by FEM *(Figure 4)*. By the slope of the diagram, the stiffness of the spring is $k \approx 5 \cdot 10^8 \ \frac{N}{m}$.

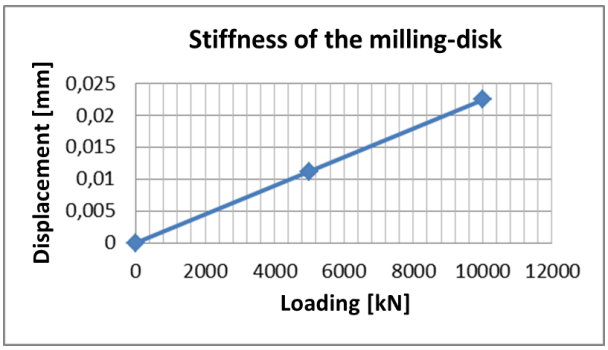


Figure 4. The stiffness (force vs. displacement) curve of the spring-like milling-disk

3.3. The stimulating force

This force is assumed to be periodic and this chapter is exploring a method for establishing the stimulating force vs. time. See *Figure 5* which depicts the geometrical and kinematical conditions of the milling-disk and the ingot at a certain time of the manufacturing process. B, D, n and z are the width of the ingot, the diameter, rotational speed and the number of the cutting edges of the milling disk respectively. Due to the cutting edges cut into the material one after the other, and each cut will act as an impulse-like stimulation the axial component of the cutting force is considered to be periodical.

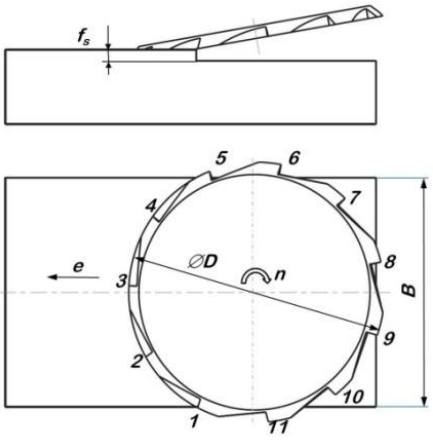


Figure 5. The milling-disk and the ingot

Using the above mentioned parameters, simple expressions can be set up to calculate the rotational time T of the disk, the time τ which is spent in cut by each edge and the time interval ΔT which takes between two neighbouring edges get into cut. In order to be able to establish the expression of the cutting force vs. time, the following numerical values are taken into consideration: $B = 1850\ mm$,

$D = 2100\ mm$, $n = 530\ 1/min$, $z = 11$, by which the unknown time intervals yields $T = \frac{60}{n}$, $\tau = \frac{T}{\pi} arcsin\left(\frac{B}{D}\right)$, $\Delta T = \frac{T}{z}$.

As regards the axial component of the cutting force, which the edges are exposed to, the followings are considered:

- The position is illustrated in *Figure 5* is assumed to be the $t = 0$ initial position, where the cutting edge 1 is about to start to cut the ingot, while the edges 2, 3 and 4 are already in cut;
- The absolute value of the axial component acting on the edges in cut is assumed to be constant while the edge is cutting, $F_0 = 6{,}5\ kN$ [2].

As an example, the axial component of the cutting force acting on edge 2 vs. time is seen in *Figure 6*.

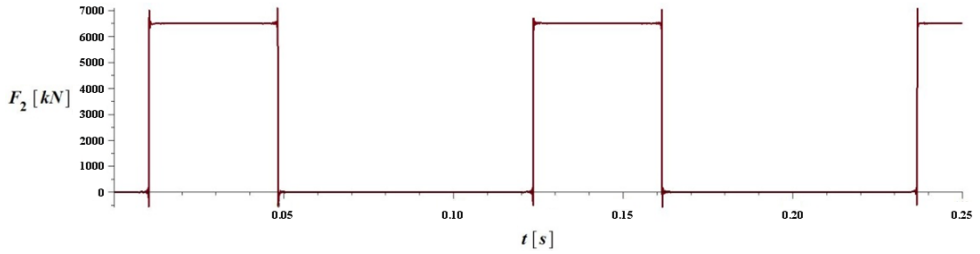


Figure 6. The axial cutting force acting on edge 2 vs. time

As it can be seen in *Figure 6*, the assumed shape of the axial force component acting on each edge in cut is rectangular with the length of τ, height of F_0 and, which is repeated by T. Due to the discontinuity, both the analytical and numerical investigation of this expression is difficult, so the application of the Fourier-series seems to be a reasonable way to overcome the problems of discontinuity. The general finite (including N members) form of the Fourier-series of the axial cutting force acting on the edge i can be written in the form of

$$\tilde{F}_i = F_0 \left\{ \frac{\tau}{T} + 2 \sum_{n=1}^{N} \left[\frac{1}{n\pi} sin\left(\frac{n\pi\tau}{T}\right) cos\left(\frac{2n\pi}{T}\left(t - \frac{\tau}{2} - (i-1)\Delta t\right)\right)\right]\right\} \tag{1.1.}$$

[3]. Summarizing all the $\tilde{F}_i$ components, the total force $F_{milling}(t) = \sum_{i=1}^{11} \tilde{F}_i(t)$ acting on the milling-disk during manufacturing is obtained. The curve vs. time can be seen in *Figure 7*, where $N = 200$ members of the Fourier-series are taken into consideration. Since there are 4 cutting edges in cut at the same time, the amplitude of the assumed cutting force is $26\ kN$.

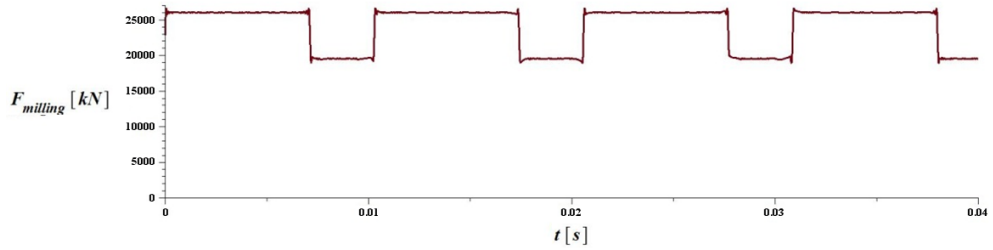


Figure 7. Periodical stimulating force approximated by Fourier-series

As it is mentioned above, besides the material damping, which is assumed to be linear, a piecewise linear damping from the Coulomb-friction is also contained by the dynamic model. The Lehr-type coefficient of the linear material damping is assumed to be $\vartheta = 0{,}0002$ [4]. The absolute value of the Coulomb-force can be estimated by the reaction forces acting at the supports of the main spindle, and can be calculated by setting up the necessary static equations, which are not detailed below. During the analysis, the dynamic coefficient of friction is assumed to be $\mu_d = 0{,}03$ [5] which multiplied by the sum of the radial components of the reaction forces, then yields the absolute value of the friction force, $S = 300\ N$, and which originally comes from the axial relative slip of the main spindle and the rolling element bearings. Thus the Coulomb-type friction is considered in the form of $F_S = S \cdot sgn(\dot{y})$, where sgn denotes the signum-function.

Hence, the

$$\dot{y} = z$$

$$\dot{z} = -\frac{1}{m}\left\{\frac{2\vartheta}{\sqrt{km}}\dot{y} + F_S\, sgn(\dot{y}) + ky + \frac{1}{2}k'\left(|y - y_0| - |y + y_0|\right) + k'y + mg - F_{maró}(t)\right\} \quad (1.2.)$$

governing equations of the dynamic model seen in *Figure 2* can be set up, where the quantity

$$s(t) = \frac{1}{k}\left[m\dot{z} + \frac{2\vartheta}{\sqrt{km}}\dot{y} + F_S\, sgn(\dot{y}) + ky + \frac{1}{2}k'\left(|y - y_0| - |y + y_0|\right) + k'y + mg\right] \quad (1.3.)$$

provides the approximate axial displacement of the edges currently in cut. For the numerical solution of (1.2.) the following parameter values are assumed: the total mass of the spindle and the milling-disk $m = 6000\ kg$, $k = 5 \cdot 10^8\ \frac{N}{m}$ the stiffness off the milling-disk which is regarded as a linear spring, the axial gap within the bearings $y_0 = 0{,}4\ mm$, and $k' = 5 \cdot 10^{12}\ \frac{N}{m}$ the stiffness of the edges which imitate the gap, and whose stiffness is assumed to be 4-order higher than that of the linear spring. Additional assumption is that $y = 0$ is the midpoint of the y_0 gap, thus the maximal displacement of m is $\pm 0{,}2\ mm$. The initial conditions are set up as

follows: $t=0$, $y(0)=-0{,}2\ mm$, $\dot{y}(0)=0$, i.e the spindle is "seated" on the gap when manufacturing about to start.

The numerical solution is obtained with the application of MAPLE 12. Due to some numerical pre-experiments, it was revealed, that from $t=10\ s$ the vibration becomes stationary, thus the investigations are ranged by the time interval of $t\in[10,11]\ s$ *(Figure 8).*

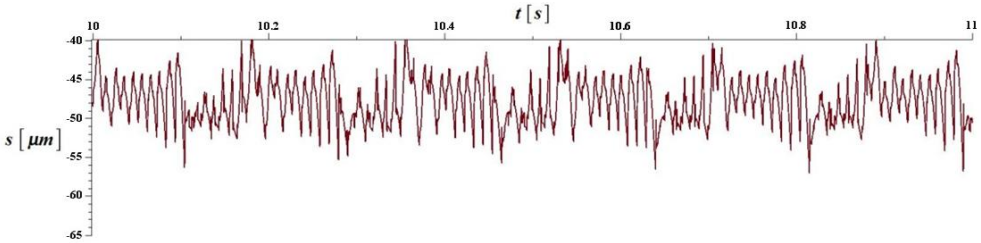


Figure 8. The axial displacement of the cutting edges in cut during the manufacturing

It is clearly seen in *Figure 8* that the cutting edges will perform a modulated vibration, which is very similar to the profilogram recorded over the manufactured surface of the ingot *(Figure 9).*

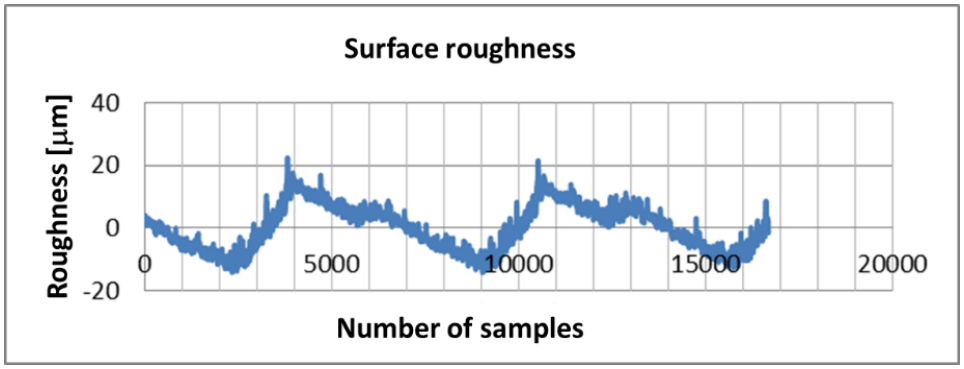


Figure 9. Profilogram registered during the surface roughness detection

It is also visible in *Figure 8*, that the maximum depth of the vibration is roughly $20\ \mu m$, and which is less than that of the profilogram, $30\ \mu m$. The difference might come from the ignorance of some other vibration source, which also active and, even amplified during manufacturing. In spite of this slight difference, the 1 DOF dynamic model provides a very good approximation about one of the main sources of vibration and a good reproduction of the pictogram obtained by the surface roughness measurement of the ingot.

4. SUMMARY

A theoretical investigation of a large-sized manufacturing device is detailed in this article by the means theoretical dynamics and numerical analysis. A 1 DOF dynamic model was created, then the piecewise linear governing equations were set up and, finally, the numerical solution of this equation system was obtained. As a consequence, a good correlation can be seen between the theoretical and the a priori experimental results.

5. ACKNOWLEDGEMENT

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REFERENCES

[1] SHAW, S. W.–HOLMES, J. P.: A periodically forced piecewise linear oscillator. *Journal of Sound and Vibration*, Vol. 90, No. 1 (1983), 129–155.

[2] *Tuskómaró forgácsolási paramétereinek fejlesztése.* Kutatási jelentés, 2009. 10. 07.

[3] FODOR, Gy.: *A Laplace-transzformáció műszaki alkalmazása.* Műszaki Könyvkiadó, Budapest, 1966.

[4] MAKHULT, M.: *Gépalapok rezgéstani méretezése.* KGM Műszaki Tájékoztató és Propaganda Intézet, Budapest, 1962.

[5] MOLNÁR, L.: *Szerszámgépek vezetékei I. Csúszóvezetékek vizsgálata tribológiai rendszerszemléletben.* Oktatási segédlet, Miskolc, 1988.