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CIRCULAR SAW BLADE VIBRATION ANALYSIS OF A RAIL CUTTING SINGLE-PURPOSE MACHINE

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Abstract: This article concerns the dynamical behaviour of a special circular saw blade operated by a rail-cutting machine. The dynamics, the eigenfrequencies and the stimulating frequencies are investigated, hence the vibrations of the circular saw blade of the machine is analysed. In this way we may draw some conclusions on the quality of the cutted surface. Since reworking has extra cost, precise cutting is expected already for the first trial. Vibrations coming from the cutting process may cause inaccurate manufacturing and, besides, may as well influence the lifetime of the saw-blade.

Keywords: *rail cutting, saw blade, vibration, analytical and numerical analysis*

1. INTRODUCTION

Rails are flexible support which made for running railway carriages and exposed to variable, heavy loads. Cutting rails is necessary clue to the following reasons. There are defined standards for the length of the rails; later segments are built in the fields with alominothermic welding. Besides we must use sidings at crossing tracks that allows turning from original way to another track. Using these ones, we have to join segments exceptionally accurately, because one may think that what would happened when a railway carriage with $50\frac{km}{h}$ would gone off the rails because of bad joints. Cutted surface quality is determined by dynamic behaviour and frequences of the saw blade, so eigenfrequences are important to keep prescribed quality.

2. CIRCULAR SAW BLADE FOR CUTTING RAILS

We have to use suitable tool for cutting which is able to cut the special Mn-alloyed steel and it has proportional cutting speed during processing due to the manufacturing time reduction. A saw blade material is expected to be carbide-tipped and relatively maintainable. HSS steel would be proper, although cutting speed can be increased by employing carbides. However these tools have high costs, we use them for cutting due to the productivity. The tool is a $\varnothing D = 630mm$

diameter segmental circular saw blade for this purpose by *GSP-High Tech Saws (Zborovice).*

2.1. Segmental circular saw blade

This tool composed of chrome-vanadium alloyed steel with hardened steel segments *(Figure 1).* There are teeth on this segments which do the cutting. Segments are fixed with bolts to the steel body. The main advantage of this special saw blade is that it has short maintenance and repair time. It means, when the tool is damaged or it is overuse, replacing the segments with new ones we can use it further and continue cutting process. The number of the teeth is permanent in segments, although we have opportunity to expand application. The hardness of this tool is around $63-65$ HRC.

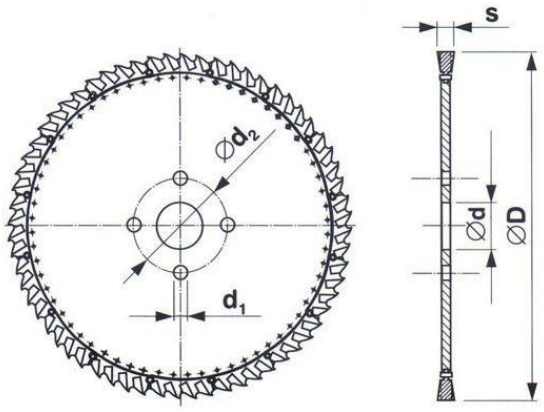


Figure 1. Segmental circular saw blade [1]

For the dynamic analysis and establishing the stimulating frequences, we apply the following details offered by the said company. $\varnothing D = 630mm$, $s = 6mm$, $\varnothing d_1 = 20mm$, $\varnothing d_2 = 160mm$.

3. ANALYTICAL VIBRATION ANALYSIS OF THE TOOL

First we create the saw blade dynamic model which is a circular disk. We constrained by its centerhole. We deal with such plane disk, whose thickness is much smaller the other two dimensions and it has permanent thickness, too.

3.1. The bending vibrations of a circular disk [2][3][4]

The general differential equation of plates in case of static load and small displacements is

$$\nabla^2\nabla^2\zeta = \frac{p}{D}, \tag{1}$$

where ζ is the transverse displacement of the disk, p is the external pressure, $D = \frac{E \cdot h^3}{12(1-\nu^2)}$ is the flexural stiffness and h is thickness of disk.

In *Equation (1)* differential operator means

$$\nabla^2\nabla^2\zeta = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}) = \frac{\partial^4\zeta}{\partial x^4} + 2\frac{\partial^4\zeta}{\partial x^2\partial y^2} + \frac{\partial^4\zeta}{\partial y^4}, \tag{2}$$

where x, y and z are the cartesian coordinates.

The part of inertial force on interface unit is

$$p_1 = -\rho h\frac{\partial^2\zeta}{\partial t^2}, \tag{3}$$

so disk's equation of motion takes the form of

$$\nabla^2\nabla^2\zeta + \frac{12\rho(1-\nu^2)}{Eh^2}\frac{\partial^2\zeta}{\partial t^2} = \frac{p}{D}\ . \tag{4}$$

During vibration the points of x, y coordinates on the surface of the disk perform displacement w at right angles to the disk plane. When $p = 0$, motion equation is

$$\nabla^2\nabla^2\zeta + \frac{12\rho(1-\nu^2)}{Eh^2}\frac{\partial^2\zeta}{\partial t^2} = 0 \tag{5}$$

and whose solution is

$$\zeta(x, y, t) = w(x, y)\cos\alpha t, \tag{6}$$

where α is angular frequency, t is time, x, y are displacement coordinates.

Differential equation of w amplitude function is

$$\nabla^2\nabla^2 w - \kappa^4 w = (\nabla^4 - \kappa^4)w = 0\ , \tag{7}$$

where

$$\kappa^4 = \frac{\rho\alpha^2}{D} = \alpha^2\frac{12\rho(1-\nu^2)}{Eh^2}. \tag{8}$$

After the conversion of (6), the form of

$$(\nabla^2 - \kappa^2)(\nabla^2 + \kappa^2)w = 0 \tag{9}$$

is obtained, so the particular solutions of (9) can be reduced to the following simple differential equation's solutions

$$\begin{matrix}(\nabla^2-\kappa^2)w=0\\(\nabla^2+\kappa^2)w=0\end{matrix}\quad . \tag{10}$$

Although there are infinite number of solutions of (7), we have to choose however those, which correspond to the current boundary conditions.

In the case of clamped parts the transverse displacements and rotation of the clamped surfaces satisfy conditions of (11)

$$\begin{matrix}w=0\\\frac{\partial w}{\partial r}=0\end{matrix}\quad , \tag{11}$$

and on the unconstrained surfaces

$$\begin{matrix}\frac{\partial^2 w}{\partial r^2}+\nu\frac{\partial^2 w}{\partial s^2}=0\\\frac{\partial^3 w}{\partial r^3}+(2-\nu)\frac{\partial^3 w}{\partial r\partial s^2}=0\end{matrix}\quad , \tag{12}$$

where r is the radial, s is the tangential coordinate.

In simple cases we can solve the above mentioned vibration equations exactly, however usually we have to use approximate solutions.

3.2. Circle plates vibrations [2], [3], [4]

To define vibration shapes and eigenfrequencies, the solutions of (7) needs to be set up in polar coordinate system. So the *Laplace operator*'s form in r,φ polar coordinate system is

$$\Delta=\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{\partial^2}{r^2\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}, \tag{13}$$

then the vibration equation takes the form of

$$(\frac{\partial^2}{r^2\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2})(\frac{\partial^2 w}{r^2\partial\varphi^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial r^2})-\kappa^4 w=0\ . \tag{14}$$

Solution of (14) is a vibration with n nodal diameter

$$w=f(r)\cos n\varphi\ . \tag{15}$$

Substituting (15) into (14), we get a normal fourth-order differential equation

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-(\frac{n^2}{r^2}-\kappa^2)\right]\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-(\frac{n^2}{r^2}+\kappa^2)\right]f(r)=0\,. \tag{16}$$

The solution of (16) (which at a place $r=0$ is not infinite) can be received from the following expression

$$f(r)=C_1J_n(\kappa r)+C_2J_n(\kappa r)\,, \tag{17}$$

where J_n is a first kind *Bessel-function* of order n and solutions can be specified depending on the boundary conditions. The C_1 and C_2 coefficients, depending on the boundary conditions, will give the shape modes during vibration. Due to the boundary conditions, a homogenous equation system for these coefficients is yielded. If this equation system's determinant is zero, we can get the frequency equation. In case of $n=1$ the nodal diameter

$$\begin{vmatrix} J_0(\lambda) & Y_0(\lambda) & AI_0(\lambda)-BI_1(\lambda) & -K_0(\lambda)-\dfrac{2(1-\nu)}{\lambda}K_1(\lambda) \\ J_1(\lambda) & Y_1(\lambda) & I_1(\lambda) & -K_1(\lambda) \\ J_0(\alpha\lambda) & Y_0(\alpha\lambda) & I_0(\alpha\lambda) & K_0(\alpha\lambda) \\ J_1(\alpha\lambda) & Y_1(\alpha\lambda) & I_1(\alpha\lambda) & K_1(\alpha\lambda) \end{vmatrix}=0\,, \tag{18}$$

where $A=-1+\dfrac{4(1-\nu)}{\lambda^2}$, $B=\dfrac{8(1-\nu)}{\lambda^3}$, J_n and Y_n are the Bessel functions of the first and second kinds, I_n and K_n are modified Bessel functions of the first and second kinds.

To analyse the circular saw blade we need some of the lowest eigenfrequencies, that can be defined on the basis of the roots of the homogenous equation system (18). To define solution we need previously described frequency parameter, which depends on disk's form, size and mode of supporting

$$\lambda^2=\omega\cdot a^2\cdot\sqrt{\frac{\rho\cdot h}{D}}\,. \tag{19}$$

where ω is circular frequency, a is the radius of the disk.

Knowing this, the following

$$f_{own}=\frac{\alpha}{2\pi} \tag{20}$$

relation between the angular and the natural frequencies can be defined

$$f_{nat}=\frac{\lambda^2}{2\pi\cdot a^2\cdot\sqrt{\dfrac{\rho\cdot h}{D}}}=\frac{\lambda^2}{2\pi\cdot a^2\cdot\sqrt{\dfrac{\rho h}{\dfrac{E\cdot h^3}{12(1-\nu^2)}}}}\,. \tag{21}$$

Based on [3] $\frac{b}{a} = \frac{40mm}{315mm} = 0{,}126984$ thus, in case of $n = 1$ nodal diameter, $\lambda^2 = 4$, as it can be approximated by simple calculations.

n	λ^2 for values of b/a of—							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0	4. 235	5. 244	6. 739	7. 036	13. 05	20. 63	36. 60	81. 45
1	3. 482	4. 814	----------	9. 096	----------	20. 93	----------	45. 09
2	5. 499	6. 345	----------	10. 37	----------	21. 63	----------	67. 65

Figure 2. Frequency parameter values [3]

We consider some most often used vibration modes in practice *(Figure 3).* A detailed calculation for $n = 1$ case is performed in the followings.

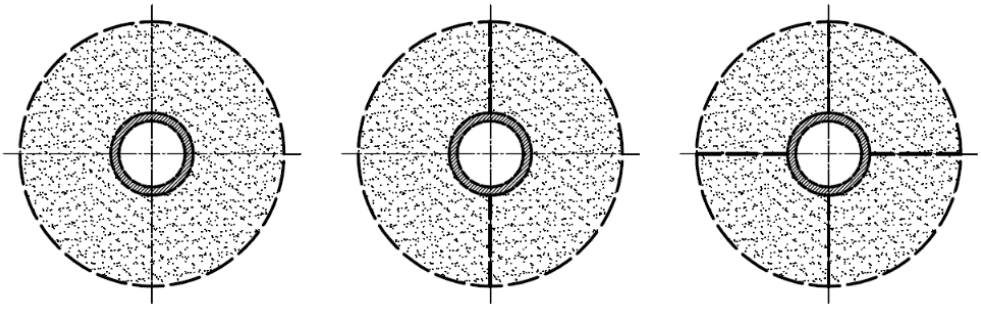


Figure 3. Permanent thickness plane disks vibration pictures [4]

3.3. Establishing one of the natural eigenfrequency of a circular saw blade – analytical method

In this chapter we define eigenfrequence of segmental circular saw blade with previously described method, which is used for analytical dynamic analysis. Based on the following consistencies

$$f_{nat} = \frac{\lambda^2}{2\pi \cdot a^2 \cdot \sqrt{\frac{\rho \cdot h}{D}}} = \frac{\lambda^2}{2\pi \cdot a^2 \cdot \sqrt{\frac{\rho h}{\frac{E \cdot h^3}{12(1-\nu^2)}}}} \tag{22}$$

$$f_{nat} = \frac{4}{2\pi \cdot (0{,}315m)^2 \cdot \sqrt{\frac{7827\frac{kg}{m^3} \cdot (6 \cdot 10^{-3} m)}{\frac{199{,}948 \cdot 10^9 Pa \cdot (6 \cdot 10^{-3} m)^3}{12(1-0{,}27^2)}}}} = 58{,}333Hz$$

3.4. The stimulating frequency of the tool

Due to the toolmaker company's brochure $v_c = 50\frac{m}{\min}$ is the optimal cutting speed for our circular saw blade. The rotational speed is

$$n = \frac{v_c}{r \cdot 2\pi} = \frac{50\frac{m}{\min}}{0,315m \cdot 2\pi} = 25,263\frac{rotations}{\min}, \tag{23}$$

The stimulating frequency while, as regards the number of the cutting teeth $z = 160$ is

$$f_{stimulating} = z \cdot \frac{n}{60} = 160 \cdot \frac{25,263\frac{turn}{\min}}{60} = 67,368Hz\,. \tag{24}$$

Resonance occurs when one of the eigenfrequencies of the saw blade and the stimulating frequency are equal or almost equal. In this case vibrations are formed in the tool and it is vibrating with relatively large amplitude. It may cause inaccuracy on cutted surface or may as well influence the lifetime of the saw blade.

3.5. The critical speed of the circular saw blade

Based on previously described calculations, it is clearly seen, that the danger of resonance is avoided employing by the suggested cutting speed, although the saw blade has a critical speed value, when the eigenfrequency and the stimulating frequency are equal. The

$$n' = \frac{60 \cdot f_{own}}{z} = \frac{60 \cdot 58,333Hz}{160} = 21,875\frac{turn}{\min} \tag{25}$$

$$v_c' = \frac{d \cdot \pi \cdot n'}{1000} = \frac{630mm \cdot \pi \cdot 21,875\frac{turn}{\min}}{1000} = 43,295\frac{m}{\min} \tag{26}$$

cutting speed is not allowed to be applied, since vibrations may be formed. During cutting process speed is much more smaller than the critical speed so we do not need to consider resonance, although at nearby the critical speed we also must not use the saw blade.

In this article we define eigenfrequency and critical speed for one vibrational mode; however there are endless vibration modes for each disc shape that we would have to analyse. Further, vibration modes have higher eigenfrequences, so they differ largely from stimulating frequencies, so danger of resonance decreases. Although it may occurs the stimulating frequence is subharmonic of one higher eigenfrequence which also may causes resonance.

4. VIBRATION ANALYSIS OF THE TOOL WITH AN FEM (FINITE ELEMENT METHOD)

After some analytical results, in this section we are going to analyse the tool with a FEM software. Previously could be observed, when we clamp the saw blade on the middle of the disk, there is slight chance for the resonance at the specified cutting speed. As with the analytical method, we calculate both the natural and stimulate frequencies of the tool. Based on these values, we can be informed about the dynamic behaviour of the circular saw blade and the critical cutting speed of it.

We analyse three different cases with FEM. In first case like previous ones we use the model a disc clamped on the central hole. Diameter of the hole is $\varnothing d = 80mm$. Besides we also analyse a model which has 4 more holes with $\varnothing d_1 = 20mm$ on a $\varnothing d_2 = 160mm$ bolt circle. Due to these holes, dynamic stiffness of the tool is higher since the mass, which could be put into vibration decreases. Finally we analyse the case of the rotating disc. We expect the natural frequencies to increase, which would decrease the risk for the resonance. Although we can solve this case using analytical analysis, it is rather complicated, so we use FEA to solve this problem instead.

The density of disc material in case of steel is $\rho = 7827 \frac{kg}{m^3}$, the Poisson's ratio is denoted by ν with the value of $\nu = 0,27$. Young modulus is $E = 199,948GPa$.

4.1. First case – clamping the tool on the central hole

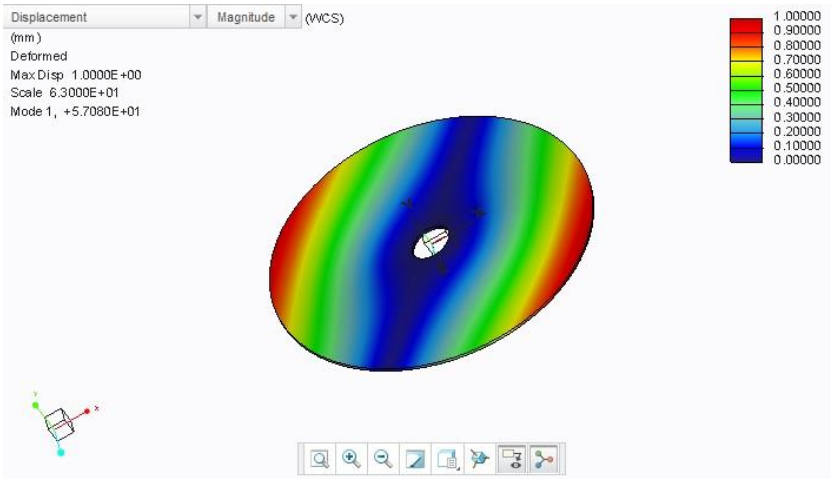


Figure 4. $\kappa = 1$ *shape mode of the saw blade – first case*

In *Figure 4* we can see displacements of the tool and mode 1 vibration picture. It means that there is $n = 1$ nodal diameter and $m = 0$ nodal circles. The lowest eigenfrequency is $57,080Hz$. Previously solved analytical analysis was successful, since eigenfrequency results are close to each other.

4.2. Second case – clamping the tool on the central hole of disc and on more 4 holes along a bolt circle

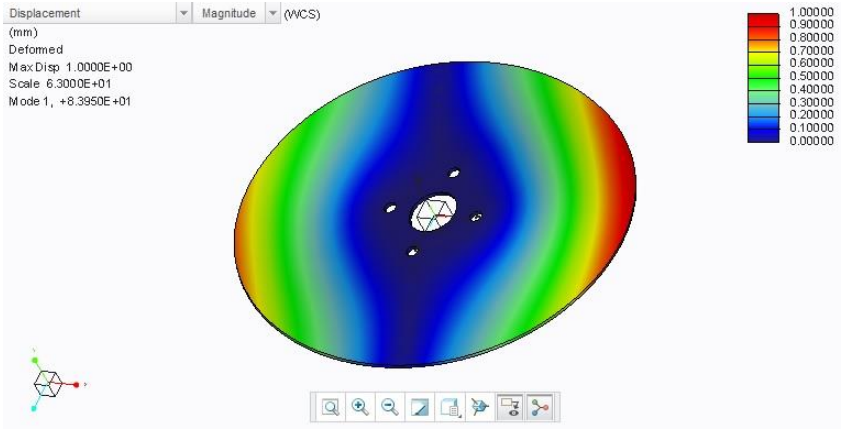


Figure 5 Displacements of the saw blade – second case

In *Figure* 5 we also can see displacements of the tool and the lowest eigen-frequency. In this case it is $83,950 Hz$. It is also mode 1 vibration picture. Due to the 4 more bores, we obtained higher eigenfrequency, so the risk of the danger of resonance is sharply decreased. It means that cut surface is more precise because eigenfrequencies and stimulating frequency are far from each other.

4.3. Third case – clamping the tool like in case 2, but the rotation of the disc is considered

In this case the lowest eigenfrequency is $83,9502 Hz$. As the rotation of disk is low, centrifugal force will not influence the previously calculated eigenfrequency.

5. COMPARISON OF THE ANALYTICAL AND FEA RESULTS

In this article we created a model of circular saw blade and we did analytical analysis and FEA. We have calculated eigenfrequencies and the stimulating frequency. Based on them, we have considered geometries, the dynamical behaviour of the cutting tool and the danger of resonance. Due to the complexity, we solved only one case with analytical method. This case was when we clamped the saw blade by the central hole. This analysis seems to be successful, since eigenfrequency results are in correlation with those of the FEA. Eigenfrequency with analytical method was $58,333 Hz$ and with finite element method was $57,080 Hz$. We analyse more cases with FEM like holding tool with more 4 holes. The result was that eigenfrequency increased, since the tool has clamping higher dynamic stiffness. In the last case our analysis involved the rotating disc. The influence of the centrifugal force however, does not seem to be significated at such a low rotational speed.

6. SUMMARY

In this article was performed analytical and FEA analysis of a special circular saw blade of a rail cutting machine. We performed optimal tool for cutting rails and their features and we created approximately dynamic model for this. Based on vibration consistencies, the saw blade as a circular plate, we calculated the natural and the stimulating frequencies. We explored that vibration shape where resonance may occur most likely at suggested cutting speed. Finally we calculated that – for the smallest eigenfrequency – critical speed where vibrations with large amplitude would occur in the tool. In this case there may occur inaccuracy, form-and size problems on cutted surface. We should avoid this case all time.

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