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ON THE SOLUTION OF THE TAU-TRANSFORMATION USING MONTE CARLO METHOD: AN APPLICATION FOR A POLE-DIPOLE TDIP DATASET MEASURED IN MONGOLIA

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Abstract: Geophysical data processing in time-domain induced polarization data (TDIP) has proven to be an important and effective tool in ore exploration. The recovered images from the induced polarization survey are interpreted by geologists in order to understand the near-surface geological structures and to guide further exploration activities such as the optimal determination of well locations. However, this does not always provide an unique near-surface image that reliably reflects the structural and physical properties of the target due to many factors. The aim of our study is to investigate the combination of the TAU transformation and the Monte Carlo method in the analysis of induced polarization (IP) data. We also developed a stable algorithm to solve TAU transformation through random generation based on the Monte Carlo method. In this solution, the algorithm of TAU transformation has been extended by the Monte Carlo technique to define the spectral amplitude (W) and time constant (TAU) of IP components. Weighted Amplitude Value (WAV) was also calculated based on this solution. This algorithm is useful for the quick processing of TDIP and may be an aid to accuracy improvement of the interpretation of geoelectric survey data in ore exploration.

Keywords: *Time Domain Induced Polarization (TDIP), TAU transformation, Monte Carlo method, spectral amplitude, time constant*

1. INTRODUCTION

The phenomenon of induced polarization [1] was first reported by Conrad Schlumberger as early as 1913. The induced polarization (IP) method has been used since 1942, having been developed during the Second World War as part of a US Navy project [2]. In the 1950s there was a rapid increase in interest in the IP method in both the mining and petroleum exploration sectors. In the 1970s significant progress was made in instrumentation [3] which also led to the development of the spectral IP (SIP) method, especially leading into the early 1980s [4]. More detailed histories of the IP method have been provided by Collett [5] and Seigel et al. [6]. Induced polarization is a very useful geophysical method not only in ore exploration [7], [8]) but in the detection zones where clay and other chargeable minerals are located within the host rock. IP measurements can be made both in the frequency (FDIP) and the time domain (TDIP). In a broader perspective, our research is more focused on time domain induced polarization. TDIP has many applications, not only in ore exploration – principally of disseminated sulphides – but also in connection with geotechnical engineering and environmental problems [9]. On the other hand, the development of forward- and inverse modelling methods plays an important role in advanced geophysical data processing. Interpretations and subsequent decision-making processes can be done more conveniently and potentially more reliably if they are based on geophysical data analysis. Geophysical data analysis methodology has become an effective and standard tool for interpreting geophysical surveys in ore exploration, groundwater engineering and environmental application. These were developed by many scientists and mathematicians with various datasets and goals [7], [10], [11], [12]. Nevertheless, mathematical techniques may not represent subsurface structures well for many reasons: the non-uniqueness of mathematical solutions, limitations imposed by underlying physics, incomplete data coverage, limited measurement, technical problems, noise in the field data, etc. Despite all of the progress made in the direction of reducing the model uncertainty and enhancing the developing interpretation of the geophysical data processing, there are still some open questions and challenges to be addressed. Those problems directly impact the data processing of time-domain induced polarization (TDIP) [13], [14], [15].

The case study is focused on the geoelectric method development for ore exploration (with special emphasis on the solution of TAU transformation and Monte Carlo method). Our dissection aims to apply a combination of data processing and the Monte Carlo technique to develop new methods of scientific value. The results of the novel method will be tested by using field data sets to examine its validity. A theoretical framework of the TAU transformation was introduced by Endre Turai for the interpretation of IP data [16]. IP measurements are widely used in ore exploration. The TAU transformation was developed to process time-domain induced polarization datasets measured in a Schlumberger electrode array. The successful applicability of the series expansion based inversion method has been confirmed in both in-field application and laboratory measurements [17], [11], [18]. An additional purpose of this paper is to show the practical application of the Monte Carlo solution in a TDIP dataset.

Monte Carlo solutions were first used by geoscientists more than 30 years ago. Since then the method has been applied to a wide range of problems, from the inversion of free oscillation data of the whole Earth structure to studies at the meter-scale lengths encountered in exploration seismology. And, more considered about the development and application of Monte Carlo methods for inverse problems in the geosciences and particularly in geophysics [19]. In this paper, we will study the development of the combination between the Tau transformation and Monte Carlo solution results as well as the calculated WAV (Weighted Amplitude Value). The algorithms have been tested on a field measurement dataset of the Yamaat area. The Yamaat gold deposit is in the Republic of Mongolia, some 110 kilometers to the southwest of the capital city of Ulaanbaatar and about 230 kilometers to the south of the border with Russia, at coordinates 705057 N and 5236792 E on the geographic map.

2. METHODOLOGY

The apparent polarization curves ($\eta_a(t)$) can be constructed from the measured time-domain IP data. The strictly monotonously decreasing function can generally be written as an integral transform of ($w(\tau)$) functions. The forward modeling of the problem was formulated by Turai [16]:

$$\eta_a(t) = \int_o^\infty w(\tau) e^{-t/\tau} d\tau. \tag{1}$$

In *Equation (1)*, t is the time after turning off the excitation current, τ is the time-constant, and $w(\tau)$ is the time-constant spectrum fulfilling

$$\int_o^\infty w(\tau) d\tau = 1. \tag{2}$$

The aim is to determine the time-constant spectrum that contains all of the basic polarization effects and can be obtained with the TAU transformation

$$w(\tau) = TAU[\eta_a(t)]. \tag{3}$$

The TAU transformation is solved as an inverse problem. Since the forward modeling [see *Equation (1)*] is nonlinear, so the inverse problem will be solved by linearization.

In the framework of the Monte Carlo method the main steps are:

1. Discretization, resulting in the following form:

$$\eta_a(t_k) = \sum_{i=1}^{I_{max}} w_i \exp\left(\frac{-t_k}{\tau_i}\right), \tag{4}$$

   where t_k is reference time of the k^{th} polarizability sample, w_i is the amplitude of the i^{th} component, τ_i is the time constant of the i^{th} component, and $\boldsymbol{I_{max}}$ is the number of exponential components.
2. Generating I_{max} random numbers in the interval ($\tau_{min} - \tau_{max}$), we consider these random numbers the values of the time constants ($\tau_{i\,rnd}$).
3. For each $\tau_{i\,rnd}$ value, let an M large number of random numbers be generated in the interval (0-w_{max}). We consider these random numbers as the $w_{i\,rnd}$ amplitudes of the $\tau_{i\,rnd}$ time-constant component.
4. Repeat the above N times from $\tau_{i\,rnd}$ generation. In this way, we perform ***(I_{max} x N x M)*** random generations.
5. For each pair ($\tau_{i\,rnd}$, $w_{i\,rnd}$), calculate the polarizability curve obtained by random generation.

$$\eta_a(t_k)^{n,m}_{rnd} = \sum_{i=1}^{I_{max}} w^{n,m}_{i,rnd} \exp\left(\frac{-t_k}{\tau^n_{i,rnd}}\right), n=1, \ldots, N; m=1, \ldots, M. \tag{5}$$

6. From the randomly generated polarizability curves, select the one with the smallest data distance (D_{min}) from the measured polarizability curve. The time constants and amplitudes of this measured curve give the discrete time constant spectrum by

$$\left(\tau^{D_{min}}_{i,rnd}, w^{D_{min}}_{i,rnd}\right), i = 1, \ldots, I_{max}\ . \tag{6}$$

In the paper, we use the Weighted Amplitude Value (WAV) defined by [12] based on his field measurement experience. Various polarization types are separated by the time constants: filtration and membrane polarization has a time constant of less than 1 second, while the time constants of redox and metallic polarization (which occur in ore deposits) are greater than 1 second. The main components of ore deposits are connected to higher time constants (chemical and metallic components) and similarly, the lower time constant is connected to the non-ore components: water and dispersed clay. Using the simple weighting procedure in *Equation (7)*, the WAV is randomized as

$$WAV(\tau_{i\ rnd}) = \tau_{i\ rnd} * w_{i\ rnd}. \tag{7}$$

The Monte Carlo algorithm with the above steps will be used to interpret the IP data set measured along Line 2 in Mongolia shown in *Figure 1*. In the exploration area, the geoelectrical survey data by the pole-dipole in TDIP belonging to four lines were measured. The main purpose was to define the gold mineralization zone using the method.

3. THE TESTING RESULTS ON FIELD MEASUREMENTS

To test the Monte Carlo solution, first the measured data set along Line_2 is applied. The data set contains apparent polarizability data collected at 1443 measurement points. The IP decay curve was sampled in 20 time reference points:

t_k = (0.28 0.36 0.44 0.52 0.6 0.68 0.76 0.84 0.92 1
1.08 1.16 1.24 1.32 1.4 1.48 1.56 1.64 1.72 1.8) (sec)

along which the values $\eta_a(t_k)$ were measured.

For the sake of simplicity, we determined the time constant data ($\tau_{i,rnd}$) by the random generation in the Monte Carlo solution shown in *Table 1*. The measured apparent polarizability values at the first measurement point were as follows:

$\eta_a(t_k)$= (7.66 6.92 6.18 5.77 5.26 4.64 4.36 4.1 3.87 3.61
3.47 3.35 3.16 3.02 2.85 2.68 2.71 2.56 2.34 2.29) in mV/V.

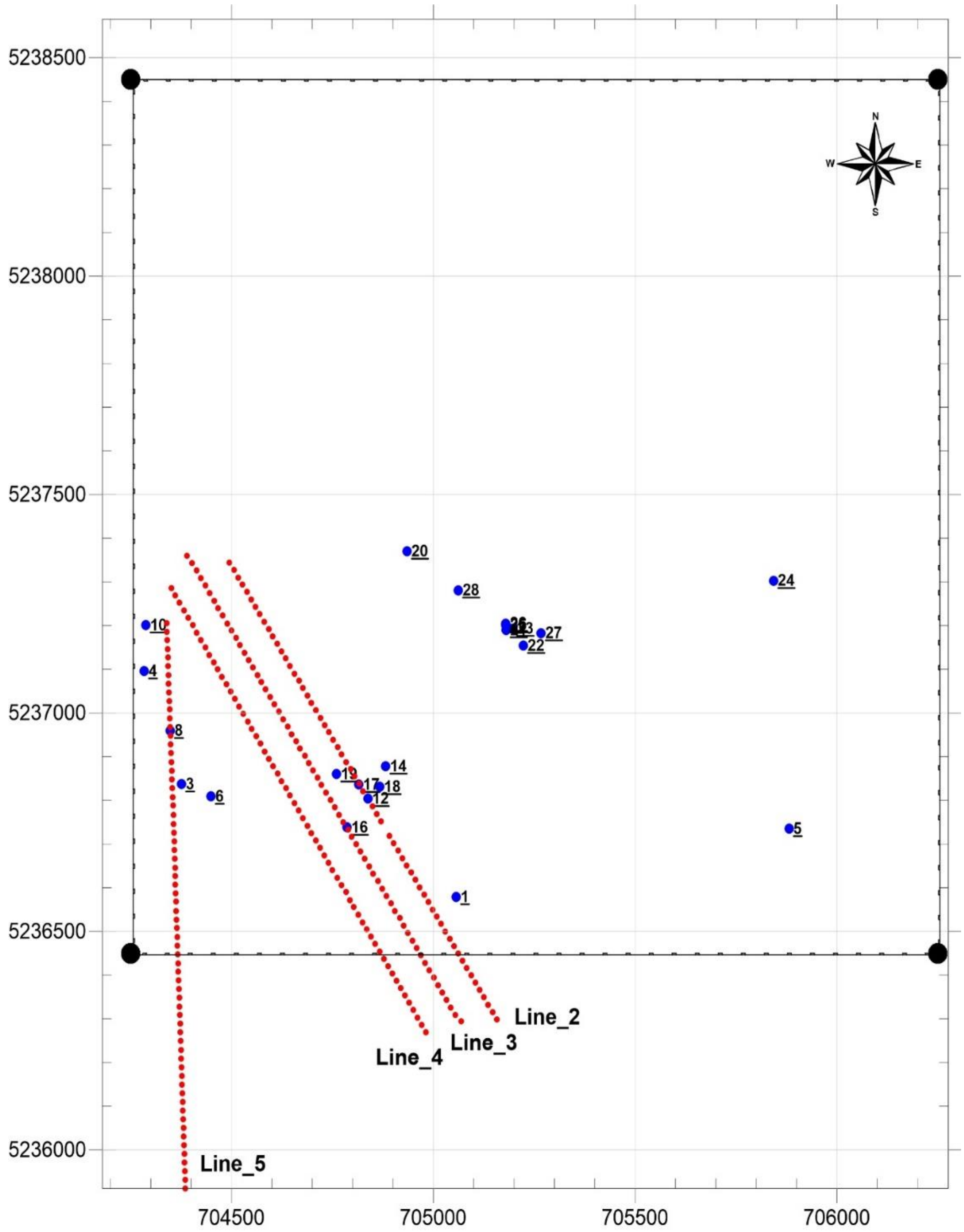


Figure 1

The location map of the Yamaat gold deposit in Mongolia. The red points show pole-dipole profiles from 2 to 4, blue points demonstrate drill points (giving core samples) and the black square is the research area (2 × 2 km).

Table 1

The results of the time constants in seconds (τ_{max} =20 sec)
by the Monte Carlo solution

Serial number of TAU	$I_{max} = 2$	$I_{max} = 3$	$I_{max} = 4$	$I_{max} = 5$	$I_{max} = 6$	$I_{max} = 7$	$I_{max} = 8$	$I_{max} = 9$	$I_{max} = 10$
1	0.95	0.70	0.88	0.19	0.97	0.28	3.93	0.59	0.49
2	5.93	5.74	12.40	0.40	0.98	0.62	6.08	0.76	3.78
3		6.13	14.76	3.20	5.87	1.53	7.60	1.12	5.35
4			15.20	5.58	7.02	4.43	13.97	1.17	7.68
5				8.92	9.20	12.89	14.47	1.99	7.82
6					15.27	13.89	16.48	2.68	8.90
7						17.08	17.44	3.95	10.68
8							17.47	4.44	12.57
9								10.71	13.50
10									14.73

To define the components of the spectral amplitude, for each $(\tau_{i,rnd})$ value, *M* large number of random numbers were generated in the interval (0-w_{max}). The amplitudes $w_{i,rnd}^{D_{min}}$U were randomly generated I_{max} × 1000 × 2000 times. The best fitting values are shown in *Table 2*.

Table 2

The results of the spectral amplitudes in mV/V (w_{max} =10 mV/V)
by the Monte Carlo solution

Serial number of w	$I_{max} = 2$	$I_{max} = 3$	$I_{max} = 4$	$I_{max} = 5$	$I_{max} = 6$	$I_{max} = 7$	$I_{max} = 8$	$I_{max} = 9$	$I_{max} = 10$
1	7.50	7.73	7.57	10.88	0.71	0.06	0.55	1.32	0.27
2	1.38	1.88	0.87	0.45	0.64	0.78	0.23	1.29	2.29
3		0.27	0.49	9.85	0.55	1.44	0.71	0.71	0.46
4			0.06	0.81	0.37	5.44	1.44	0.96	0.12
5				1.59	0.17	1.84	6.63	4.58	1.10
6					6.40	0.08	1.75	0.21	0.31
7						1.42	0.03	0.43	0.10
8							0.99	0.39	3.85
9								0.45	0.83
10									1.07

WAV parameters have been calculated by the combination between the time constants $(\tau_{i,rnd})$ and the spectral amplitudes $(w_{i\ rnd})$ are shown in *Table 3*.

Table 3

The results of the WAV by the Monte Carlo solution

The serial number of the compo-nent	$I_{max} = 2$	$I_{max} = 3$	$I_{max} = 4$	$I_{max} = 5$	$I_{max} = 6$	$I_{max} = 7$	$I_{max} = 8$	$I_{max} = 9$	$I_{max} = 10$
1	7.12	5.42	6.69	2.11	0.69	0.02	2.17	0.78	0.13
2	8.18	10.79	10.83	0.18	0.63	0.49	1.37	0.97	8.68
3		1.65	7.24	31.47	3.24	2.20	5.37	0.79	2.47
4			0.97	4.53	2.58	24.08	20.07	1.13	0.95
5				14.18	1.61	23.69	96.03	9.12	8.60
6					97.68	1.09	28.81	0.56	2.72
7						24.18	0.53	1.70	1.11
8							17.30	1.73	48.34
9								4.84	11.20
10									15.71

Using each pair ($\tau_{i,rnd}, w_{i\,rnd}$), we calculated the polarizability curves and found that the best fit between the measured and calculated data was at 3 components (giving D = 3.86 at $I_{max} = 3$), as is shown in *Figure 2*. We suggesting choosing it on all measured points in the exploration area.

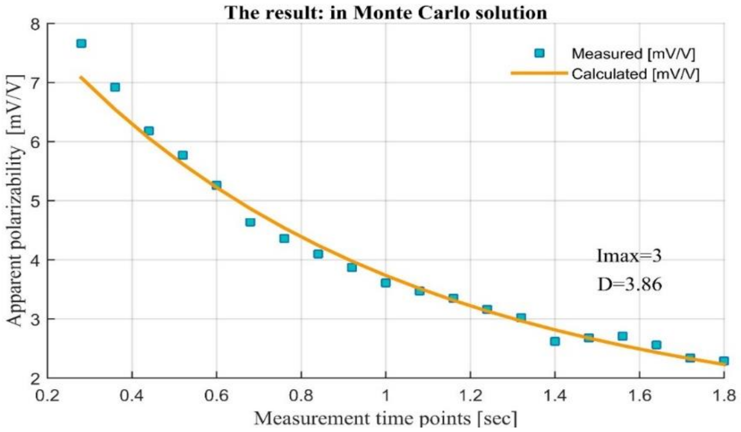


Figure 2

Result of the Monte Carlo solution: $I_{max} = 3$ is the number of components, N = 2000 is the number of the randomly generated time constant, M = 1000 is the number of the random generated w amplitude, $\tau_{max} = 20$ (sec) and $\tau_{min} = 0.01$ (sec) are used generating the time constant, the amplitude range of the random generation is set at $w_{max} = 10$ (mV/V/sec.)

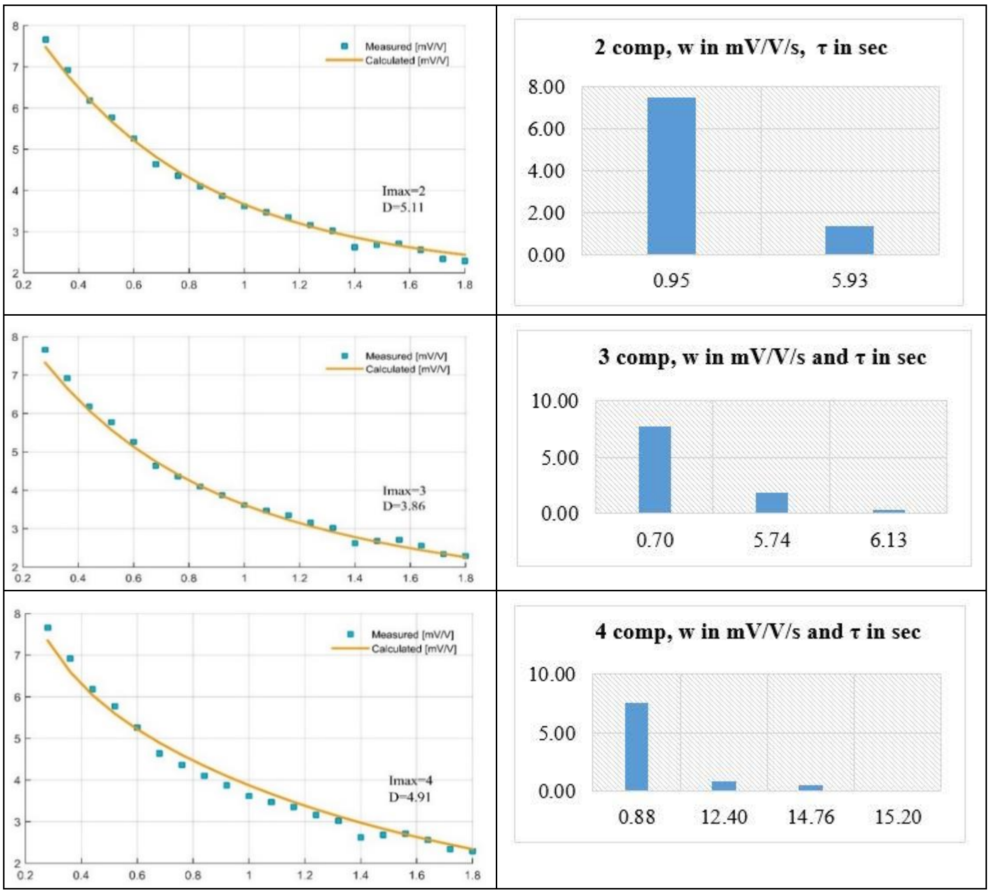
The quality of inversion results is characterized by the distance between the measured and calculated data, as presented in *Table 4*.

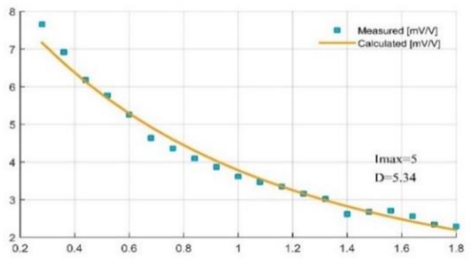
Table 4

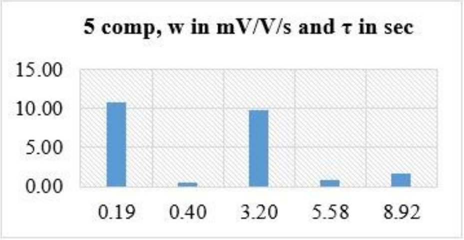
The result of the distance between measured and calculated polarizability data in percentage

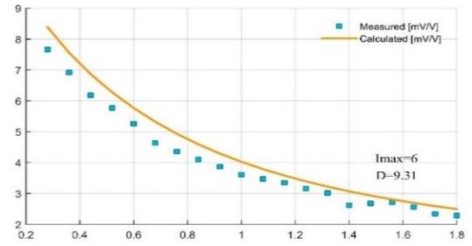
D [%]	$I_{max} = 2$	$I_{max} = 3$	$I_{max} = 4$	$I_{max} = 5$	$I_{max} = 6$	$I_{max} = 7$	$I_{max} = 8$	$I_{max} = 9$	$I_{max} = 10$
	5.118	**3.86**	4.910	5.347	9.310	9.960	16.519	38.560	42.250

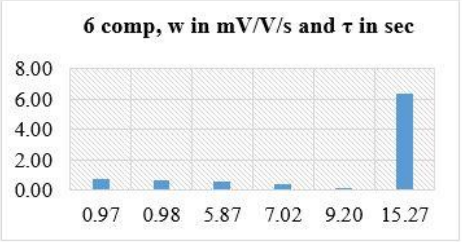
The result of the research is shown in *Figure 3*. The IP decay curves of the frist measurement point were calculated from $I_{max} = 2$ to $I_{max} = 10$, the fit between the measured and the calculated data is D% between $\tau_{min} = 0.01$ sec to $\tau_{max} = 20$ sec and $w_{min} = 0$ mV/V/s to $w_{max} = 10$ mV/V/s. $I_{max} = 3$ is the best number of IP components because here is the minimum error between calculated and measured data. Also $I_{max} = 9$ and $I_{max} = 10$ are deficiently had the higest error in the physical values.

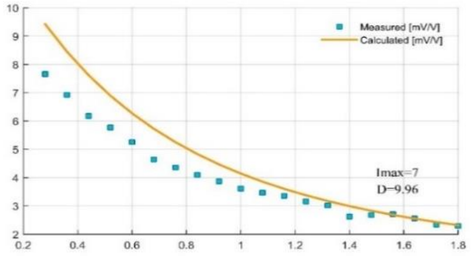


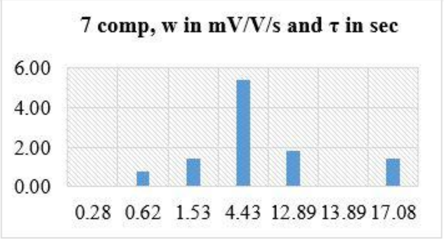

Measured [mV/V]
Calculated [mV/V]
Imax=5
D=5.34
2
3
4
5
6
7
8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

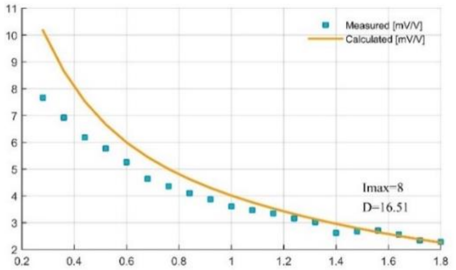

5 comp, w in mV/V/s and τ in sec
15.00
10.00
5.00
0.00
0.19
0.40
3.20
5.58
8.92


Measured [mV/V]
Calculated [mV/V]
Imax=6
D=9.31
2
3
4
5
6
7
8
9
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


6 comp, w in mV/V/s and τ in sec
8.00
6.00
4.00
2.00
0.00
0.97
0.98
5.87
7.02
9.20
15.27


Measured [mV/V]
Calculated [mV/V]
Imax=7
D=9.96
2
3
4
5
6
7
8
9
10
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


7 comp, w in mV/V/s and τ in sec
6.00
4.00
2.00
0.00
0.28
0.62
1.53
4.43
12.89
13.89
17.08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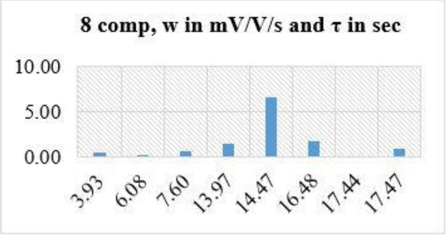
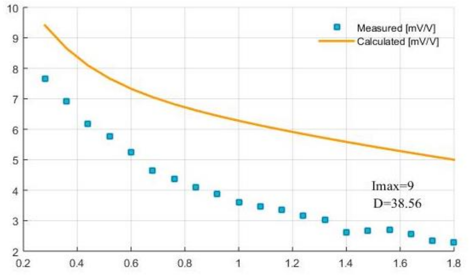
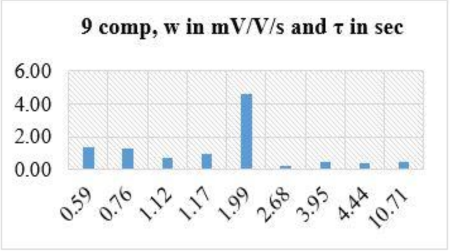
Calculated [mV/V]
Imax=8
D=16.51
2
3
4
5
6
7
8
9
10
11
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


8 comp, w in mV/V/s and τ in sec
10.00
5.00
0.00
3.93
6.08
7.60
13.97
14.47
16.48
17.44
17.47


Measured [mV/V]
Calculated [mV/V]
Imax=9
D=38.56
2
3
4
5
6
7
8
9
10
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


9 comp, w in mV/V/s and τ in sec
6.00
4.00
2.00
0.00
0.59
0.76
1.12
1.17
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2.68
3.95
4.44
10.71

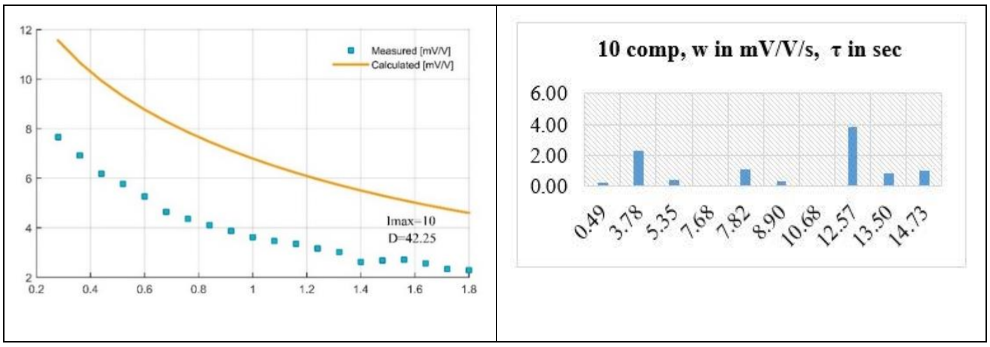

Figure 3

Result of the Monte Carlo solution: on the left are the IP decay curves from I_{max} = 2 to I_{max} = 10, the horizontal axes show measurement time points (sec) and the vertical axes are the apparent polarizability (mV/V), the fit between the measured and the calculated data is D% between τ_{min} = 0.01 sec to τ_{max} = 20 sec and w_{min} = 0 mV/V/s to w_{max} = 10 mV/V/s; on the right are *the time constant spectra from I_{max} = 2 to I_{max} = 10, the horizontal axes are the time constants in sec (TAU), vertical axes are the spectrum amplitudes in mV/V/s (IP components).*

4. CONCLUSION

We investigated the TAU transformation using the Monte Carlo method. As the research has demonstrated, the time constants and spectral amplitudes were successfully determined by this method on the field dataset (TDIP). In this study, the solution was tested on the $\boldsymbol{\tau_{max}}$ = 20 in sec, $\boldsymbol{\tau_{min}}$= 0.01 in sec and $\boldsymbol{w_{max}}$ = 10 in mV/V/s, $\boldsymbol{w_{min}}$ = 0 in mV/V/s was also searched by the random generation with components from I_{max}=2 to I_{max}=10. The results given by the Monte Carlo method, based on the fit between the measured and the calculated data show that I_{max}=3 and I_{max}=4 components are the main polarization with characteristic spectral amplitudes and time constants. For instance, the data distance value at I_{max} = 10 is very high (*D* = 42.25%). However, the value at I_{max} = 3 is the smallest, showing the optimal number of polarization components with the lowest error (*D* = 3.86%).

In the research area, cataclastic granite, diorite porphyry, rhyolite and quaternary sediment rocks were distributed in the lithological information of drillhole 19 those rocks have been very activily altered by silica, silicate, chlorite, and biotite. In the future, we suggest applying the 3 components based on data analysis using by the Monte Carlo solution to two-dimensional interpretation for the TDIP in the Yamaat area.

The use of the Monte Carlo method to perform the TAU transformation was introduced to geoelectric data processing in the time domain IP data set measured in pole-dipole arrays. The WAV parameters were also calculated based on the result of the solution. As a consequence, we suggest applying the investigated solution to data processing for the multichannel array in TDIP. In addition, the results prove the applicability of the method to data analysis in field surveys and may aid the interpretation of induced polarization surveys in ore exploration.

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