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CHEBYSHEV POLYNOMIALS BASED INTERVAL INVERSION APPROACH FOR THE ANALYSIS OF BOREHOLE GEOPHYSICAL DATA: A CASE STUDY FROM EGYPT

MAHMOUD ABDELLATIF[1, 2*] – NORBERT P. SZABÓ[1, 3]

[1]*Department of Geophysics, University of Miskolc*
3515 Miskolc-Egyetemváros, Hungary
[2]*Department of Geology, South Valley University*
83523 Qena, Egypt.
**m.ali@sci.svu.edu.eg*
[3]*MTA-ME Geoengineering Research Group, University of Miskolc*
3515 Miskolc-Egyetemváros, Hungary

Abstract: In the last few years, the interval inversion approach has shown significant success in evaluating and characterizing the hydrocarbon-bearing zones, providing an optimal overdetermination ratio for more accurate and reliable solutions. In this study, the method is used with an alternative basis function to reveal the petrophysical properties of the reservoir rock in Komombo Basin, Upper Egypt. First, the depth-dependent response functions are utilized to formulate the forward problem. Then we expand the petrophysical parameters into a series by using Chebyshev polynomials as a basis function. The Marquardt algorithm is used to solve the inverse problem. Eventually, the petrophysical parameters – which include porosity, clay content, and water saturation in the invaded and uninvaded zone of W. Al Baraka-2 well – are derived from a relatively small number of expansion coefficients. The results are evaluated by computing the errors of the estimated parameters and measuring the misfit between the observed and calculated data. The reliable estimation of the petrophysical parameters assisted in figuring the hydrocarbon potentiality of the reservoir formation in the investigated area.

Keywords. *Interval inversion, Chebyshev polynomials, Petrophysical parameters, Komombo Basin*

1. INTRODUCTION

Borehole geophysics is an aspect of geoscience whose measurements represent a key role in explorations and engineering geophysics. It deals not only with the vertical analysis of subsurface conditions but the fluid content as well. It provides detailed in-situ information about the geometry and physical properties of geological structures [1]. For the oil and gas industry, the evaluation of petrophysical properties of borehole measurements has always been crucial for the assessment of hydrocarbon-bearing formations.

Nowadays, the inversion methods – local and interval inversion – play a vital role in the valuable analysis of borehole data. These methods can be used for characterizing reservoir rock via estimating the petrophysical parameters such as porosity, water saturation in both the uninvaded and the invaded zone, content of clay, and volume of rock matrixes. Regarding the local inversion approach, the model parameters are predicted point by point in a marginally overdetermined inverse problem. Although the method succeeded in setting the unknown model parameters [2], it is rather sensitive to the uncertainty of measured data and limited in its estimation accuracy. Given that the precise computation of hydrocarbon reserves requires the most reliable estimations of the petrophysical parameters by reducing the harmful effect of data noise, the interval inversion method has been introduced [3]. Consecutively, the method was developed by the research work of the inversion and tomography research team of the Department of Geophysics, University of Miskolc and is still of interest. The interval inversion method inverts the data of a relatively long depth interval to reveal the vertical distribution of the unknown model parameters in a joint inversion procedure [4, 5, 6, 7]. In addition, it has been used for automating estimation of layer thickness [8], determining textural parameters and cementation exponents [9; 10], respectively, and evaluating organic-rich shale formations [11]. In addition, a 2D interval inversion procedure is modified for the determination of lateral variation of formation boundaries along with a profile of boreholes together with petrophysical parameters [6].

In mathematics it is known that the most widely used classical orthogonal polynomials are the orthogonal ones, these include the Hermite, Laguerre and Jacobi polynomials. Jacobi polynomials includes Genebauer, Chebyshev, and Legendre polynomials [12]. All the previously mentioned studies for reservoir characterization have been conducted by applying the interval inversion-based Legendre polynomials as a discretization technique with the aim of increasing the vertical resolution in hydrocarbon-bearing formations. In the recent study, we tried to reveal the efficiency of applying Chebyshev polynomials as an alternative basis function for characterizing the reservoir rock in Komombo Basin, Upper Egypt.

2. Methodology

2.1. Interval inversion method

At first, let us formulate the forward modeling for computing the calculated data. In the case of the local inversion the model vector column (**m**) of the model parameters given as

$$\boldsymbol{m} = (\emptyset, S_{x0}, S_w, V_{sh})^T, \quad (1)$$

where $\emptyset$ is the porosity, S_{x0} and S_w are the water saturation in the invaded and uninvaded zones respectively, and the volume of shale is given as V_{sh}.

The volume of sand can be calculated by using the following material balance equation:

$$V_{sh} + V_{sd} + \emptyset = 1, \quad (2)$$

The l-th calculated data ($\boldsymbol{d}_l^{(calc)}$) is obtained by connecting the model vector (**m**) to a set of response functions (g_t) as follows:

$$\boldsymbol{d}_l^{(calc)} = g_t\,(\boldsymbol{m}). \quad (3)$$

The following response functions of [13], [14] and [15] are used to derive the calculated data

$$GR = \rho_b^{-1}(V_{sh}GR_{sh}\rho_{sh} + V_{sd}GR_{sd}\rho_{sd}), \quad (4)$$

$$\frac{1}{Rd} = \frac{\emptyset^2 S_w^2}{aR_w(1-v_{sh})} + \frac{v_{sh}\,s_w}{R_{sh}}, \quad (5)$$

$$\frac{1}{Rs} = \frac{\emptyset^2 S_{x0}^2}{aR_w(1-v_{sh})} + \frac{v_{sh}\,s_{x0}}{R_{sh}}, \quad (6)$$

$$\emptyset_N = \emptyset(\emptyset_{N,mf} - (1-s_{x0})C_{cor} - 2\emptyset(1-s_{x0})S_{hf}(1-2.2\rho_{hc}).$$
$$[1-(1-s_{x0})(1-2.2\rho_{hc})] + V_{sh}\emptyset_{N,sh} + V_{sd}\emptyset_{N,sd}, \quad (7)$$

$$\rho_b = \emptyset\left[\rho_{mf} - 1.07(1-s_{x0})\left(\alpha_0 - \rho_{mf} - 1.24\rho_{hc}\right)\right] + V_{sh}\rho_{sh} + V_{sd}\rho_{sd}. \quad (8)$$

The common parameters represented in *Equations (4)–(8)* such as porosity ($\emptyset$), volumes of sand (V_{sd}) and shale (V_{sh}), water saturation in invaded (S_{x0}) and uninvaded zones (S_w) are the volumetric fractions of the rock's solid and fluid constituents. The physical properties of mud filtrate (*mf*), hydrocarbon (*hc*), shale (*sh*), and sand (*sd*) beside the textural parameters in *Equations* (*5* and *6*) such as cementation exponent (*m*), saturation exponent (*n*) and tortuosity factor (*a*) are treated as constants. These zone parameters can be set by core information, from values in the literature or determined by the interval inversion method [9].

In the interval inversion method, *Equation (3)* is modified to be a depth-independent function for estimating the f-th calculated data ($\boldsymbol{d}_f^{(calc)}$)

$$\boldsymbol{d}_f^{(calc)} = \boldsymbol{d}^{(calc)}(z) = g_t[\emptyset(z), S_{x0}(z), S_w(z), V_{sh}(z)], \quad (9)$$

where z denotes the depth coordinate.

The i-th model parameters in equation (9) are discretized by using a series expansion approach:

$$m_i(z) = \sum_{q=1}^{Q_i} B_q^i \Psi_q(z), \quad (10)$$

where m_i denotes the i-th petrophysical parameter, B_q is the q-th expansion coefficient and Ψ_q is the q-th basis function (up to Q number of additive terms). In our study instead of using Legendre polynomials as a basis function we apply Chebyshev polynomials for approximating the variation of model parameters.

2.2. Chebyshev polynomials

The Chebyshev polynomials are a sequence of orthogonal polynomials that are related to De Moivre's equation. There are four kinds of Chebyshev polynomials, all have the same recurrence relation *Equation (11)* but with different starting polynomials *p1,* where *p1 = z, 2z, 2z – 1, and 2z + 1* for first, second, third, and fourth kinds, respectively. The first and second kinds are more common comparing with the other ones [16].

$$p_0 = 1\,, \quad p_n(z) = 2zp_{n-1}(z) - p_{n-2}(z). \tag{11}$$

In our study we used the Chebyshev polynomials of the first kind which are orthogonal over the interval [1, –1]

$$\int_{-1}^{1} T_m(z)\, T_n(z)\, w(z)\, d(z) = 0 \;\; if\; n \neq m, \tag{12}$$

where w(z) is the weight function and equal to $(1 - z^2)^{-1/2}$.

In terms of (z) the Chebyshev polynomials can be computed by the following recurrence relations

$$T_0\,(z)=1, \tag{13}$$

$$T_1\,(z)= z, \tag{14}$$

$$T_n(z) = 2zT_{n-1}(z) - T_{n-2}(z), \quad n \geq 2. \tag{15}$$

The priority of using first kind of Chebyshev polynomials is that with a large possible managing coefficient its absolute value over the interval [–1 1] is bounded by one, unlike the second kind or in other words the larger the index *n* the more zeros these polynomials have. This is confirmed by illustrating the Chebyshev polynomials of the first and second kind for fourth and up to fourth degree in *Figure 1*. Another advantage of the $T_n(x)$ is that they are orthogonal with respect to the inner product. For more clarification, the polynomials of the first kind are orthogonal to each other, which gives a more valuable property. By using the Chebyshev polynomials of the first kind the series expansion in *Equation (10)* becomes

$$m_i(z) = \sum_{q=1}^{Q_i} B_q^i \mathrm{T}_{q-1}(z). \tag{16}$$

Finally, the inverse problem is solved by the damped least squares method [17] for a small number of expansions coefficients to compute the vertical distribution of the petrophysical parameters

$$\mathbf{B} = \mathbf{G}^{-g}\, \mathbf{d}^{(m)} \tag{17}$$

where $\boldsymbol{G}^{-g}$ is the generalized inverse matrix. The relative data distance between the calculated data and the measured data is minimized to obtain the optimal value of the expansion coefficients

$$D_d = \sqrt{\frac{1}{F}\sum_{k=1}^{F}\left(\frac{d_f^{(m)}-d_f^{(c)}}{d_f^{(m)}}\right)^2} = \min, \tag{18}$$

where F denotes the number of inverted data points, $\boldsymbol{d}_f^{(m)}$, $\boldsymbol{d}_f^{(c)}$ are the f-th measured and calculated data, respectively.

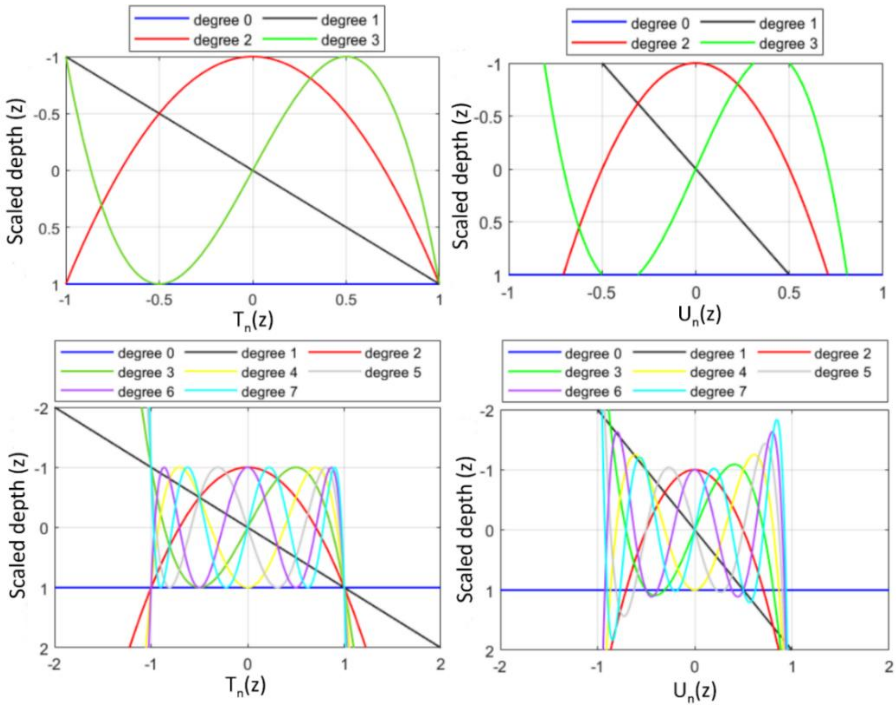

Figure 1. *Chebyshev polynomials of the first T_n (z) and the second U_n (z) kinds for nth degree (n = 0, …, 3) over the interval [–1 1] and nth degree (n = 0, ...,7) over the interval [–2 2].*

2.3. Assessment of the inversion procedure

The interval inversion method permits us to derive petrophysical parameters more precisely provided that the observed data are reliable. The source of errors was studied by [18], who provided an estimation of the uncertainty of different types of borehole data. Several ways have been proposed to quantify the quality of the inversion results. One of these is given by [19] where the covariance matrix of the estimated petrophysical parameters from the linear optimization techniques is related to the covariance matrix of the measured data, including their variances. Unlike the local inversion method, the petrophysical parameters are derived by interval inversion for a small number of expansion coefficients (**B**) thus the covariance matrix of series expansions can be expressed as follow:

$$cov\boldsymbol{B} = \boldsymbol{V} \quad cov(\boldsymbol{d})^m \, \boldsymbol{V}^T, \tag{19}$$

where **V** is the general inverse matrix ($\boldsymbol{G}^{-g}$) of the actual inversion method. Hence, the depth-dependent model covariance matrix of the estimated parameters is used [2]:

$$[cov\boldsymbol{m}(z)]_{ij} = \sum_{n=1}^{Q^{(i)}} \sum_{m=1}^{Q^{(i)}} T_{n-1}(z)(cov\boldsymbol{B})_{hh'} T_{m-1}(z). \tag{20}$$

The error of the estimated parameters is obtained by the main diagonal of the previous covariance matrix

$$\sigma[m_i(z)] = \sqrt{cov\boldsymbol{m}_{ii}(z)}. \tag{21}$$

On the other hand, the estimated parameters from the inversion method are well correlated to each other so the correlation matrix is used to count the strength of correlation between the parameters. In our inverse problem, the correlation matrix of the estimated expansion coefficients [2] is used as follows:

$$(corr\boldsymbol{B})_{ll'} = \frac{(corr\boldsymbol{B})_{ll'}}{\sigma(\boldsymbol{B})_l \sigma(\boldsymbol{B})_{l'}}. \tag{22}$$

Otherwise, the scalar S is used to characterize the correlation matrix $\boldsymbol{B}$ and it is given as

$$S(\boldsymbol{m}) = \sqrt{\frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} (corr_{ij}^{(m)} - \delta_{ij})^2}. \tag{23}$$

where δ is the Kronecker delta.

3. Study area and the inverted dataset

The investigated area is Komombo Basin which lies west of the River Nile in the southern part of Western desert, north of Aswan city. The area ranges from latitudes 24°10′00″ & 24°42′00″ N and longitudes 32°40′00″ & 32°55′00″ E. The Komombo area comprised the first oilfield discovery outside Egypt's conventional producing areas, known as the Al Baraka oilfield. The stratigraphic sequence of the Southern Western Desert has been recorded in depth by several authors [20, 21] and mainly is described in terms of Late Jurassic, Cretaceous, and Paleocene succession. About 20 wells have been drilled in the study area as illustrated in *Figure 2*. In our study, we used the data set of W. Al Baraka-2 well.

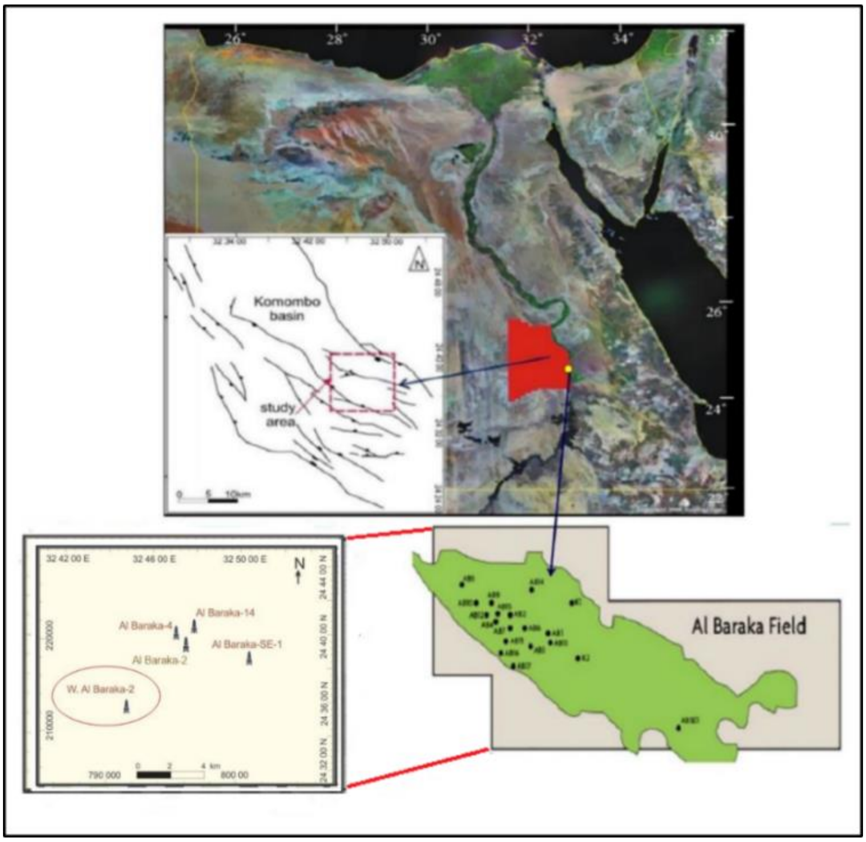

Figure 2. *Location map of the study area indicating the drilled wells*

4. RESULTS

The modified method is utilized for the analysis of the applicable data set of well W. Al Baraka-2 which contains five types of probes. The measured data are natural gamma-ray intensity (GR in API), Shallow resistivity (R_S in ohm-m), Deep resistivity (R_d in ohm-m), Bulk Density (ρ_b in g/cm^3), and Neutron-Porosity ($\emptyset_N$ in v/v). The environmentally corrected data logs are illustrated in *Figure 3* with a depth matched to the range from 0 to 14 m. Since we have five types of logs with a sampling interval of 0.25, the total number of data is 280 points. The four model parameters ($\emptyset$, S_{x0} , S_w and, V_{sh}) are discretized by the orthogonal Chebyshev function, setting the degree of polynomials up to 14. Consequently, the total number of unknowns becomes 60 where $4(Q^* + 1) = 60$, Q^* is the maximum degree of Chebyshev polynomials, thus the ratio of data to unknowns is 4.6.

To obtain more consistent results the uncertainty of the observed data comparably to [18] is quantified by studying the effect of data variance on the solution of the inverse problem. The standard deviations of input data are set as σ_{GR}= 0.05, σ_{RD} = 0.04, σ_{RS} = 0.04, σ_{CN} = 0.03, and σ_{DEN} = 0.03. The confidence intervals of measured log types are illustrated in *Figure 3*.

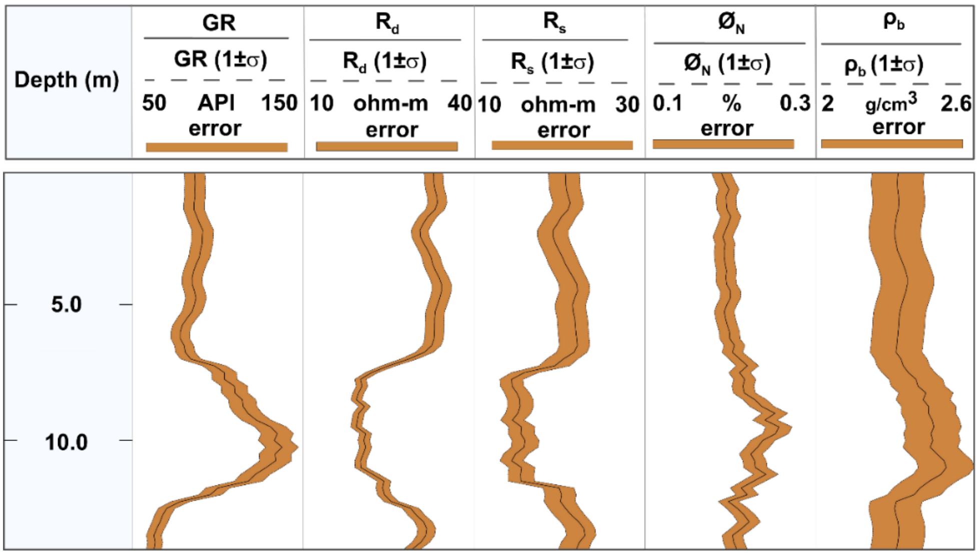

Figure 3. *Input well logs measured in W. Al Baraka-2 well and uncertainty ranges of log readings for the interval inversion procedures*

The data distance between the calculated data and the measured one is computed by representing the convergence plot of the interval inversion method as illustrated in *Figure 4*. Based on these results, the maximum number of iterations during the inversion process is 50. the relative data distance in the zero-th iteration $D_d(0)$ = 71.7 % which is reduced to D_d = 3.8 % in the last iteration. Based on these results we can conclude that, despite the data noise, a steady and rapid convergence to the optimum can be seen which leads to reliable estimation of the petrophysical parameters.

The petrophysical parameters are derived from 60 expansion coefficients which are illustrated with their errors in *Figure 5*. The initial values of the expansion coefficient are set at 0.01 for porosity, 0.5 for water saturation in invaded and uninvaded zones, and 0.3 for the volume of shale.

The resulting petrophysical parameters with their estimated errors are demonstrated in Figure 6. The average values of the estimated parameters of the reservoir rock in the investigated area range between 14–25% for porosity, 36–81% for clay content, 36–100, and 28–100 for water saturation in the invaded zone and water saturation in the uninvaded zone, respectively. The relative errors of the estimated parameters are 0.02 v/v for porosity, 0.1 v/v for water saturation in invaded and uninvaded zones, and 0.03 v/v for the volume of shale.

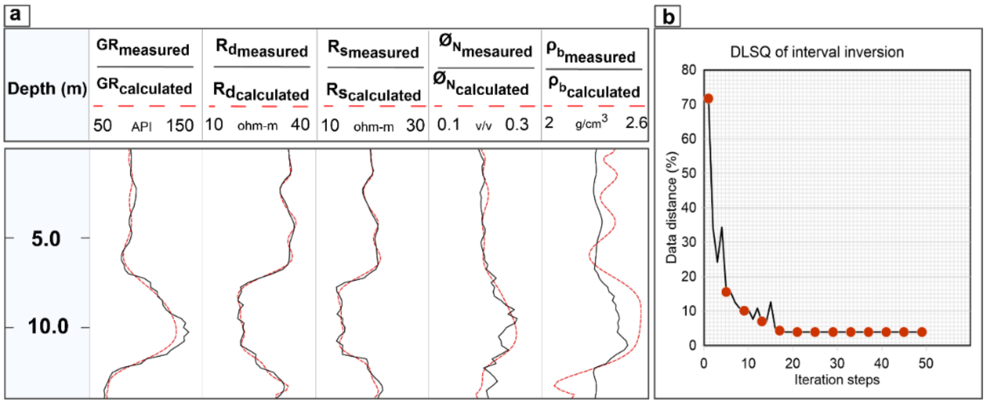

Figure 4. *a) Misfit between the measured and calculated data,*
b) Convergence plot in the subsequent DLSQ inversion procedure

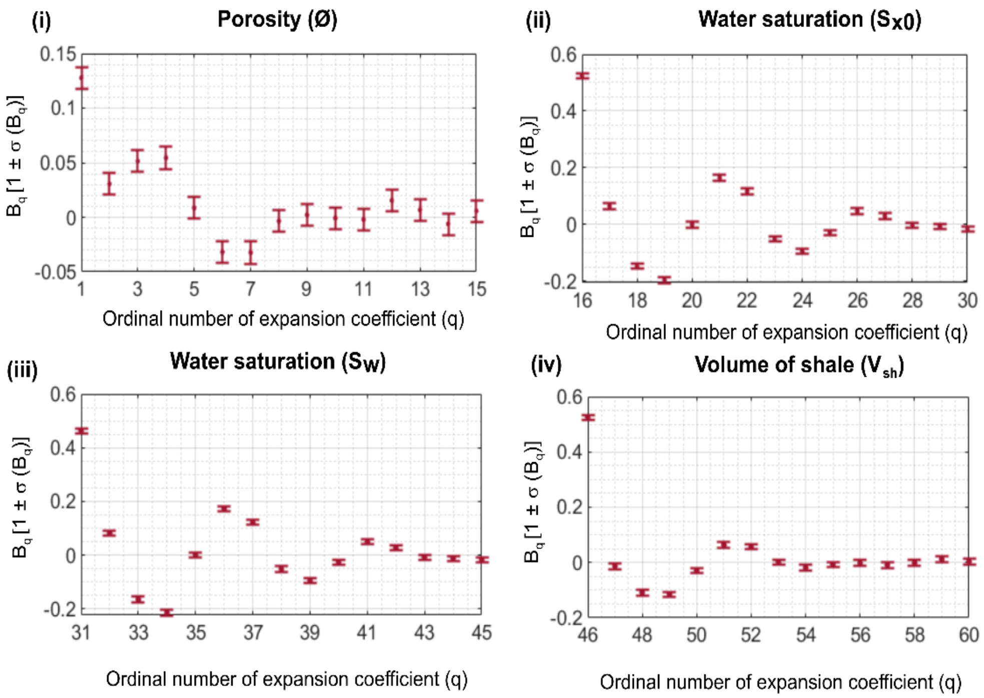

Figure 5*. Results of interval inversion procedure using Chebyshev polynomials of 14 degrees as basis functions in W. Al-baraka well. Estimated values of expansion coefficients for (i) porosity, (ii) water saturation of uninvaded zone, (iii) water saturation of invaded zone, (iv) volume of shale, and their estimation error ranges versus ordinal number of expansion coefficients in the model vector.*

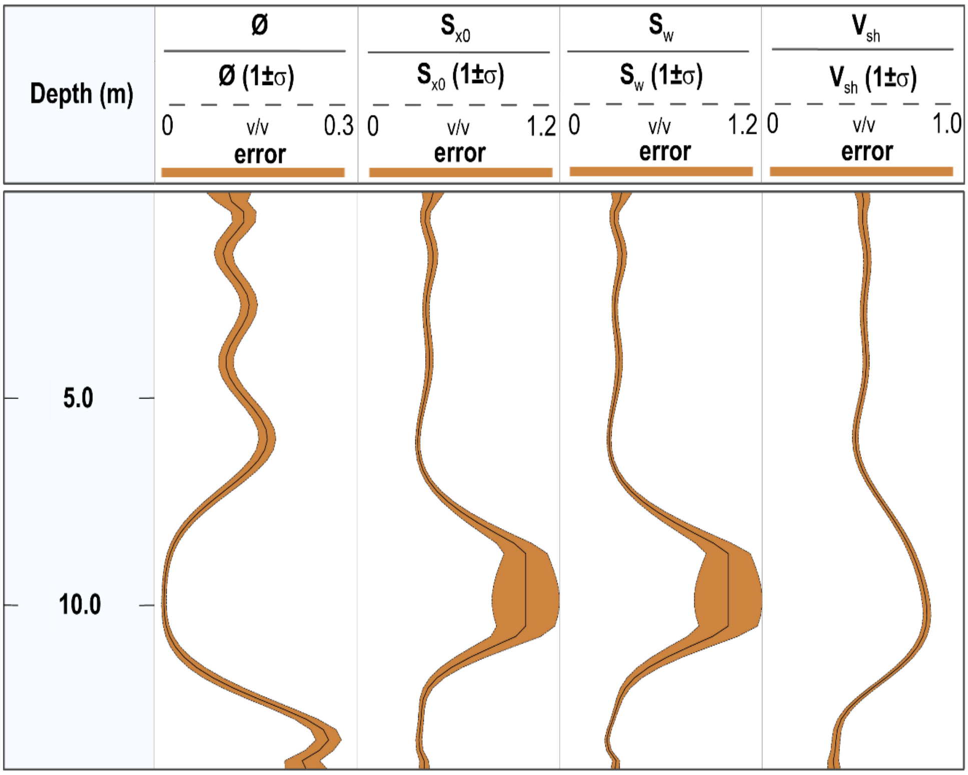

Figure 6. *Well logs of the estimated petrophysical parameters by interval inversion method with their calculated errors*

The correlation between the estimated model parameters is quantified as another check for the quality of our results. The mean value of the correlation coefficients is equal to 0.3 which indicates poorly correlated expansion coefficients and highly accurate results.

The reliable estimation of the petrophysical parameters by the modified interval inversion method helped to quantitatively throw light over the hydrocarbon potentiality. Considerable oil saturation of the reservoir rock is exhibited in the study area. The hydrocarbon saturation is derived by subtracting the estimated values of water saturation from unity. In more detail, the irreducible and movable hydrocarbon saturations are computed using the below equations. The detailed interpretation of the reservoir rock using the modified interval inversion technique is illustrated in *Figure 7*

$$S_{hc,irr} = 1 - S_{x0}, \tag{24}$$

$$S_{hc,mov} = S_{x0} - S_w. \tag{25}$$

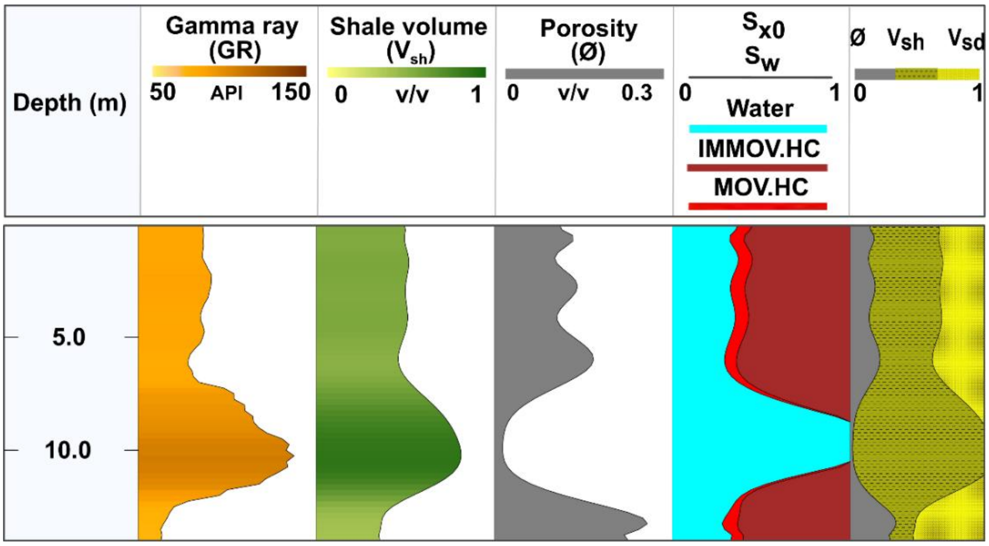

Figure 7. *Interval inversion interpretation plot of the reservoir rock in the investigated area*

5. Conclusions

Interval inversion approach-based Chebyshev polynomials have been used to characterize the reservoir rock in Komombo Basin, upper Egypt. The modified method shows a reliable estimation of the petrophysical parameters such as porosity, water saturation in invaded and uninvaded zones, and the volume of shale. A variety of checking quality techniques have been applied in our study. Furthermore, the estimated parameters allow the calculation of the hydrocarbon saturation in the investigated area.

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