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THE ACCURACY OF DEPTH ESTIMATION IN GPR SURVEYS

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Abstract: The determination of the reflection depth can be very important in the ground penetrating radar (GPR) interpretation. The precise depth estimation is not always necessary but in certain cases, it can be crucial to solve the geological or engineering problem. Being aware of the possible anomaly depth ranges and their errors is even more important. The difference between the observation depth assuming dielectrics and lossy medium is compared in this study. The possible depth estimation error ranges are presented in realistic conditions. Forward modeling examples show the characteristics and differences of the dielectric and of the lossy medium. Parameter sensitivity calculations were performed in order to characterize the effect of slight EM parameter changes on other GPR parameters (e.g. wave propagation velocities).

Keywords: *georadar, reflection depth, modeling, parameter sensitivity*

1. INTRODUCTION

GPR measurements are widely used in detecting various near-surface anomalies. These can be cavities, underground utilities, the water table, etc. Even if the accurate depth estimation is hopeless, the GPR user should know the realistic ranges of wave propagation velocity. Thus, a well determined depth interval of the anomaly can be given.

In GPR practice, the wave propagation velocity (v_d) is usually estimated assuming dielectrics *[Equation (1)]*. This is an appropriate approximation in most cases, but sometimes the equations of the lossy medium *[Equation (2)]* have to be used

$$v_d = \frac{1}{\sqrt{\mu\varepsilon}} \cong \frac{c}{\sqrt{\varepsilon_r}}, \tag{1}$$

$$v_l = \frac{1}{\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}}+1\right)}}, \tag{2}$$

where $\mu \cong \mu_0$ is the absolute magnetic permeability of the medium,
μ_0 is the absolute magnetic permeability of vacuum,
v_d is the wave propagation velocity calculated in the dielectric,
v_l is the wave propagation velocity calculated in lossy media,

ε	is the absolute dielectric constant of the medium,
ε_r	is the relative dielectric constant of the medium with respect to vacuum,
c	is the velocity of light in vacuum,
σ	is the specific electrical conductivity of the medium,
ω	is the angular frequency.

Since the increase of conductivity enhances the attenuation of the electromagnetic (EM) waves, GPR measurements cannot be used entirely efficiently in low-resistivity environment. But with certain limitations and careful data processing, useful information can be gained from media with lower resistivity (e.g. under 100 Ωm) as well. The possible change of magnetic permeability is not discussed in this paper.

2. Theoretical curves for depth estimation in lossy media

Several theoretical curves are published in [1, 2] which deal with the conductivity dependence of different GPR parameters (resolution, skin depth, wavelength, wave propagation velocity, etc.). *Figures 1* and *2* show the connection between the EM parameters and reflection depth.

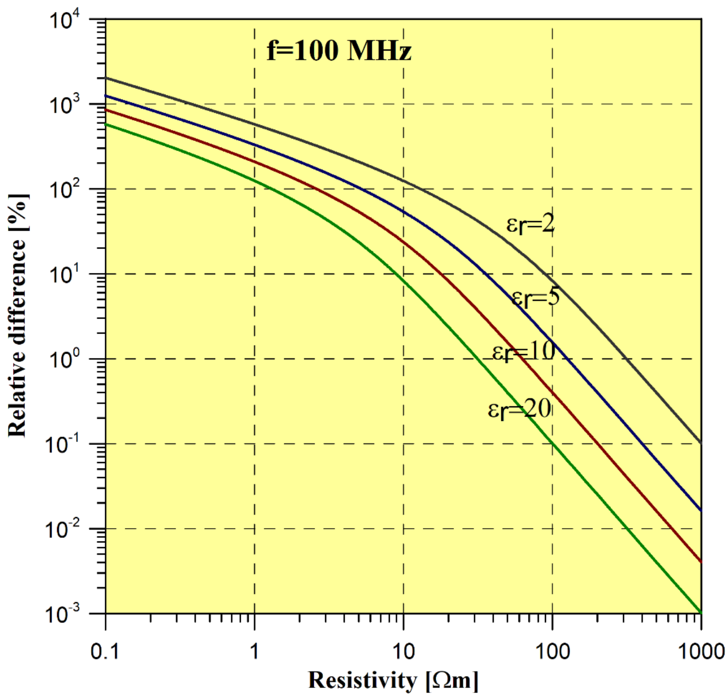

Figure 1

Relative differences between reflection depth values calculating from the dielectrics and the lossy media, as a function of resistivity (f = 100 MHz)

Figure 1 shows the relative difference of calculated reflection depth values between dielectric and lossy media. These differences are the same as if the wave propagation velocities would be expressed *[Equation (3)]*

$$E_d[\%] = \frac{d_d - d_v}{d_v} \cdot 100 = \frac{\frac{v_d \cdot t}{2} - \frac{v_v \cdot t}{2}}{\frac{v_v \cdot t}{2}} \cdot 100 = \frac{v_d - v_v}{v_v} \cdot 100, \quad (3)$$

where E_d is the relative reflection depth difference between dielectric and lossy media,
d_d is the reflection depth calculated in the dielectric,
d_v is the reflection depth calculated in lossy media,
t is the two-way-time.

It can be seen how the differences decrease with increasing resistivity. The velocity and the calculated depth also depend on the relative dielectric constant. The most probable values for relative permittivity are between 5 and 15. The errors are negligible above 100 Ωm resistivity but can be higher than 10% between 100 and 10 Ωm. Calculating with the equation for dielectrics, the reflection depth is overestimated, as well as the wave propagation velocity. The presented curves are calculated for 100 MHz antenna frequency. These relative differences decrease with increasing frequency.

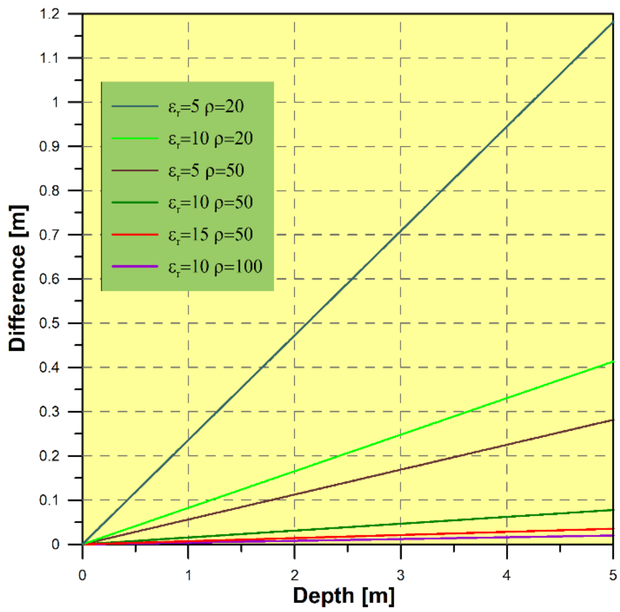

Figure 2
Absolute difference between reflection depths of the two models, with various relative dielectric constant and resistivity values, as a function of true depth (f = 100 MHz)

Figure 2 is a simplified version of *Figure 1*, which represents the absolute depth difference. It shows the linear connection between the depth and the depth difference. Six different relative dielectric constant and resistivity pairs are demonstrated. The light green line represents a soil with $\varepsilon_r = 10$ relative dielectric constant and $\rho = 20$ Ωm resistivity and it can be seen that the depth difference is higher than 15 cm even at 2 m depth. This is almost a 10% error. It could increase up to 20% if the ε_r were to decrease to 5 at the same resistivity. The slope of the curves depends on the product of the relative permittivity and the resistivity. For example, a medium with $\varepsilon_r = 10$ and $\rho = 50$ Ωm and another one with $\varepsilon_r = 5$ and $\rho = 100$ Ωm have the same curve. This means that the difference between the reflection depth calculated from the equations of dielectrics and lossy media is the same. The values of both *Figure 1* and *2* were calculated at 100 MHz frequency and at 1 relative magnetic permeability ($\mu_r = 1$).

3. MODELING EXAMPLE

In order to model the effect of resistivity change, MATGPR modeling software [3] was used. It can generate synthetic GPR B-scans with the Split-step methods of Bitri and Grandjean [4]. 2D EM models were created. The geometry and the EM parameter values need to be set up. Choosing the appropriate grid size is also an important step before forward modeling. The model dimensions and sampling rate are specified as well.

In the presented models *(Figure 3)* the anomalous body is a rectangle shape cavity with $\varepsilon_r = 1$ and $\rho = 10{,}000$ Ωm. The difference between the three simple models is the background resistivity values (10^4, 50 and 20 Ωm). The relative permittivity equals 10 in both cases ($\varepsilon_r = 10$). The relative magnetic permeability ($\mu_r = 1$) and the antenna frequency (100 MHz) are also the same.

Because of the lower resistivity of the background (model) on the right side (and in the middle), the image of the radargram is quite blurred. The bottom of the anomalous rectangle cannot be seen as clearly as it can be on the left side. The edges are not sharp at all in the lower resistivity backgrounds. The top of the rectangle looks thicker and a dark patch spreads towards the surface. A full hyperbola can be observed in the middle radargram. Although the B-scan on the left side has better quality, the middle *(Figure 3b)* one still contains the necessary information. Only the bottom of the cavity is uncertain, because a definite reflection is not received and the image itself is quite blurred. The radargram in *Figure 3c* contains the least information and has the lowest resolution. Under real circumstances the quality and the information content of the B-scan could be even worse because of noise and other reflection interferences.

The first break can be read out at about 42 nsec on the left image, at about 35 nsec in the middle, and at about 28 nsec on the right side. Besides the two-way traveltime there is an obvious difference in the resolution.

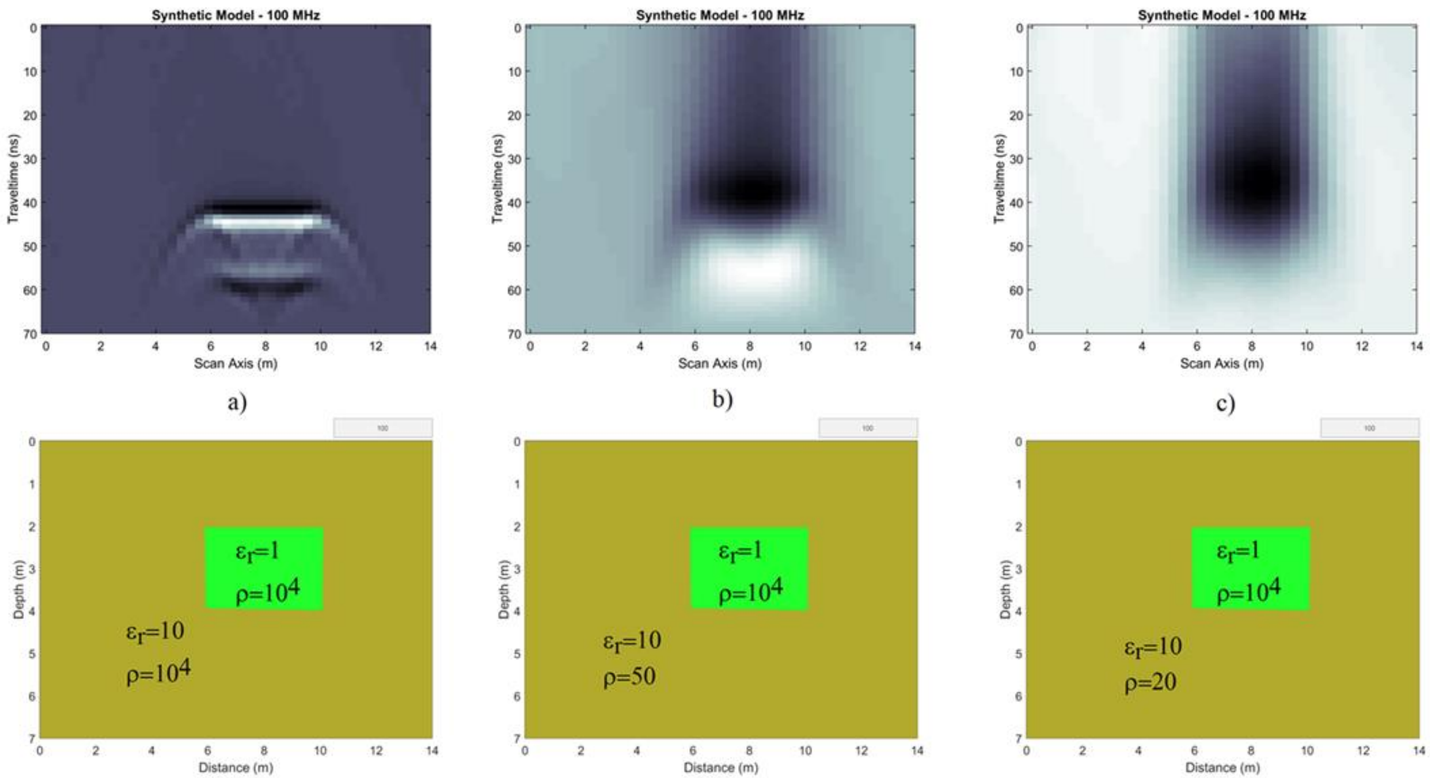

Figure 3

Split-step modeling results of one simple rectangle shaped cavity at 2–4 meters depth in case of a very resistive (ρa = 104 Ωm) background (a) and two lower resistivity (ρb = 50 Ωm and ρc=20 Ωm) backgrounds (b–c). True models are below and the synthetic B-scans can be seen above. The relative permittivity equals 1 (úr = 1) in each part of the three models

4. Parameter sensitivities

The slight changes of the EM parameters affect the GPR parameters in a complex manner. These can be characterized by the so-called parameter sensitivities. They can be defined different ways. The wave propagation velocity is one of the key GPR parameters that is directly related to the reflection depth. Based on analogs from DC geoelectrics [5], the velocity and conductivity sensitivities can be defined as follows:

$$\psi = \frac{\partial v_l}{\partial \sigma}\frac{1}{v_l} = \frac{\partial(\ln v_l)}{\partial \sigma}, \tag{4}$$

or

$$\psi^* = \frac{\partial v_l}{\partial \sigma}\frac{\sigma}{v_l} = \frac{\partial(\ln v_l)}{\partial(\ln \sigma)}. \tag{5}$$

The velocity and permittivity sensitivities can be calculated similarly:

$$\psi = \frac{\partial v_l}{\partial \varepsilon_r}\frac{1}{v_l} = \frac{\partial(\ln v_l)}{\partial \varepsilon_r}, \tag{6}$$

or

$$\psi^* = \frac{\partial v_l}{\partial \varepsilon_r}\frac{\varepsilon_r}{v_l} = \frac{\partial(\ln v_l)}{\partial(\ln \varepsilon_r)}. \tag{7}$$

The partial differentiation of the wave propagation velocity can be performed analytically, the chain rule has to be applied. The sensitivity function is negative in this case *(Figure 4)* because the velocity decreases if the conductivity increases.

In this case, the wave propagation velocity is the one assuming a lossy medium (v_l). The velocity for dielectrics (v_d) would not be sensitive to the conductivity, only the relative dielectric constant.

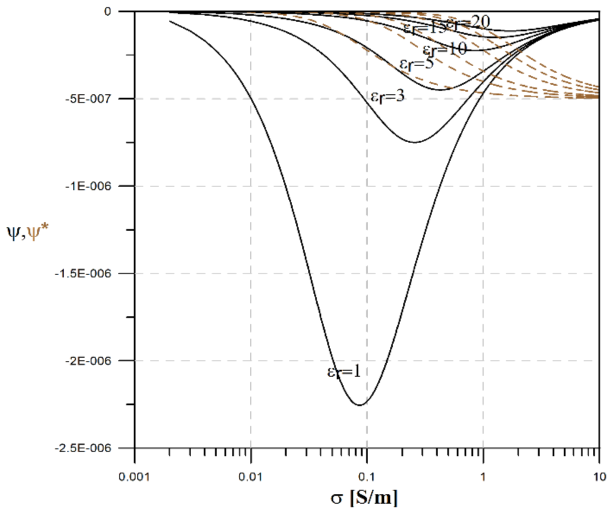

Figure 4

Wave propagation velocity-conductivity sensitivity curves assuming lossy medium. Black lines indicate ψ, brown dashed lines indicate ψ^*.

The wave propagation-conductivity sensitivity [based on *Equation (4)*] clearly has an absolute maximum value. In case of $\varepsilon_r = 1$ this extreme value is larger than 10 Ωm. With increasing dielectric constant, it decreases below 10 Ωm. Ψ^* *[Equation (5)]* can be produced from Ψ with a simple multiplication of conductivity.

The lower the relative permittivity, the more sensitive the velocity is to the conductivity changes *(Figure 5)*. If ε_r is between 1 and 10 the sensitivity is dependent on the resistivity but above 10 it is independent of it.

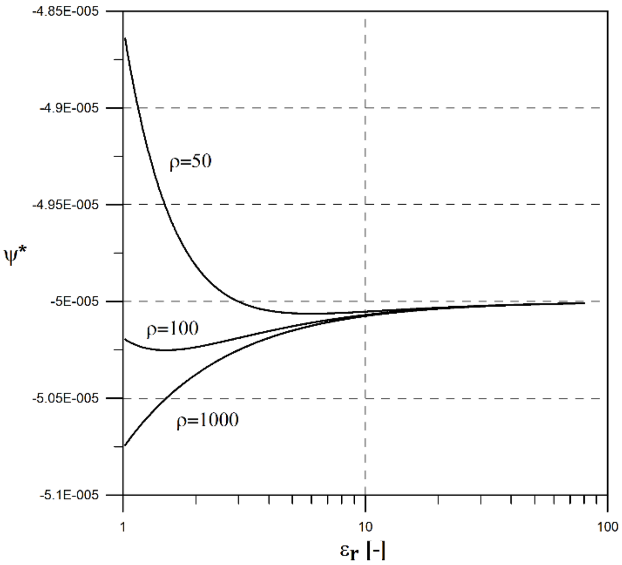

Figure 5

Wave propagation velocity-relative permittivity sensitivity (Ψ^*) curves assuming lossy medium. Three resistivity values are presented.

5. CONCLUSION

The most important GPR parameter which we want to determine or estimate is the reflection depth. If an anomaly can be pointed out, based on the wave propagation velocity distribution (or average value), the depth can be estimated. The relative and absolute reflection depth differences between lossy medium and dielectrics were presented. The comparison of the lossy and dielectric models has significance below 100 Ωm. Realistic EM parameter values were plotted that can be very similar to the parameters (σ, ε_r) of real soils.

The modeling results show that the possibilities for giving accurate depth estimation depend not only on the wave propagation velocity but also on the resolution (which can be derived from the wavelength). When the resistivity is lower, the reading of the time cannot be as accurate because the vertical (and horizontal) resolution is degraded.

The characteristics of the wave propagation velocity-conductivity and the velocity-relative dielectric constant sensitivities were described. The investigation of other GPR sensitivity curves (e.g.: resolution-conductivity) can be also described in the future as well.

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REFERENCES

[1] Nádasi, E., Turai, E. (2017). Analysis of electromagnetic petrophysical parameters in GPR survey (in Hungarian). *Magyar Geofizika*, 58/4, pp. 253–258.

[2] Nádasi, E., Turai, E. (2018). Increasing the accuracy of GPR measurements. *Geosciences and Engineering: A Publication of the University of Miskolc*, 6 (9), pp. 142–150.

[3] Tzanis, A. (2010). matGPR Release 2: A freeware MATLAB® package for the analysis & interpretation of common and single offset GPR data. *FastTimes*, 15 (1), pp. 17–43.

[4] Bitri, A., Grandjean, G. (1998). Frequency – wavenumber modelling and migration of 2D GPR data in moderately heterogeneous dispersive media. *Geophysical Prospecting*, 46, pp. 287–301.

[5] Gyulai, Á. (1989). Parameter sensitivity of underground DC measurements. *Geophysical Transactions*, 35 (3) pp. 209–225.