Geosciences and Engineering, Vol. 12, No. 1 (2024), pp. 88–99.
https://doi.org/10.33030/geosciences.2024.01.006

COMPARISON OF DIFFERENT ROCK PHYSICS MODELS FOR ACOUSTIC VELOCITY

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Abstract: Unlocking the secrets of rocks is an important task to explore, we require a deep understanding of rock physics. By describing the influence of rock physics on seismic velocities, we attempt to provide a comprehensive understanding of rock physics models and their application to predicting velocities. Throughout the discussion of the most used rock physics models. Moreover, we summarize the results of models using synthetic data to represent relationships between rock physics and elastic moduli. Knowing rock physics models for acoustic waves enhances the interpretation of seismic data, improves reservoir characteristics, aids in fluid identification, and supports uncertainty analysis. However, the choice of model depends on the specific properties of the rock and the application.

Keywords: *rock physics, model, acoustic velocity, elastic moduli, porosity*

1. INTRODUCTION

Rock physics characterizes the performance and physical properties of rocks and geological materials (Mavko et al., 2009). The rock physics aims to construct P-wave velocity (Vp), S-wave velocity (Vs), density (ρ), and their relationships to elastic moduli such as bulk modulus (K) and shear modulus (G), quality factor (Q), porosity (ϕ), pore fluid, pressure, for given lithology and fluid types (Wyllie et al., 1958), (Wyllie et al., 1956). Rock physics is frequently called "velocity-porosity" science. The concept behind this name is to estimate elastic-wave velocities in porous rock based on its porosity (Nur et al., 1998), or to perform an inverse operation and interpret velocity reported in a well (Nolen-Hoeksema and Richard, 2000). In the field of geophysics, acoustic measurements are often used to study the properties of rocks and the subsurface, thus rock physics talks about velocities and elastic parameters because these are what link physical rock properties to seismic expressions which can provide us with valuable information about exploration and production of natural resources, such as water, gas, and oil. To interpret acoustic measurements, rock physics models are used. Rock physics models (RPMs) are used to describe the relationship between the physical properties of rocks and the acoustic properties (Mavko et al., 2020). These models are important for a wide range of applications, including oil and gas exploration, geothermal energy, and earthquake seismology. However, by using these models, geophysicists can better understand the subsurface and make more accurate predictions about the location and characteristics of subsurface resources. The most used rock physics model is the elasticity model which is widely used for describing the acoustic behavior of the rock (Berryman, 1995). Several rock physics models can be used to describe acoustic measurements, depending on the specific properties of the rock being. In this paper, we will review some of the most used rock physics models for describing acoustic measurements, based on the literature available up to now such as Biot model (Biot, 1956), Gassmann model (Gassmann, 1951), Biot–Gassmann model (Lee, n.d.), Wyllie model (Wyllie et al., 1956), Raymer model (Raymer et al., 1980), the pressure dependence of acoustic velocity and quality factor – new petrophysical models – by Dobróka (Dobróka and Molnár, 2012). These models provide an approach for describing the acoustic behavior of the rock based on its properties. We constructed to test these different models using synthetic data for the physical properties of quartz, and it is assumed to be filled with water. These data are essential in understanding the behavior of acoustic velocity propagation through the different used rock physics models.

2. Materials and methods

The secrets of the rock are so complex that cannot be described without using the exact key to unlock the hidden properties of the rock, moreover, the exact key will not represent all physical properties of the rock it will only show what we search for. Hence, we will go through the most important properties and neglect the others which are dependent on the assumption used for the different models and the parameters used in the different equations to estimate the acoustic velocity.

2.1. Biot model

Biot model (Biot, 1956) is used to describe elastic wave propagation in fluid-saturated porous media. Biot (1941) assumes that the rock formation is porous and saturated with a single fluid, while the rock is isotropic, homogeneous, linearly elastic, and contains well-connected and small pores (Chandrasekaran et al., 2022) with an incompressible fluid.

The Biot theory (Biot, 1956) uses the concept of linear continuum mechanics, connects the elastic moduli and density to the P- and S-wave velocity of the elastic waves in rock matrix as

$$V_P = \sqrt{\frac{K+\frac{4}{3}\mu}{\rho}} \tag{1}$$

$$V_S = \sqrt{\frac{\mu}{\rho}} \tag{2}$$

assuming that the wavelength is much higher than the grain size and the porosity is isotropic and uniform. (Here K is the bulk modulus, μ is the shear modulus, ρ is the density.) Biot's equation of motion couples the displacement of the rock matrix and that of the fluid constituents. The Biot model has limitations, including assumptions about hypotheses, homogeneous media, static properties (Thomsen, 1985), liquid characteristics (Borregales et al., 2019), and non-uniqueness. These assumptions may lead to inaccurate predictions, as they assume porous media are homogeneous and isotropic. Additionally, static properties do not reflect the dynamic behavior of porous mediums, and the model does not consider the compaction of liquids. Furthermore, the model is not unique, making it difficult to determine the true properties of porous media from velocity data alone. Despite its limitations, Biot's theory has a wide range of applications used in various fields, it is utilized in geophysics (Sahay, 2008), petroleum engineering (Suvorov and Selvadurai, 2019), and civil engineering for interpreting seismic data, predicting fluid flows, and evaluating soil and rock mechanical behavior. However, it has limitations and requires more advanced models to overcome.

2.2. Gassman model

The Gassmann model (Gassmann, 1951) is a widely used theoretical model in geophysics that describes the behavior of fluid-saturated porous rocks (Ciz et al., 2007) which is used to predict seismic velocity when fluid content changes in rocks. Gassmann assumes that the porous material is isotropic, elastic, homogeneous, and composed of one type of mineral. The Gassmann model predicts that the seismic velocity of the rock decreases with increasing fluid saturation, while the attenuation of seismic waves increases (Han and Batzle, 2004). To predict the seismic velocity using the Gassmann model, we first need to calculate the bulk modulus of the saturated rock using the Gassmann equation (Al-Khateb, 2013; Berryman, 1999) which is a key component of the Gassmann model and describes the bulk modulus of the fluid-saturated porous rock (in a low-frequency approximation) as

$$K_s = K^* + \frac{K_0\left(1-\frac{K^*}{K_0}\right)^2}{(1-\phi)\frac{K^*}{K_0}+\phi\,\frac{K_0}{K_f}} \tag{3}$$

where $K_0, K_f, K^*,$ and K_s are the bulk moduli of the mineral, fluid, dry rock, and saturated rock frame, respectively, ϕ is the porosity. The Gassmann model predicts that the P-wave and S-wave velocities decrease as the fluid saturation of the rock increases. The Gassmann model (Berryman and Milton, 1991) is a widely used tool in geophysics, reservoir engineering, and rock physics. It assumes isotropy, homogeneity, and incomprehensibility of fluids, neglecting frequency and attenuation effects, and ignoring pore liquid chemistry effects. It also assumes that fluid is balanced with external fluid, which may not be the case in all cases. The model is widely used in seismic exploration, reservoir characterization, monitoring of reservoirs, geomechanics, rock physics modelling, and unconventional reservoirs like shale gas and tight oil formations. However, it should be used with caution and consider its limitations when interpreting seismic data.

2.3. Wyllie model

The Wyllie model (Wyllie et al., 1956) is used to estimate the porosity and P-wave velocity of a rock formation. It is assumed that the density of the rock's solid minerals and its bulk density are connected to the rock's porosity. The Wyllie time-average equation, proposed by Wyllie, Gregory and Gardner (Wyllie et al., 1956), relates sonic velocities to rock porosity. The equation states that total travel time is the sum of the time spent on the rock matrix and that in the pores giving for the propagation velocity

$$\frac{1}{V} = \frac{1-\phi}{V_m} + \frac{\phi}{V_f} \tag{4}$$

where V_m, V_f, ϕ are the matrix velocity, the fluid velocity, and the porosity, respectively. The expression also can be written in terms of interval travel time as

$$\Delta t = \phi \Delta t_f + (1-\phi)\Delta t_m \tag{5}$$

where Δt_f and Δt_m are the values of acoustic travel-time in fluid and rock matrix, respectively.

The Wyllie model has limitations, including its assumption of constant density and velocity for solid minerals, and the rock's internal fluid, as well as its assumption of uniform rocks, and its assumption of isotropic properties. However, it is widely used in petrophysics for estimating porosity, interpretation of seismic data, prediction of rock properties, and reservoir simulation. The model is particularly useful for simple, homogeneous, and water-saturated rocks, but should be considered when applying it to more complex geological settings. By combining Wyllie models with other data, reservoir representations can be more accurate and production strategies optimized.

2.4. Raymer model

Raymer model (Raymer et al., 1980) estimates compressional wave velocity in rock formations using porosity and mineralogy measurements from wireline logs. It assumes matrix and pore space rocks, filled with fluid, and uses density and resistivity measurements to determine reservoir productivity. The Raymer–Hunt–Gardner (RHG) formula proposed a new empirical equation to calculate P-wave velocity in the porous rock sample as a function of the porosity, using matrix- and fluid velocity

$$V_P = (1-\phi)^2 V_m + \phi V_f \tag{6}$$

and for S-wave velocity in dry rocks

$$V_S = (1-\phi)^2\, V_s \tag{7}$$

where V_S is the S-wave velocity in the solid constituent.

The Raymer model has limitations, such as assuming homogeneous and elastic matrix rock and incompressible fluid filling pore space. This can lead to mistakes in porosity and compressional wave velocity estimates. It also has limited accuracy in complex formations, as it is most accurate in simple rock formations with uniform characteristics. The model relies on empirical correlations to determine compressional wave velocity, which may not apply to all types of rocks and fluid mixtures. Additionally, it ignores shear wave velocity, which is crucial in geo-mechanical studies. The Raymer model has several applications in the oil and gas industry, including reservoir characterization, well planning, completion design, production optimization, and seismic interpretation. However, it is essential to consider these limitations when interpreting reservoir data and adjusting production strategies.

2.5. Rock physical model to describe pressure dependence

To describe the pressure dependence of acoustic velocity and quality factor in porous materials (Dobróka and Molnár, 2012) proposed a mathematical model based on the idea that microcracks in rocks open and close under the change of pressure. The model assumes that the material is isotropic and homogeneous. These assumptions are reasonable for materials with homogeneous compositions and structures but may not apply to all materials or conditions. It focuses on uniaxial stress states and longitudinal acoustic waves, providing a theoretical link between propagation speed and rock pressure. The model equation shows that the propagation speed – as a stress function – starts at V_0 (at zero pressure) and increases to $V_{max} = V_0 + \Delta V$ value (at high pressure). The mathematical model gives a velocity vs. pressure expression according to the function 1–exp(-λσ), which specifies the range of speeds in which the propagation speed can vary from stress-free states to states characterized by high rock pressure as

$$V(\sigma) = V_o + \Delta V(1 - e^{-\lambda\sigma}) \tag{8}$$

The three model parameters ($V_o, \Delta V, \lambda$) can be estimated in the inversion of the acoustic velocity dataset measured on rock samples at various pressures. As an example, we show the results sound in the case of a fine-grained sand sample. The measured dataset is shown in *Table 1*.

Table 1

The measured sonic velocity dataset

σ	0	0.45	0.9	1.8	2.7	3.6	4.5	5.4	6.3
V	2.719	2.733	2.761	2.837	2.934	3.021	3.081	3.13	3.186
σ	7.2	8.1	9	10.8	12.6	14.4	16.21	18.01	19.81
V	3.219	3.248	3.293	3.349	3.402	3.446	3.496	3.536	3.56

Starting from an initial model $\vec{m}_0$ an iteration procedure is used to improve the model as

$$\vec{m}_{new} = \vec{m}_{old} + \delta\vec{m} \tag{9}$$

with

$$\delta\vec{m} = G^{-g}\delta\vec{d} \tag{10}$$

Here $\delta\vec{d}$ is the data correction,

$$G^{-g} = (\underline{\underline{G}}^T\underline{\underline{G}})^{-1}\underline{\underline{G}}^T \tag{11}$$

is the generalized inverse matrix where $\underline{\underline{G}}$ is the Jacobi matrix, T is the symbol of matrix transpose. The errors of the estimated parameters are computed using the model covariance matrix

$$\underline{\underline{\mathrm{cov}}}^{(m)} = \underline{\underline{G}}^{-g}\underline{\underline{\mathrm{cov}}}^{(d)}(\underline{\underline{G}}^{-g})^T \tag{12}$$

where $\underline{\underline{\mathrm{cov}}}^{(d)}$ in the data covariance matrix. The estimation error of the i-th model parameter is given by

$$\sigma_i^{(m)} = \sqrt{\mathrm{cov}_{ii}^{(m)}} \tag{13}$$

In this procedure, *Equation (8)* serves as a forward problem formula. The estimated model parameters with their estimation errors are shown in Table 2.

Table 2

Model parameters estimated by inversion and their estimation errors

	$\mathbf{m_{new}}$	**estimation error**
$\mathbf{V_0}$	2.6936	0.0028
$\boldsymbol{\Delta V}$	0.9767	0.0064
$\boldsymbol{\lambda}$	0.1063	0.0017

The fit between the measured and calculated data is shown in *Figure (1)*. To characterize the accuracy of the estimation, the relative data distance (D [%]) is used

$$D = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{d_k^{(meas)} - d_k^{(calc)}}{d_k^{(calc)}}\right)^2} \quad *100\% \tag{14}$$

where $d_k^{(meas)}$ and $d_k^{(calc)}$ are the k-th measured and calculated data, respectively, N is the total number of data. In our example D = 0.47%.

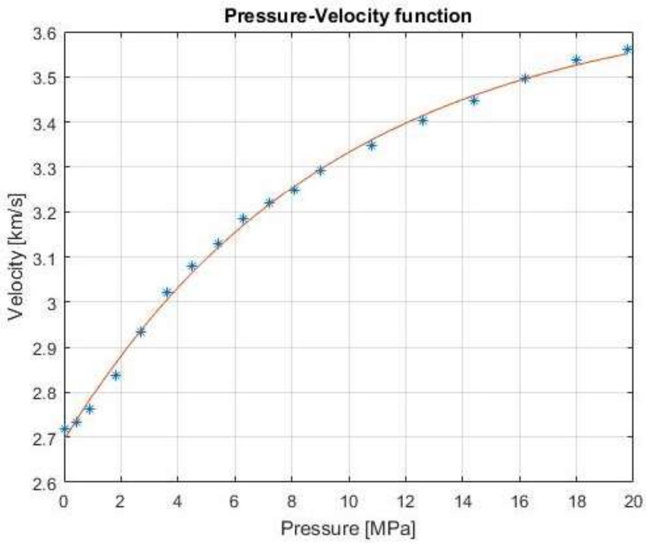


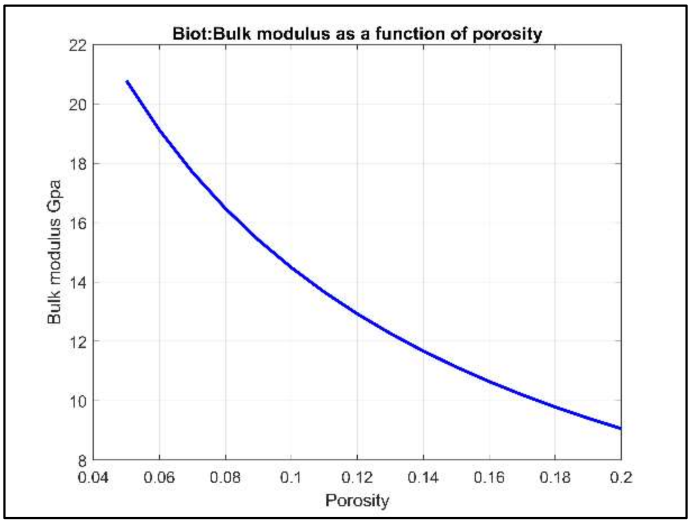
Figure 1

The fit between the measured data given in Table 1 and the calculated ones in LSQ inversion using the forward model in Equation (8)

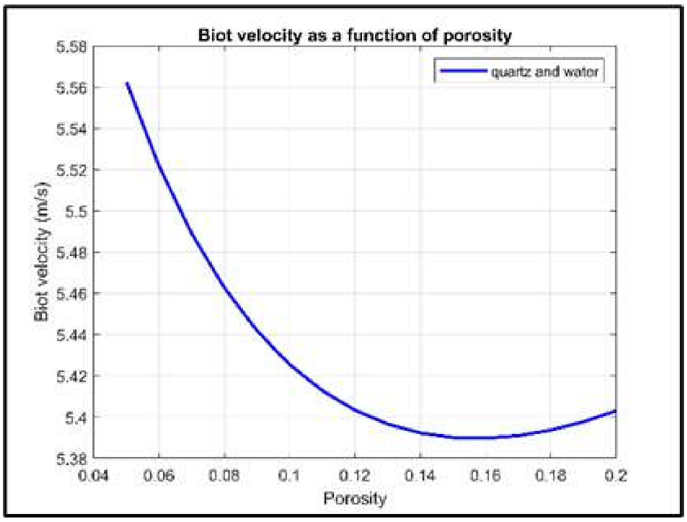
The above-discussed rock-physical models have several limitations when studying the pressure dependence of acoustic velocity and quality factor in materials. The model can provide useful insights into the pressure dependence of acoustic properties in materials, but its limitations should be carefully considered when interpreting its predictions. Experimental validation and careful parameter selection are essential to ensure the model's accuracy. The model has various applications in materials science, geophysics, and engineering. It is useful for studying the pressure dependence of acoustic properties in rocks and minerals, interpreting seismic data, and making predictions about the behavior of rocks and minerals under high-pressure conditions. It is also useful in materials science and engineering for analyzing material behavior under various pressure conditions, as well as in seismology for a better understanding of the Earth's crust and mantle.

3. Results and discussion

MATLAB program has been developed to test these different models using synthetic data. The Biot model states that increasing porosity decreases wave velocity due to a decrease in effective bulk modulus *[Figure 2(a)]*. This model showed the inverse relationship between porosity and velocity in a porous medium *[Figure 2(b)]*.



(a)

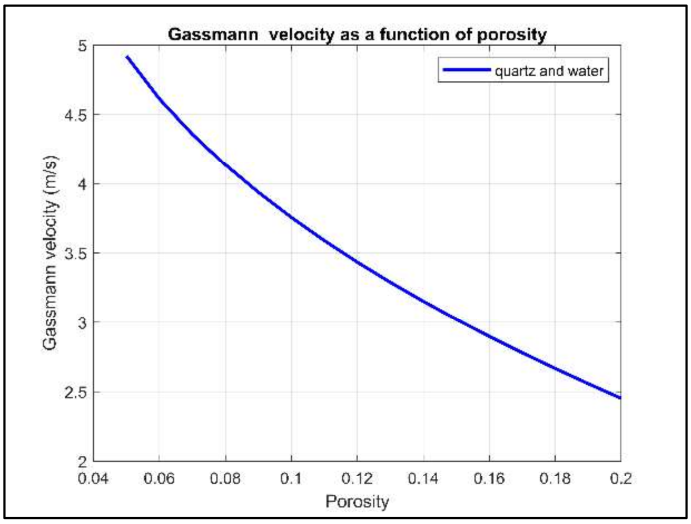


(b)

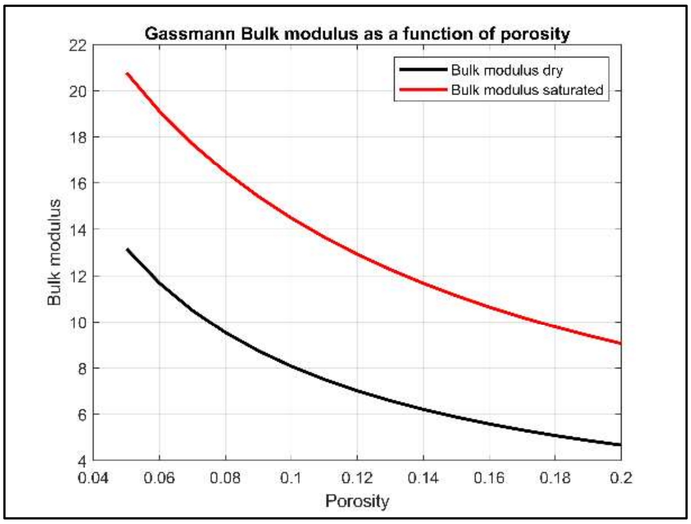
Figure 2

Biot model (a) Bulk modulus as a function of porosity,
(b) velocity as a function of porosity

The Gassmann model describes the relationship between porosity and elastic properties in fluid-saturated porous mediums. *Figure 3(a)* demonstrates that the bulk modulus decreases with increasing porosity, as the effective bulk modulus decreases. *Figure 3(b)* illustrates an inverse relationship between porosity and velocity.



(a)



(b)

Figure 3

Gassmann model (a) velocity as a function of porosity,
(b) Bulk modulus as a function of porosity

One of the earliest and most widely used transforms is the Wyllie time-average equation (Wyllie et al., 1958) which is used to determine a relationship between compressional velocity (or acoustic velocity) and porosity. This relationship is only reliable for consolidated sandstones over a small porosity range of 25%–30% (Raymer et al., 1980). The Wyllie model states that seismic wave velocity is directly proportional to the compressional modulus and inversely proportional to porosity. As porosity increases, the velocity decreases, while an improved version of the time-average equation, developed by Raymer et al. (1980), is shown in *Figure 4*.

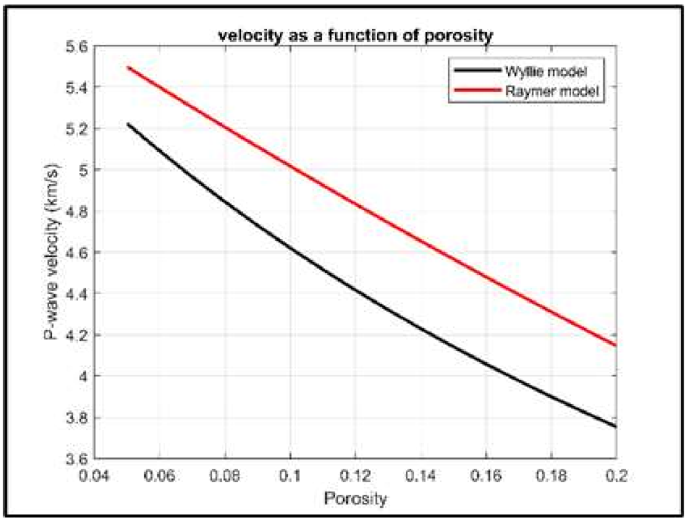


Figure 4

Velocity as a function of porosity in Wyllie and Raymer models

4. CONCLUSIONS

Theoretical modeling of velocity plays an important role in describing the acoustic behavior of rocks. Furthermore, we can recognize the inverse relation between porosity and velocity in all the discussed models. For porosities greater than the critical porosity, velocity is not strongly dependent on porosity. For values below the critical porosity, velocity depends strongly on porosity and increases significantly with a small decrease in porosity and the models described in this paper are just a few examples of the many models that have been developed over the years. The choice of model depends on the specific properties of the rock. By understanding these models, scientists and engineers can better predict the behavior of rocks and improve our understanding of the subsurface.

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