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ANALYSIS OF THE FULL WAVE SPECTRA OF LABORATORY ACOUSTIC MEASUREMENTS

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Abstract
The paper presents and analyzes the Fourier spectra of the complete waveforms of the P and S waves measured on the drilling core samples in the laboratory. Measurements were performed on sandstone and aluminum samples at 12 different pressures. For measurements on an aluminum sample, the spectra show significantly higher amplitude values at all pressures and at all frequencies than for sandstone sample. In the case of sandstone sample, the energy of the P wave is concentrated in the frequency band between 100,000 Hz and 300,000 Hz, while the energy of the S wave is predominantly in the frequency band between 300,000 Hz and 800,000 Hz. Based on acoustic measurements, it can also be stated that the maximum of the energy spectrum of the P wave measured on the same sample is significantly higher than that of the S wave.

Keywords: *acoustic measurements, P wave, S wave, Fourier spectra, Energy spectra.*

1. Introduction

Seismic-acoustic laboratory measurements are widely used in petrophysical practice to help solve geological-geophysical tasks in hydrocarbon exploration, ore exploration, building material exploration, and environmental studies. It has been well known that in acoustic measurements the rate of propagation is strongly dependent on pressure (Wyllie et al., 1958; Nur and Simmons, 1969; Stacey, 1976). In the range of low pressures, the velocity increases strongly which gradually increases to a maximum saturation value as the pressure increases (Birch, 1960; Yu et al., 1993). This phenomenon is linked to changes in the structure of rocks, compression of pores (Birch, 1960; Jones and Wang, 1981) and cracks (Walsh and Brace, 1964; Sengun et al., 2011). In laboratory tests, the propagation velocities (Vp and Vs) of P waves (primary or pressure waves) and S waves (secondary or shear waves) as a function of pressure are measured on the drilling core samples. The Department of Geophysics at the University of Miskolc is intensively researching and developing in this field. A new petrophysical model was introduced (Dobróka and Somogyi Molnár, 2012). The new model has been used to describe both the pressure dependence of P waves (Somogyi Molnár et al., 2015) and the pressure dependence of S waves (Kiss, 2018). The model was further developed for two-component (Somogyi Molnár et al., 2019) and multi-component (Dobróka et al., 2022) cases. Another line of the research is the spectral study of the full waveform of P and S waves. In this case the time domain signals of the acoustic waves are transformed into the frequency domain and the characteristics of the spectra thus produced are examined. With these studies, the spectra and spectral ratios of the core samples (Diallo et al., 2003) as a function of frequency can be calculated relative to the spectra of aluminum reference sample. The quality factor of rocks can be calculated from the spectral ratios (Knopoff, 1965; Toksöz et al., 1979). In this paper, in connection with this direction of research, we recorded the total waveform P and S of the aluminum reference sample and a sandstone sample at 12 different pressures. Fourier spectra of acoustic signals (Bracewell, 1978; Brigham, 1974) and energy spectra were calculated and analyzed.

2. Mathematical bases of spectrum calculation

Denote the time domain signal of the P wave by p(t) and the signal of the S wave by s(t). The spectra of acoustic measurements can be generated by the Fourier transform (Bracewell, 1978).

For the P waves:

$$P(f) = \int_{-\infty}^{+\infty} p(t) e^{-j2\pi f t} dt, \tag{1}$$

where t – the time,
f – the frequency,
$j=\sqrt{-1}$ – the imaginary unit,
p(t) – the time domain sign of the P wave,
P(f) – the Fourier spectrum of the P wave.

For the S waves:

$$S(f) = \int_{-\infty}^{+\infty} s(t) e^{-j2\pi f t} dt, \tag{2}$$

where - *s(t)* – the time domain sign of the S wave,
- *S(f)* – the Fourier spectrum of the S wave.

From the Fourier spectra, the energy spectra of the P and S waves (frequency distribution of the energy of the waves) can be calculated using the following equations.

For the P waves:

$$E_P(f) = |P(f)|^2, \tag{3}$$

where $E_P(f)$ – the energy spectrum of the P wave.

For the S waves:

$$E_S(f) = |S(f)|^2, \tag{4}$$

where $E_S(f)$ – the energy spectrum of the S wave.

3. Laboratory measurements

The measurements were performed in the acoustic laboratory of the Department of Geophysics at the University of Miskolc (Figure 1). In this laboratory, it is possible to determine longitudinal (P) and transverse (S) wave propagation velocities on 35 mm diameter cylindrical specimens under different pressure conditions. The digitally controlled measuring system includes a pressure vessel, a load frame, and are connected to a two-channel acoustic measuring instrument and software for controlling the above and measuring velocity. Table 1 summarizes the most important technical data of the measuring instrument. A pair of pressure heads with piezoelectric crystals (1MHz and 0.5MHz) capable of generating and detecting P and S waves are available for the instrument. During our tests, we loaded the samples in only one axial direction. At 12 different pressures (0.26 MPa, 1.04 MPa, 4.16 MPa, 8.32 MPa, 12.48 MPa, 16.64 MPa, 20.8 MPa, 24.96 MPa, 29.12 MPa, 33.28 MPa, 37.44 MPa and 41.6 MPa), the total waveform of the P and S waves was recorded on an aluminum reference sample and a sandstone sample. Lengths of sample used in the experiment was less than 5 cm to achieve a high diameter to length ratio (diameter 35 mm, length 47,5 mm, weight 111 g). By using a high diameter to length ratio, the risk of the onset time of the direct acoustic pulse passing through the length of the sample being obscured by reflections from sides can be reduced (Diallo et al., 2003). For the duration of the measurement, we fixed the sample in a rubber o-ring between the transmitter and the receiver. Before jacketing the sample with the rubber o-ring, to achieve good acoustic coupling at the contact points between the sample and the transmitter and receiver side, special gel was smeared in thinly. The frequency of the used transducers was 0,5 MHz. We have used stacking (256 count of average) during the acoustic measurements. Figure 2 shows an image of a P wave measured on an aluminum sample at a pressure of 1.04 MPa, and Figure 3 shows an image of an S wave at same pressure.

Table 1. *Technical data of the measurement system*

Pressure Vessel	
Max. Operating Overpressure	80 MPa
Max. Axial Load	400 MPa
Max. Axial Deformation under pressure	30 mm
linearity	0.05 %
Load Frame	
Max. Compression Force	300 kN
Pressure Plate diameter	300 mm
Pressure Plate hardness	55 HRC
Max. Pressure Plate path (stroke)	100 mm
Load Frame stiffness	250 kN/mm
accuracy up to 30 kN	grade 1
accuracy above 30 kN	grade 0.5
Digital resolution of Displacement Measurement System	0.05 micron
Pressure Generator	
Max. pressure	80 MPa
Max. volume	500 ml
Pressure measurement accuracy	grade 1

Stoke measurement accuracy	grade 0.5
Ultrasonic Unit	
Frequency filter	0.5 MHz, 1 MHz, 2 MHz, broadband
Run time of the strike pulse	<10 ns
Excitation voltage	-350 V, -550 V
Gain	-20 dB – 90 dB
Stacking (count of average)	2, 4, 8, …, 256

Figure 1. *The acoustic laboratory.*

4. Examination of spectra

Fourier spectra were determined for each record. Figure 4 shows the Fourier spectra of the P wave measured at 1.04 MPa on the aluminum sample (the real spectrum in blue, the imaginary spectrum in green and the amplitude spectrum in red), and Figure 5 shows the Fourier spectra of the S wave at the same pressure. Comparing the figures, it is clear that the spectrum of the S wave (Figure 5) shifts towards higher frequencies compared to the spectrum of the P wave (Figure 4). In Figure 6 we plotted together the amplitude spectrum of the P wave measured at 1.04 MPa for the sandstone sample (blue) and the aluminum sample (red). The figure shows that the peak of the amplitude spectrum of the P wave measured on the aluminum sample occurs at a frequency of 480 000 Hz and is much higher than the spectral maximum of the sandstone sample at 320 000 Hz. (Since the range of the spectra varies by several orders of magnitude the vertical axis of Figure 6 and 7 is logarithmically scaled.) The same can be said for the spectra of the S wave shown in Figure 7. Here again, the maximum of the amplitude spectrum of the aluminum sample (at a frequency of 640 000 Hz) is higher than the maximum of the spectrum of the sandstone sample at a frequency of 390 000 Hz. Now compare the energy spectra of the

P and S waves measured on the sandstone sample. Figure 8 shows the energy spectrum of the P wave measured at a pressure of 1.04 MPa, and the energy spectrum of the S wave is shown in Figure 9.

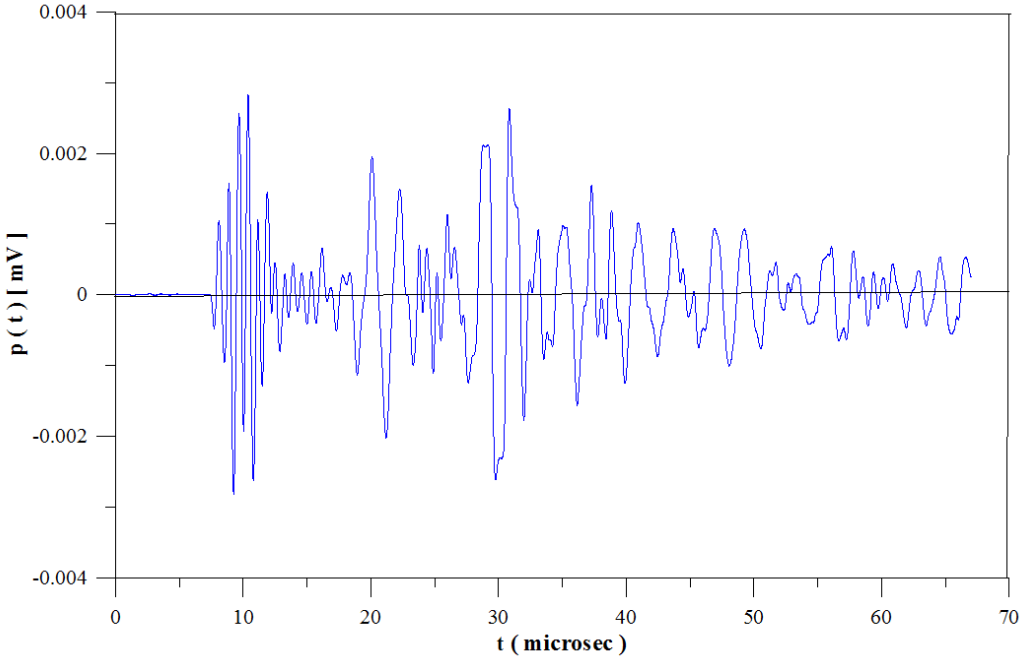

Figure 2. *P wave measured on an aluminum sample.*

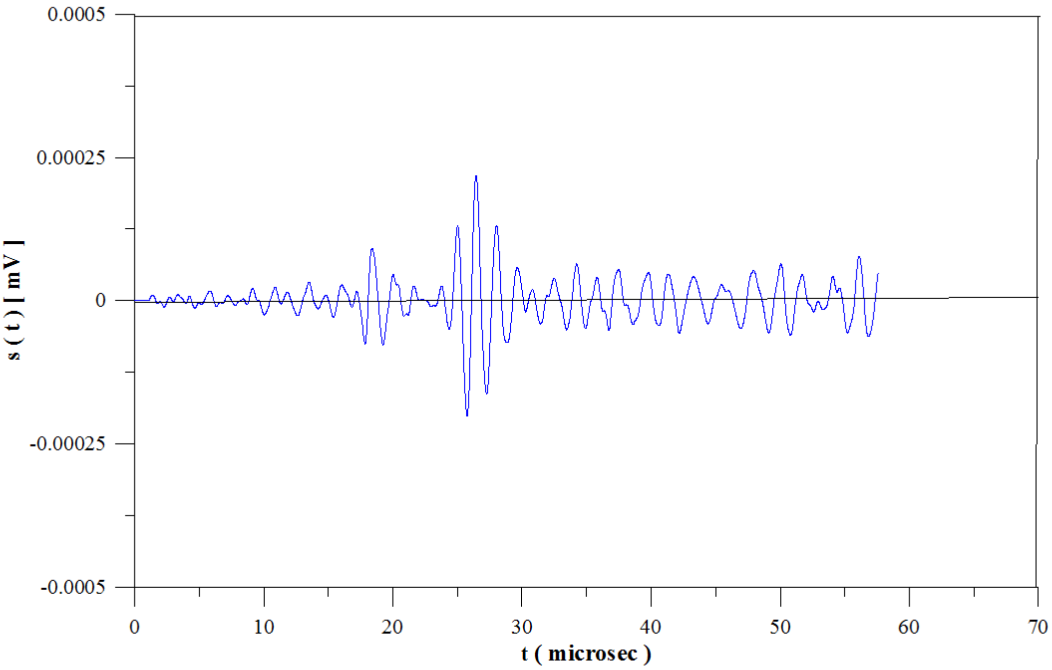

Figure 3. *S wave measured on an aluminum sample.*

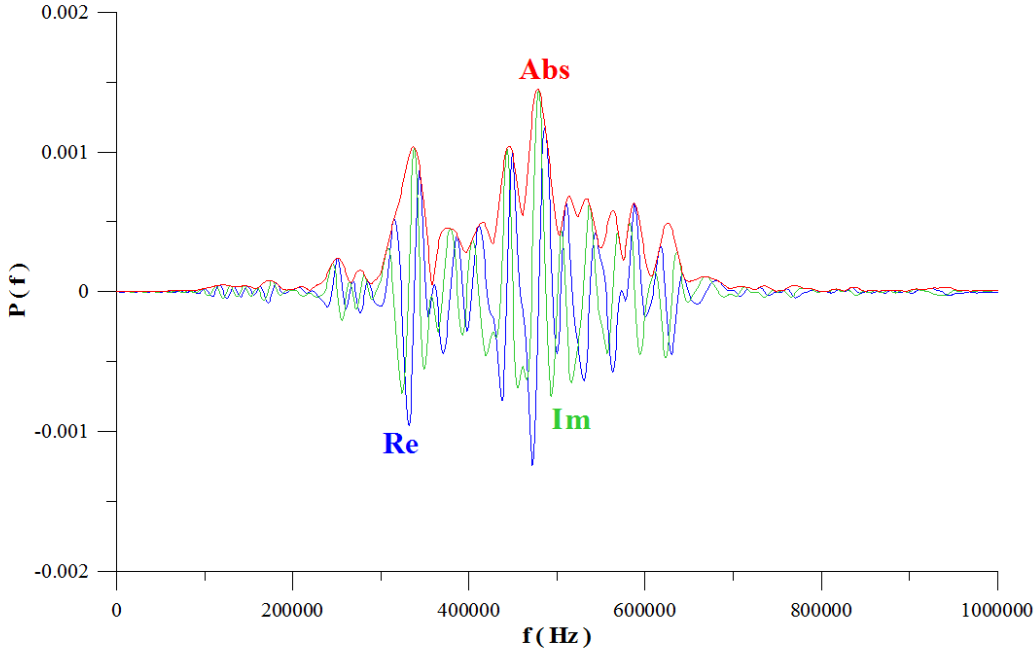

Figure 4. *Fourier spectra of the P wave (the real spectrum in blue, the imaginary spectrum in green, and the amplitude spectrum in red).*

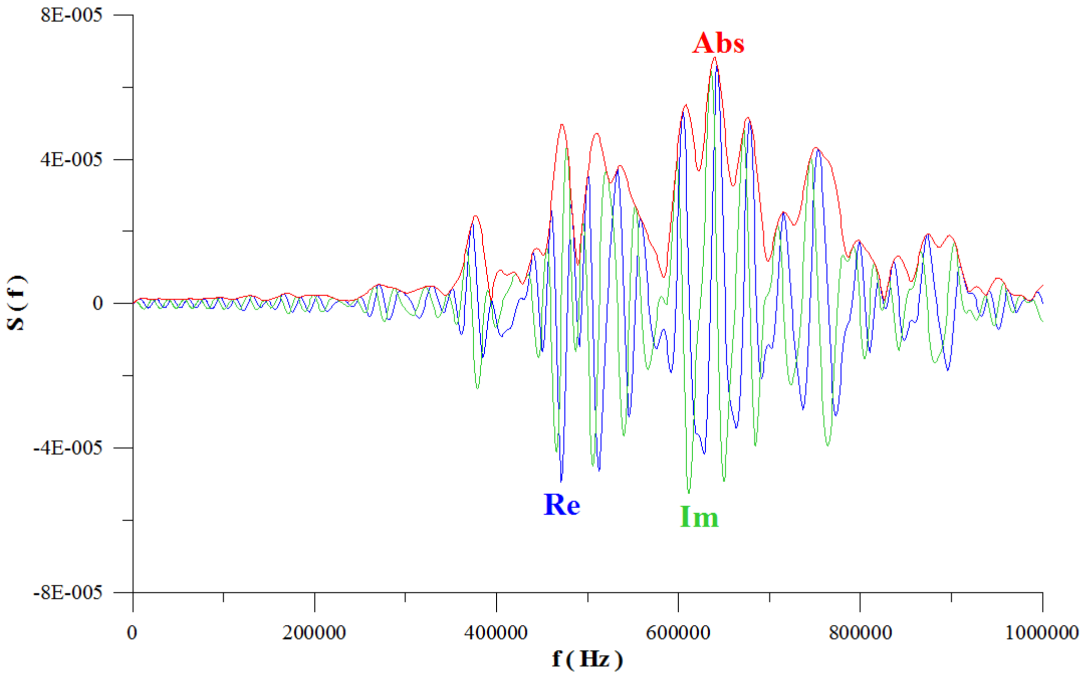

Figure 5. *Fourier spectra of the S wave (the real spectrum in blue, the imaginary spectrum in green, and the amplitude spectrum in red).*

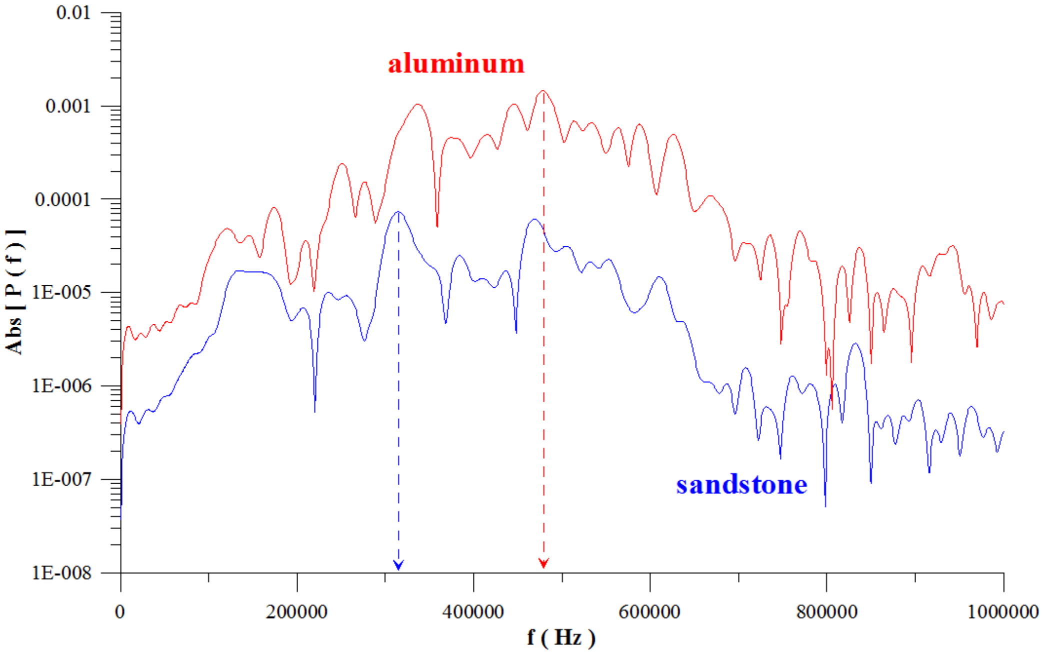

Figure 6. *Amplitude spectrum of the P wave measured at 1.04 MPa for the sandstone sample (blue) and the aluminum sample (red).*

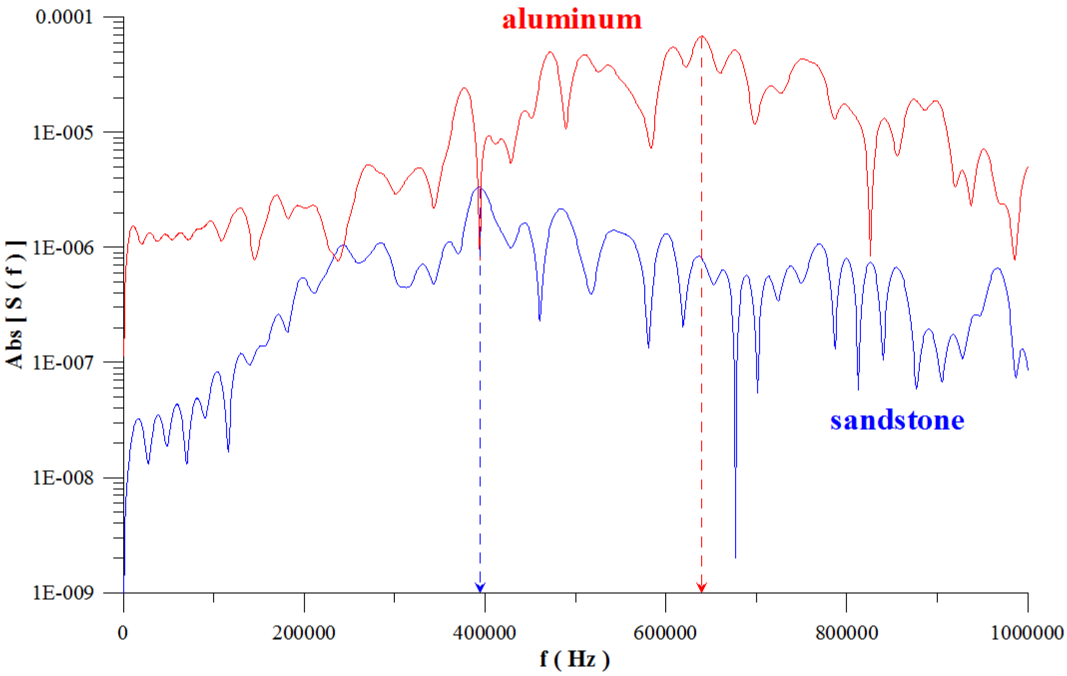

Figure 7. *Amplitude spectrum of the S wave measured at 1.04 MPa for the sandstone sample (blue) and the aluminum sample (red).*

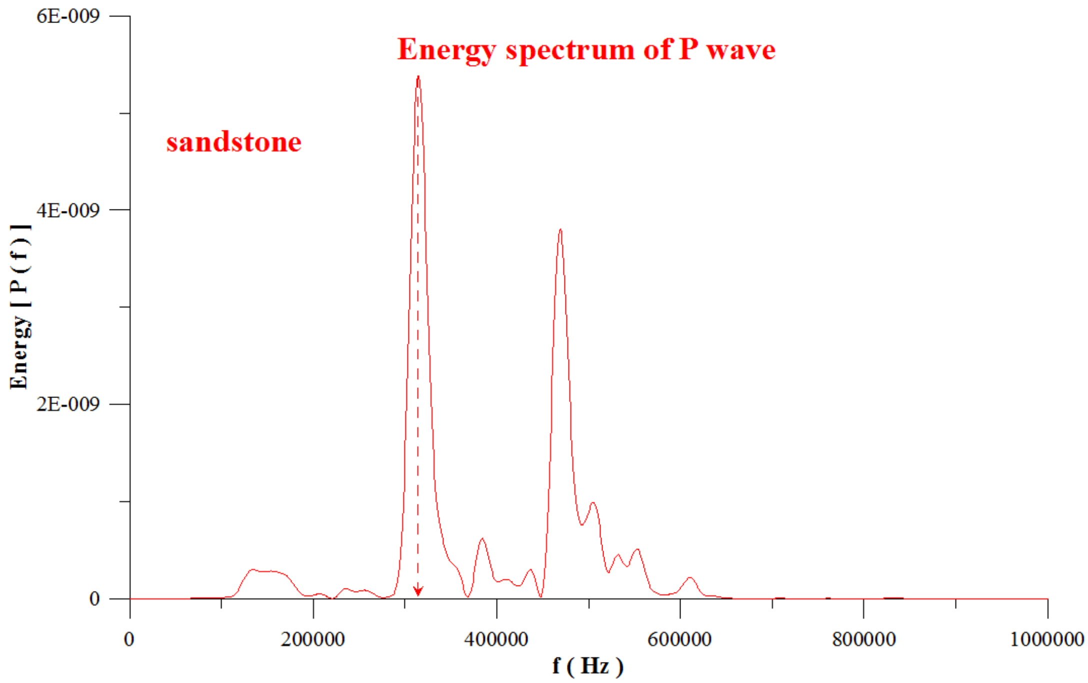

Figure 8. *Energy spectrum of the P wave measured on sandstone sample.*

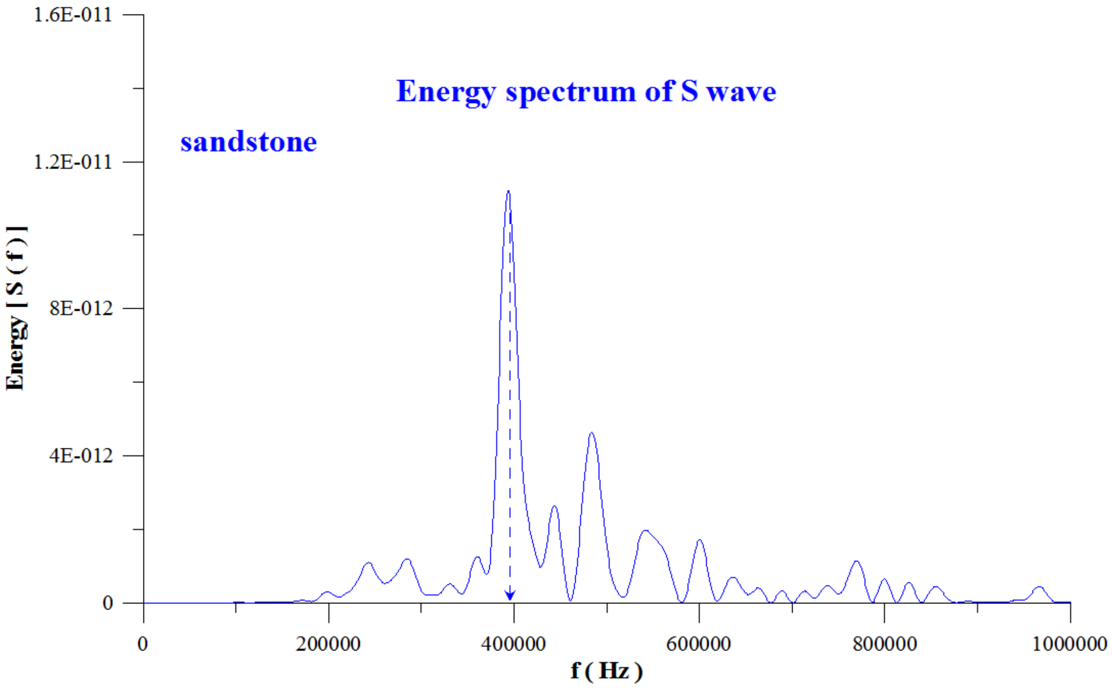

Figure 9. *Energy spectrum of the S wave measured on sandstone sample.*

It can be stated that the energy of the P wave is concentrated at lower frequencies (280 000 Hz to 550 000 Hz) than the energy of the S wave (380 000 Hz to 620 000 Hz). It can also be seen that the maximum of the energy spectrum of the P wave is 480 times higher than the maximum of the energy spectrum of the S wave. Similar findings can be made for the spectra of other measurements made under different pressures. The frequency-dependent ***SR*** spectral ratio (Diallo et al., 2003) can be calculated from the spectra of the sandstone sample and the aluminum reference sample using the following equation.

$$SR(f) = \ln\left(\frac{E_{sandstone}(f)}{E_{ref}(f)}\right), \tag{5}$$

where - $E_{sandstone}(f)$ – energy spectrum measured on a sandstone sample,
- $E_{ref}(f)$ – energy spectrum measured on the aluminum reference sample.

The process of the elastic attenuation modifies both amplitude and phase velocity of propagating acoustic waves, and an alternative measure of the ability of elastic attenuation is the quality factor (Q), defined as (Diallo et al., 2003)

$$\frac{1}{Q} = \frac{\Delta E}{2\pi E}, \tag{6}$$

where ΔE is the energy dissipated per wave cycle, and E is the total energy in a wave cycle. The quality factor (Q) of the rock can also be determined from the spectral ratio. Attenuation is estimated by comparing the recorded P and S full wave signal propagating through the sample rock and a reference sample with very high quality factor, very much higher than that of natural occurring rocks (for example aluminum sample): taking the natural logarithm of Asr/Aref ratio,

where - *Asr* – amplitude spectrum of sample rock
- *Aref* – amplitude spectrum of reference sample.

The task of the near future is to determine the quality factor of rocks, further investigations towards that end are needed.

5. Summary

Examining the spectra of the acoustic laboratory measurements, it was found that the maximum of the amplitude spectra of both the P waves and the S waves is much larger in the case of the aluminum sample than in the case of the sandstone sample. The maximum of the energy spectrum of the sandstone sample occurs at a lower frequency (320 000 Hz) for the P wave than the maximum of the energy spectrum of the S wave (390 000 Hz) and the spectral maximum value of the P wave is higher than the spectral maximum of the S wave. The energy of the P wave is concentrated in the frequency range of 280 000 Hz to 550 000 Hz, and the energy of the S wave occurs at higher frequencies (380 000 Hz to 620 000 Hz). The quality factor (Q) can be derived from the ratios of the calculated spectra of the rock sample to the calculated spectra of the aluminum sample used as reference samples. Quality factors derived from the spectra of acoustic waves at different pressures can give the pressure dependence of the quality factor.

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