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THE INFLUENCE OF THE POSITION AND SHAPE GEOMETRY OF THE COUNTER-CONICAL MOTION TRANSMITTER PIN ON THE PITCH FLUCTUATION OF THE SPIROID WORM MACHINED WITH ADJUSTED SHAFT

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Abstract
The difference between the mathematical defined shape of the conical helicoid surface and the operating surface of the spiroid worm produced with axis adjustment on a thread grinding machine results – among other things –in thread pitch fluctuation. During the manufacturing of conical helicoid surfaces the transmission of the rotation from the main spindle to the worm shaft can be realised by a driving pin and an adjusted lathe dog. This article deals with the examination of the fluctuation of the thread pitch parameter occurring when using a counter-conical drive pin. The fluctuation is described here as a function of the position and the shape geometry of the drive pin.

Keywords: *spiroid worm, driving pin, position, form, pitch fluctuation*

1. Introduction

The mathematical geometric definition of the conical helicoid surface is consisted by moving the profile curve or the creator surface along the conical screw line, as it can be seen in ***Figure 1.***

Conical helicoid surfaces have been used in many fields of the machine industry for example as operating surface of the spiroid worm. The conditions of the existing technologic environment result in serious restrictions regarding the mathematically exact surface generation. One possible solution is to create a production geometry model adapted to the mathematical creation of the helicoid surface, which involves new technological development (Balajti, 2007). The other possible solution is the exact geometric (production geometry) modelling of the technological production of the working surface of the worms, and then its development with the aim of approaching the mathematically defined helicoid surface as much as possible. The given circumstances lead mathematical geometers to formulate a kind of boundary conditions. To investigate the difference between the operating surfaces of the spiroid worm produced under the before mentioned boundary conditions and the mathematically exact conical helicoid surface, this paper intends to present a mathematical geometric procedure, offering a relatively cost-effective solution. The examination of the machining of the conical worm operating surface and the hob have been carried out in the intelligent production system of the DifiCAD Ltd. Engineering Office for the production of worms surfaces.

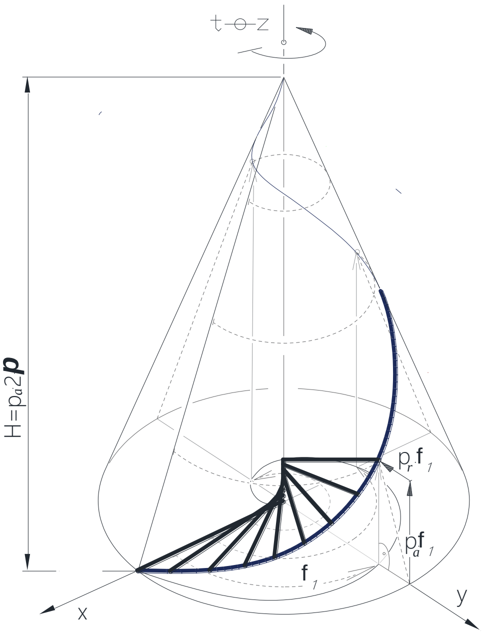

Figure 1. *Schematic sketch of the geometric definition of a flat threaded conical helicoid surface*

In accordance with the technological conditions, in order to machine the conical screw surface, the shaft of the spiroid worm must be inclined with the half opening angle of the reference cone in the direction of the path of the thread grinding machine (Dudás, 2000), as it can be seen in ***Figure 2***.

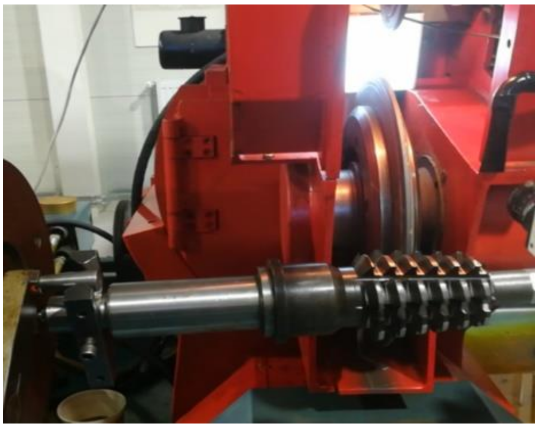

Figure 2. *Driving with a counter-conical motion transmitter pin in the case of the conical hob machining in the DifiCAD Ltd. Engineering Office.*

Figure 3 illustrates the geometric relationships of the placement of the cylindrical and conical worms as the work-piece. If machining the operating surface of the cylindrical worm on a thread grinding machine, the main spindle and the worm axis are in a line, as shown in ***Figure 3.a)***. By the other hand, the conical worms operating surface can be machined on a thread grinding machine only if the reference cone's generatrix is set parallel to the path of the thread grinder as shown in ***Figure 3. b)***.

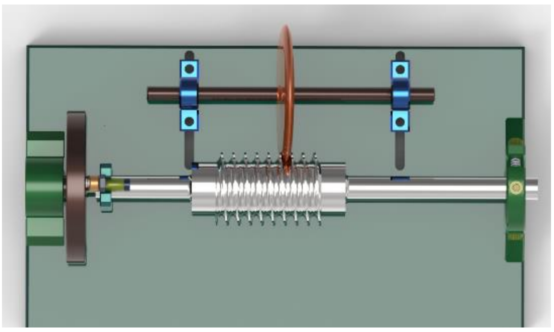

Figure 3.a) *Solid model of the grinding of the cylindrical worm in the same line with the spindle axis using the Solid Edge 2020 design software*

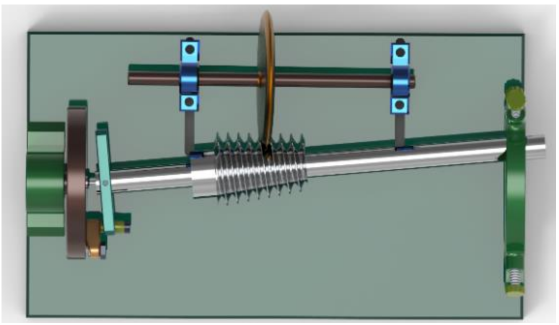

Figure 3.b) *Solid model of the conical worm grinding with an axis set at the reference cone half-angle compared to the straight line of the main spindle axis using the Solid Edge 2020 design software*

When the axis of the conical helicoid surface has been inclined to the axis of the main spindle by the half reference cone angle, the axis of the worm must be driven through a lathe dog by a driving pin. The axis of the conical worm is perpendicular to the plane of the lathe dog face. The lathe dog face has been set to form a half-cone angle with the face of the chuck. The drive fork is driven by a drive pin attached to the face of the chuck. The trajectory of the axis points of the right circular cone shaped driving pin is a circle in the plane of the face of the chuck (i.e. perpendicular to the axis of the main spindle), but an ellipse in the plane perpendicular to the conical helicoid axis inclined with the half reference cone angle (Balajti, 2007), as shown in ***Figure 4***.

Due to the motion of the axis points of the drive pin on an elliptic trajectory in the plane perpendicular to the axis of the conical worm surface, the radius changes continuously. By a constant angular velocity of the spindle the angular velocity of the worm will vary (Dudley, 1962).

Principles stated and results of the present research work integrates well with a large amount of valorous similar kinematic oriented research. The kinematical model for the motion transmission (Kelemen et al., 2021) was developed in a complete case study (Popkonstantinovic et al., 2019). The variation of the real motion law during the functioning has been studied by computer aided numerical models (Dudás L., 2010) as well as by analytical models (Bercsey et al., 2002). The problem of coupling was also approached by CAD-based simulations (Popa-Müller et al., 2019), and by consideration based on experience (Tolvaly-Rosca, 2020). The technical simulation methods have been integrated in the training of engineers (Dezső at al., 2017). The obtained results, together with the excellent results regarding the influence of profile shifting on the efficiency of the coupling (Tomori, 2021) were integrated in the educational process too (Tomori, 2021). Our investigations are based on a sophisticated mathematical background, adapted for engineering application (Vadászné Bognár, 2003).

Eliminating the thread pitch error and other errors during the production of worms has been a serious challenge from both sides:

- on the one hand, the technological background of the worm's production must be formulated realistically using the abstraction tools of mathematics,
- on the other hand, it is necessary to formulate the relationship between the mathematical geometric definition of the helicoid surface and its manufactured shape, using the language of mathematics.

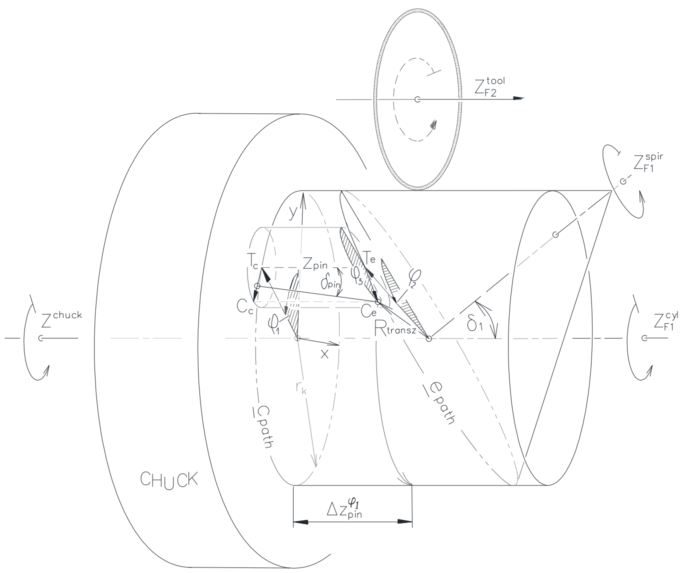

Figure 4. *Schematic illustration of the correlation between the angular rotations φ_1, φ_2 and φ_3 resulting from the displacement of the drive pin and the Δz_{csap} axial displacement of the pin.*

During the production of the working surface of the conical worm, due to its variable angular speed, the rotation angle becomes nonlinear with the time, which results in the pitch variation (Balajti at al., 2022). The motion transfer between the lathe dog and the drive pin occurs as a permanently point contact between the coupling surfaces. The correlations between the angular rotations during the driving were revealed in our previous paper (Balajti et al., 2022), which has created the basic of the geometric studies of the driving with the use of a counter-conical shaped driving pin. In the case of the manufacturing of a conical worm or a worm gear hob, the geometric relationships can be interpreted as shown in ***Figure 4*** during driving with a counter-conical driving pin with a half-opening angle δ_{pin}. The axis point of the drive pin follows a circular path with a radius of r_c in the face plane of the chuck. In the case of the adjusting of the conical worm shaft with a half-cone angle δ_1, the relationship between the angles φ_1, φ_2 a and φ_3 has been illustrated in ***Figure 4***, whose analytical relationships have been written as a function of the geometric parameters r_c and δ_1.

The points of the circle paths $\underline{c}$ can be described in the next formulas

$$x_c = -r_c \cdot sin\varphi_1 \quad \text{and} \quad y_c = r_c \cdot cos\varphi_1 \tag{1}$$

This elliptic path points e are affine to the circular path c can be written in the following forms

$$x_e = x_c / cos\delta_1 = -r_c \cdot sin\varphi_1 / cos\delta_1 \quad \text{and} \quad y_e = y_c = r_c \cdot cos\varphi_1 \tag{2}$$

The relationship between the rotation angles φ_1, φ_2 a and φ_3 has been written into the next forms

$$\varphi_2 = arctg(tg\varphi_1 \cdot cos\delta_1) \quad \text{and} \quad \varphi_3 = arctg(-tg\varphi_1 \cdot cos^3 \delta_1) \tag{3}$$

and displacement parallel to the main spindle shaft in the next form has been determined (Mándy, 2022).

$$z(\varphi_1) = tg\delta_1 \cdot r_c \cdot (1 - cos\varphi_1) \tag{4}$$

2. Motion transmission of the counter-cone shaped drive pin

The ellipse e trajectory points of the cone-shaped drive pin axis points are also the centres of the planar section ellipses of the drive pin in the lathe dog face plane V_2, as it can be seen in ***Figure 5***.

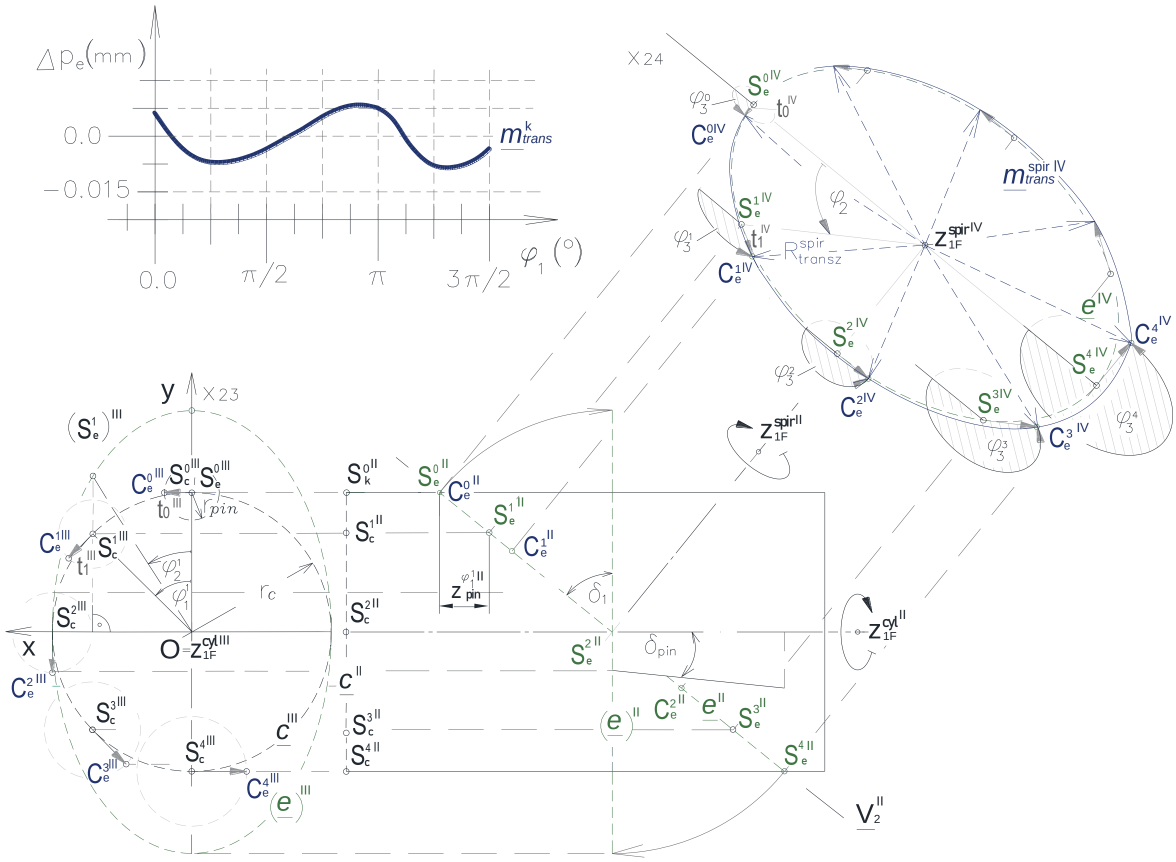

Figure 5. *The schematic representation of the relationships between the angular rotations φ_1, φ_2 and φ_3 resulting from the displacement of the driving pin, the axial displacement z_{pin}, and the half-opening angle δ_{pin}, as well as a diagram of the thread pitch fluctuation in the case of driving with a conical pin*

The section ellipses of the pin are similar to this ellipse path $\underline{e}$. The ***Figure 5.*** shows some discrete situations, among which at the lowest point, in the case of $\varphi_1 = 0°$, the minor and major axes of the plane section ellipse centred at S_e^0 are r_{pin}^0 and $r_{pin}^0/cos\delta_1$, respectively, while in the case of $\varphi_1=180°$, the section with the highest centre S_e^4, the minor and major axes of the ellipse are r_{pin}^4 and $r_{pin}^4\ /cos\delta_1$.

The minor axes $r_{pin}(\varphi_1)$ and major axes $r_{pin}(\varphi_1)\ /tg\delta_1(\varphi_1)$ of the ellipse sections of the driving pin depend on the angular rotation (φ_1), and their magnitude can be calculated using the next correlation

$$r_{pin}(\varphi_1) = tg\delta_{pin} \cdot tg\delta_1 \cdot r_c \cdot (1 - cos\varphi_1) \tag{5}$$

At the same time, the radii starting from the centres of the planar ellipses are also the tangent vectors $\mathbf{t}(\varphi_1)$ of the elliptical path $\underline{e}$.

The direction of the tangent vectors $\mathbf{t}(\varphi_1)$ of this elliptical path $\underline{e}$ depends on the value φ_3 associated with the angular rotation φ_1, while its length is a function of the size of the section ellipses of the driving pin (***Figure 5.***). The sum of the endpoints of these tangents and elliptical radii is nothing but the motion transfer curve $\underline{m}_{trans}^{spir}$. On image plane 4 shown in ***Figure 5.***, the size of the radius R_{trans}^{spir} from the centre point z_{t1F}^{spirIV} pointing to the motion transmission curve varies, and consequently the pitch fluctuation changes according to the blue curve outlined in the diagram.

3. Investigation of thread fluctuation as a dependence on the distance of the drive pin and the main spindle

Moving the cone-shaped driving pin with half opening angle δ_{pin} on the circular path with radius r_c to the circular path with radius (r_c-Δr_c) has an effect on the pitch of the conical worm, and the geometric relationships necessary for its examination has been interpreted according to ***Figure 6***.

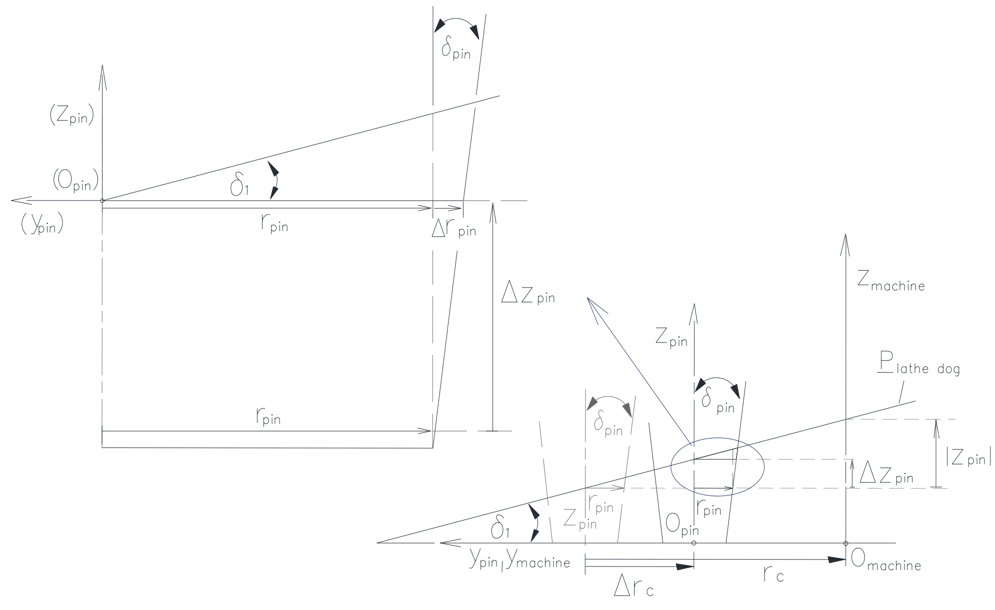

Figure 6. *Schematic illustration of the relationships resulting from the displacement of the counter-conical drive pin*

Between the geometric elements presented the following relations occur

$$|z_{pin}|/r_c = \tan\delta_1 \tag{6}$$

$$\left(|z_{pin}| - \Delta z_{pin}\right)/\left(r_c - \Delta r_c\right) = \tan\delta_1 \tag{7}$$

$$\tan\delta_{pin} = \Delta r_{pin}/\Delta z_{pin} \tag{8}$$

The plane of the lathe dog face is perpendicular to the [$y_{machine}$, $z_{machine}$] plane. The intersection of the right circular cone drive pin with the lathe dog face plane $\underline{P}_{lathe\ dog}$ results in a variable ellipse whose shape depends on the driver parameter φ_1. The minor axes of these can be primed as a function of

$$b(\varphi_1) = \left(r_{pin} + \Delta r_{pin}\right) = tg\delta_{pin} \cdot tg\delta_1 \cdot r_c \cdot \left(1 - cos\varphi_1\right) + \Delta r_{pin} \tag{9}$$

while the major axes expression

$$a(\varphi_1) = \left(r_{pin} + \Delta r_{pin}\right) \cdot cos\delta_1 \tag{10}$$

Decreasing the radius r_c of the circular path by -Δr_{pin} or increasing it by +Δr_{pin} changes the pitch fluctuation to the extent shown in ***Figure 7.a)***. The "pitch deviation" as a function of changing r_c can be seen in ***Figure 7.b)***.

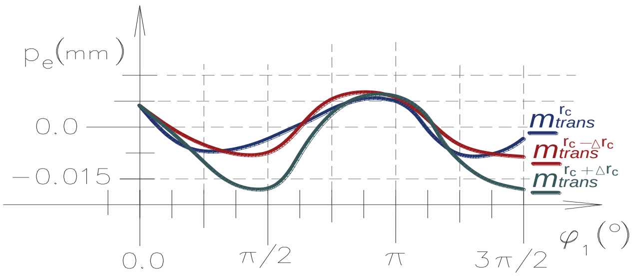

Figure 7.a) *The pitch fluctuation resulting from the displacement of the conical drive pin relative to r_c = 30mm*

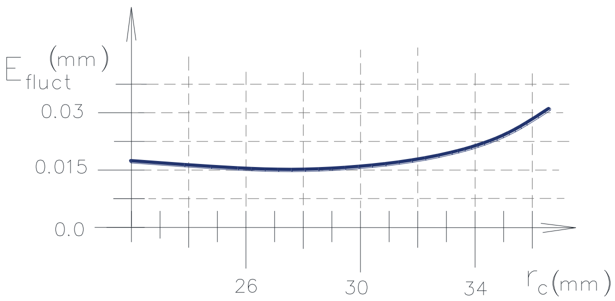

Figure 7.b) *The pitch error as a function of the displacement of the conical drive pin*

The pitch fluctuation induced by the counter-cone-shaped drive pin with a half-angle $\delta_{pin}=\delta_1$ in the case of the fixed r_c value was investigated through the motion transmission curve $\underline{m}_{trans}^{rc}$, while the pitch fluctuations induced by an $r_c+\Delta r_c$ respectively an $r_c-\Delta r_c$ value in the $\underline{m}_{trans}^{r+\Delta rc}$ and $\underline{m}_{trans}^{r-\Delta rc}$ transmission curves. It can be concluded that the fluctuation of driving with a conical driving pin decreases to a small extent with a small reduction of the r_c value, and increases to a large extent with a small increase of the r_c value in the examined case.

4. The thread fluctuation depending on the taper angle of the cone shaped drive pin

As it was shown before in the chuck face plane, the axis point of the cone shaped drive pin with a half taper angle $\delta_{pin}=\delta_1$ follows a circular path with radius r_c, but the geometric location of the pin axis points in the face plane of the lathe dog $\underline{P}_{\text{lathe dog}}$ is an elliptic path $\underline{e}$. By changing the half taper angle of the driving pin, the size of the section ellipses change as a function of δ_{pin} (***Figure 8.***). Considering the drive pin of the spiroid worm, the following relations must be fulfilled

$$r_{pin} / \left| z_{pin} \right| = \tan \delta_{pin} = tg\delta_1 \tag{11}$$

$$\left(r_{pin} + \Delta r_{pin} \right) / \left| z_{pin} \right| = \tan \delta_{pin}^{+} \tag{12}$$

Sectional ellipses that also vary as a function of φ_1, have dimensions of half minor axis ($r_{pin}+\Delta r_{pin}$) and half major axis ($r_{pin}+\Delta r_{pin}$)$\cdot cos\delta_1$.

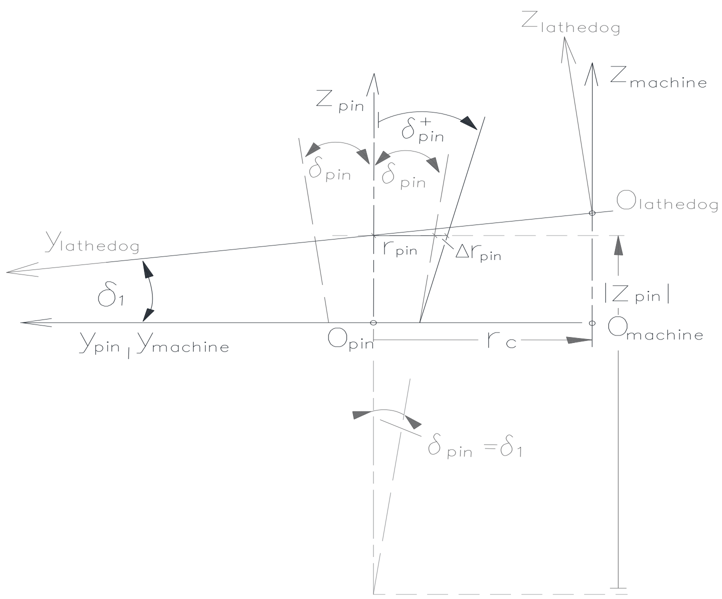

Figure 8. *Geometric relations of changing the half taper angle of the conical driving pin*

Every point on the path $\underline{e}$ is the center of a section ellipse. At the end point of the radius of the section ellipses, the pin contacts the lathe dog, which touches the path $\underline{e}$. The totality of these tangent endpoints is the motion transfer curve $\underline{m}_{trans}^{\delta pin=\delta 1}$.

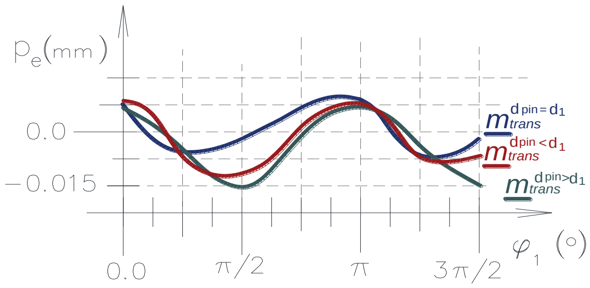

Figure 9.a) *Thread pitch fluctuation resulting from changing the half opening angle of the conical drive pin*

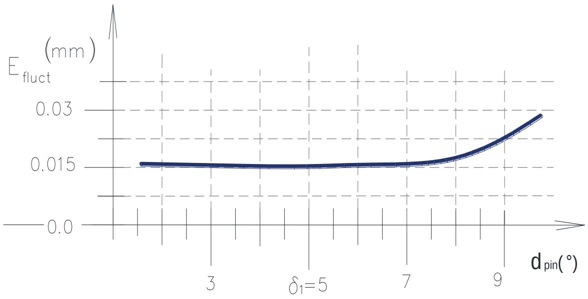

Figure 9.a) *The pitch error deviation as a function of the half-opening angle of the conical drive pin*

It can be concluded that in the case of a small reduction of the half-opening angle $\delta_1=\delta_{pin}$ of the cone-shaped driving pin, the pitch fluctuation of the spiroid screw becomes damped, while in the case of a small increase in the half-opening angle, the pitch fluctuation of the spiroid screw increases.

5. Summary

During the process of the adjusted conical worm machining, the motion transferred between the drive pin and the lathe dog occurs as a permanently point contact. The contact point of the pin and lathe dog moves on a circular path in the plane perpendicular to the main spindle axis, but on an elliptical path in the plane perpendicular to the spiroid worm axis. The correlations between the rotation angles corresponding to the two different trajectory curves and the axial displacement were established.

The function-like relationship between the displacement of the counter-cone drive pin and the pitch fluctuation was identified with the aim of enabling the cost-effective production of the spiroid worm within the required manufacturing accuracy.

The influence of the taper angle of the drive pin on the thread pitch fluctuation was presented in an analytical form. The function can be used to estimate the taper angle of the driving pin in dependence of the accepted limit of the pitch error, and thus a cost effective solution still be offered.

4. Acknowledgment

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