APPLYING THE MOST FREQUENT VALUES ASSISTED HILBERT TRANSFORM INTO SEISMIC ATTRIBUTES

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Abstract
In the present study, we developed a new method to calculate two of the most important seismic attributes that can able to give a huge impact in the interpretation field leading to a better geological and geophysical interpretation of the data. The first attribute was the Reflection Intensity with great improvement in detection of thin layers that show low reflection coefficient and enhance the ability to follow unconformity, the other one is the Instantaneous Phase to discover the discontinuity of events. The new method depends on combining the most frequent values (MFV) with inversion to calculate the Hilbert transform and applying hybrid Seismic Denoising, using also dual inversion and calculated Steiner weight from the iterative reweighted least square method (IRLS). Scaled Hermite function in a series expansion was used to discretize the spectrum of the time trace. The procedure followed with adjusting the process using multi-windows technical on the generated synthetic seismic traces loaded with Gaussian and Cauchy noise at the same time. The result of the method was shown significant effects on outliers or normal Gaussian noise when applied to two seismic attributes better than the old way using the Discrete Fourier Transform (DFT) method.

Keywords: *Seismic attributes, Inversion-based Fourier Transform, Robust Hilbert Transform*

1. Introduction

The quality of information within the interpretation process is predominantly contingent upon the efficiency of applied data processing algorithms. In the realm of applied geophysics, the interpretation of results derived from geophysical methods is highly sensitive to the data quality, making it a critical determinant of successful outcomes. This paper concentrates on the development of the applied inversion-based Fourier transformation method in the context of seismic attributes. The ameliorative procedures employed in applied geophysical data acquisition over the past two decades necessitate progressive data processing techniques. The conversion process from the time domain to the frequency domain is frequently employed in geophysical data processing due to its capacity to augment signal processing interpretation. In seismic attributes, the discrete Fourier transformation (DFT) is customarily applied to identify the frequency components that help in the characterization of complex reservoirs, for instance. Noise presents in seismic data in most surveys generating unwanted features, it can be divided into incoherent (represent the random) and coherent noise.The random noises can be shown as a temporal direction and also spatially random noise, the directional one of noises appears in late times compared to the early reflection data. About the coherent noise, it can be shown as linear noise, reverberations, and multiples. Also, maybe appear in the land data surveying which is called ground roll which dominates the reflection energy in recorded data. Coherent noise has low frequencies and large amplitudes (Yilmaz, 2001). The aforementioned noises can be a reason for significant artefacts that cause an important negative impact on interpretation outputs results, starting from simple structural attributes for example through pre-stack impedance inversion to Amplitude Vs. Angle (AVA) analysis. These noises are linearly projected into the frequency domain during Fourier transformation, which means the traditional FT algorithms are sensitive to the non-Gaussian noise most. Inverse problem theory comprises an array of methods capable of negating the noise and treating it as an overdetermined inverse problem (Menke, 1984; Dobróka et al., 1991). The 1D Fourier transformation was handled by Szegedi and Dobróka (Szegedi and Dobróka, 2014) as a robust inverse problem utilizing the IRLS algorithm with Cauchy-Steiner weights, the results appeared as a significant reduction in noise sensitivity of the continuous Fourier transform using robust inversion. Subsequently, the series expansion-based inversion method after that was used in the interpretation of borehole data (Szabó, 2011; Szabó, 2015) and also in processing induced polarization data (Turai, 2011). Marashly and Dobróka (2021) demonstrated that the IRLS inversion-based Fourier transform method yielded promising results in noise resistance when applied to a synthetic wavelet. They proposed an algorithm based on a dual inversion. The first step was applying IRLS-FT to the Fourier transform and in the second one, after the production of the transformed spectrum of Hilbert was calculated, the inverse Fourier transform is also calculated using an inversion-based robust/resistant process. Seismic attributes can be traced back to the seismic exploration methods. In most exploration, the primary objectives are the accurate imaging of structure in time and depth. This aids in visually enhanceing the appearance of seismic features in interpretation because it's directly sensitive to geological features and reservoir property (Chopra and Marfurt, 2007). Consequently, correctly characterizing the amplitudes of the reflection is crucial. We believe that the amplitudes are accurately rendered, and the features obtained from them can be used in interpretation. Those are considered seismic attributes (Taner et al., 1979). There are two methods to calculate attributes, one from typical post-stack and the other using additional information from seismic traces before stacking. The most important attribute is the seismic amplitude of the fact that the amplitude of reflection is directly related to the porosity or the saturation of the underlying formation. As per practice, the interpretation of attributes is non-unique, necessitating a calibration process. Fourier Transform has numerous applications in the field of seismic, one of which is the creation of attribute stations. Szegedi and Dobróka (Szegedi and Dobróka, 2014) proposed a novel definition of a Fourier Transform procedure (IRLS-FT) that was performed by using an inversion basis. In data processing, accuracy could be enhanced by maximizing the signal-to-noise ratio.

This paper introduces a new essential methodology designed to assist researchers in the suppression of outlier noise within seismic attributes. This approach holds the potential to enhance the signal-to-noise ratio thereby exceeding conventional techniques. We used this method in the calculation of the Hilbert transform for full syntactic systemic trance to calculate two of the most important seismic attributes. The initial stage of the process started with creating the synthetic seismic trace using the analytical signal as a presentation of a source, which then propagates through earth layers. The demonstration of the reflected recorded signals by a receiver will be present as a convolution between the pre-created reflection coefficient with the generated signal along the seismic trace containing seismic reflections data. The reflection time series embody the impulse response of the artificial layered ground. A seismic trace represents a combined response of artificial layered ground. Under the presumption that the pulse shape remains unaltered, each spike associated with the reflection coefficient function possesses a random amplitude value. This value undergoes vertical alterations in relation to spatial distribution, thus resulting in an inclined layer. Seismic traces have reflection signals not seen without applying processing algorithms to approach sufficient accuracy. These processing algorithms analyzed the input signal to correct the data and calculate the acoustic impedances. In a real environment, unwanted waves appear in the seismic traces cases considerable deformation in it. These noises are generated by different sources like surface waves, or damaged seismic records, the undesirable signals consist of incoherent noises that will demonstrate in this paper through Gaussian and Cauchy noise. Following this, the Discrete Fourier Transform (DFT) was applied to the noisy synthetic traces to compare the convolutional method with the presented one using two well-known attributes.

2. Analytical synthetic seismic traces

In the initial stage of the process, we started to create an analytical signal (u_0) in the synthetic seismic traces (analytical or complex channel) which was introduced by the Nobel Prize laureate Hungarian physicist Dénes Gábor (Gábor, 1946). His idea was to use square-integrable complex functions as elements of the so-called Hilbert space by introducing the analytical signal (Gábor, 1946). We used a 10 Hz wave packet localized at 0.1 sec with a sampling interval of 0.005 sec in the [–1,1] (sec) domain, we compare the frequency structure of the noised and original data by performing a continuous 1-D wavelet transform (CWT) (*Figures 1* and *2*). Gaussian noise was added to the data set with a standard deviation (0.05), also Cauchy noise was implemented to the seismic traces in random positions for every trace to simulate a real case for noise, 15% of data samples are affected to outliers.

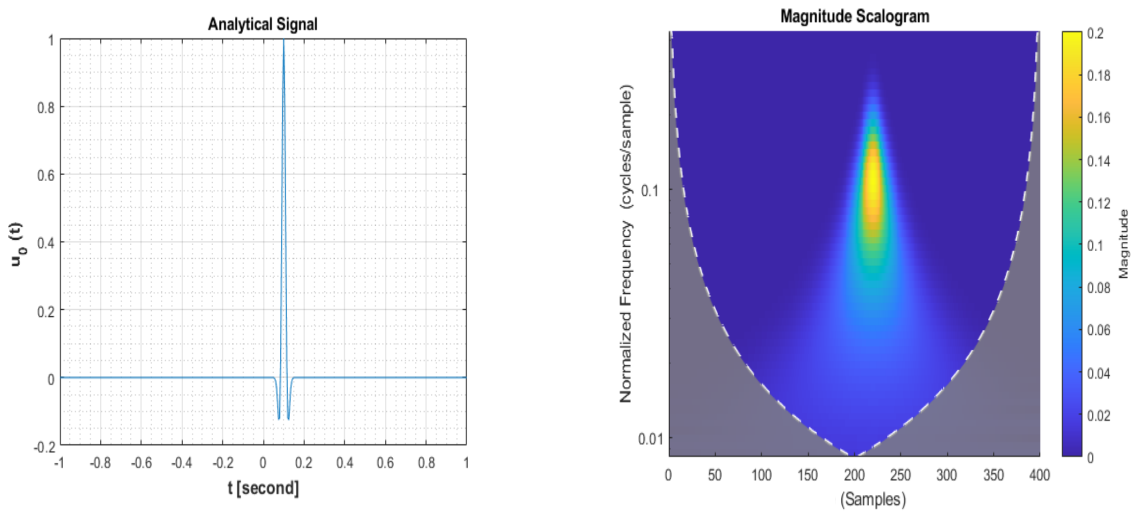

Figure 1. *Analytical Signal and Magnitude Scalogram for the noise-free data set*

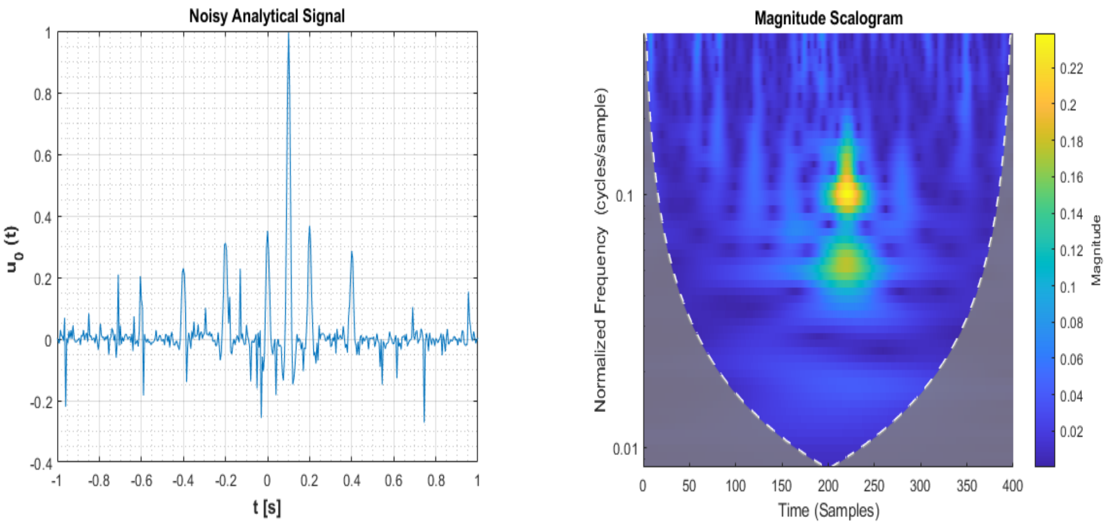

Figure 2. *Analytical Signal and Magnitude Scalogram for the noisy data set*

The convolution of the analytical signal *(Figure 3a)* and reflection coefficient (u_{rc}) in every channel in the time domain was performed to generate the seismic trace (u_{st}) *(Figure 3b)* after that random generate noise with previous properties was added to simulate the real rough environment. In *Figure 3c* magnitude scalogram generated for the first channel for compression between noise and noise-free case. Better visualization of errors added to the first channel is shown in *Figure 4*.

$$U_{st}(\mathrm{t}) = u_0(\mathrm{t}) * u_{rc}(\mathrm{t}). \tag{1}$$

3. Attributes and noise sensitivity

This paper focuses on post-stack attributes, that are computed from the stack data. These attributes are described as process data that comes from properties derived from the analytical trace. The first attribute is reflection strength, which is calculated from the complex trace using the following equation

$$E(t) = \sqrt{T(t)^2 + H(t)^2} \tag{2}$$

where E(t) is signal envelop, T(t) is the seismic trace, and H(t) is Hilbert's transform of T(t). This attribute showed the acoustic impedance contrast that is used for detecting bright spots, thin-bed tuning effects, unconformities, gas accumulation, and other applications. The signal envelope calculated using equation (1) demonstrates the instantaneous energy of the signal and the relation between magnitude and reflection coefficient.

Ricker wavelet (Ricker, 1953) shown in figure 1 was used to generate the seismic trace in order to illustrate the noise sensitivity of the attribute by convulsion it with the reflection coefficients shown in *Figure 2*. The results of the reflection strength can be seen in *Figure 5* using the conventional process for trace free-noise and noise data. Hilbert's transformed functions were calculated using the Discrete Fourier Transform process (DFT) and its inverse (IDFT). The synthetic seismic trace contains 6 layers appearing on 8 channels as *Figure 5* shows.

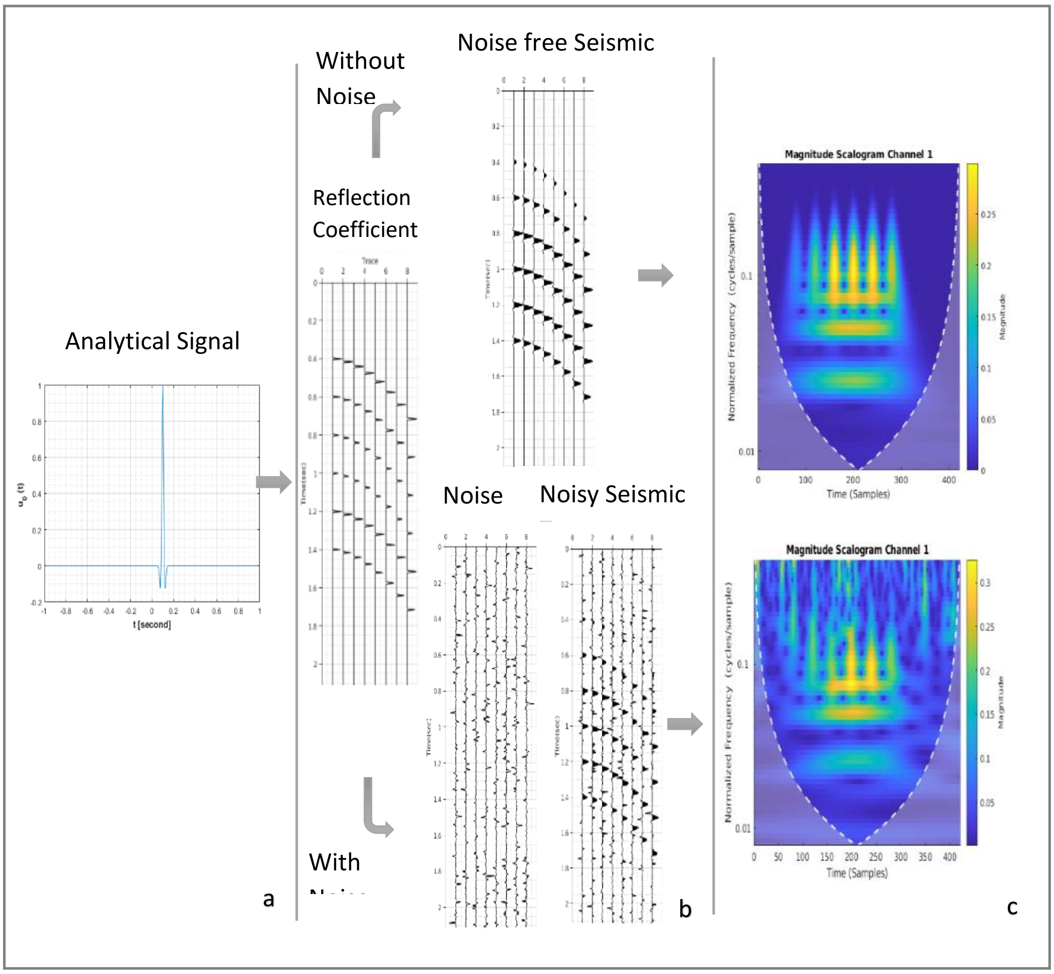

Figure 3. *Synthetic seismic trace generating process with noisy data*

The procedure is also applied to the instantaneous phase attribute shown in *Figure (6)*, it is measured in degrees (–π, π) and it's independent of amplitude. This attribute can show the lateral continuity, and discontinuity of events, and is used for calculating phase velocity. Phase change along the trace if there are only picking problem of layer changes latterly. The attribute can be calculated by the following equation:

$$\phi(t) = \arctan |H(t)/T(t)|. \tag{3}$$

The seismic trace T(t) and its Hilbert transform H(t) are related to the envelope A(t) and the phase ϕ(t) by the following relation:

$$T(t) = A(t)\cos(\phi(t)). \tag{4}$$

$$H(t) = A(t)\sin(\phi(t)). \tag{5}$$

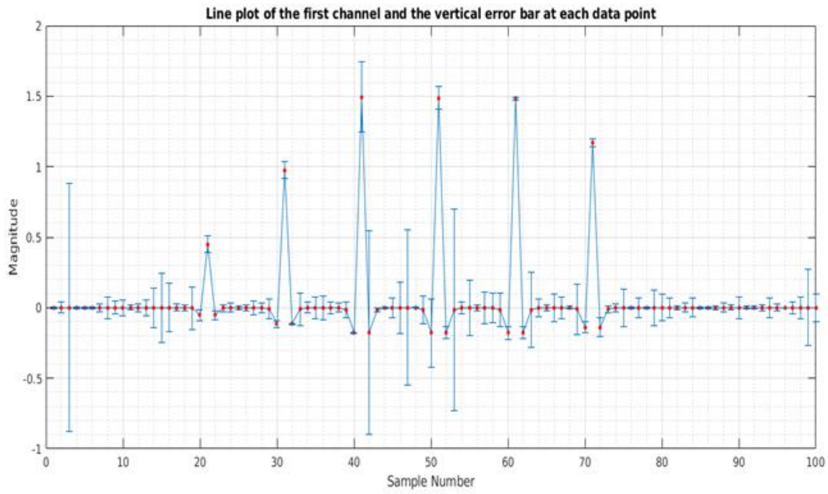

Figure 4. *Line plot of the first channel and the vertical error bar at each data point after adding noise*

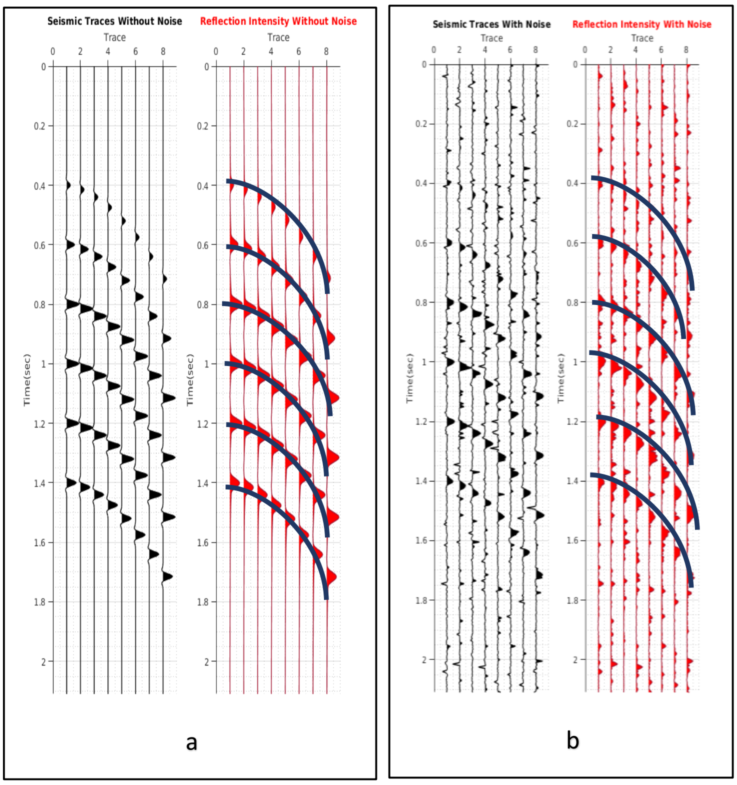

Figure 5. *The synthetic seismic trace with noise on the left side the first seismic attribute (Reflection Intensity on the right) calculated conventionally [(a) Before adding noise, (b) After adding noise]; dark blue lines represents the layers*

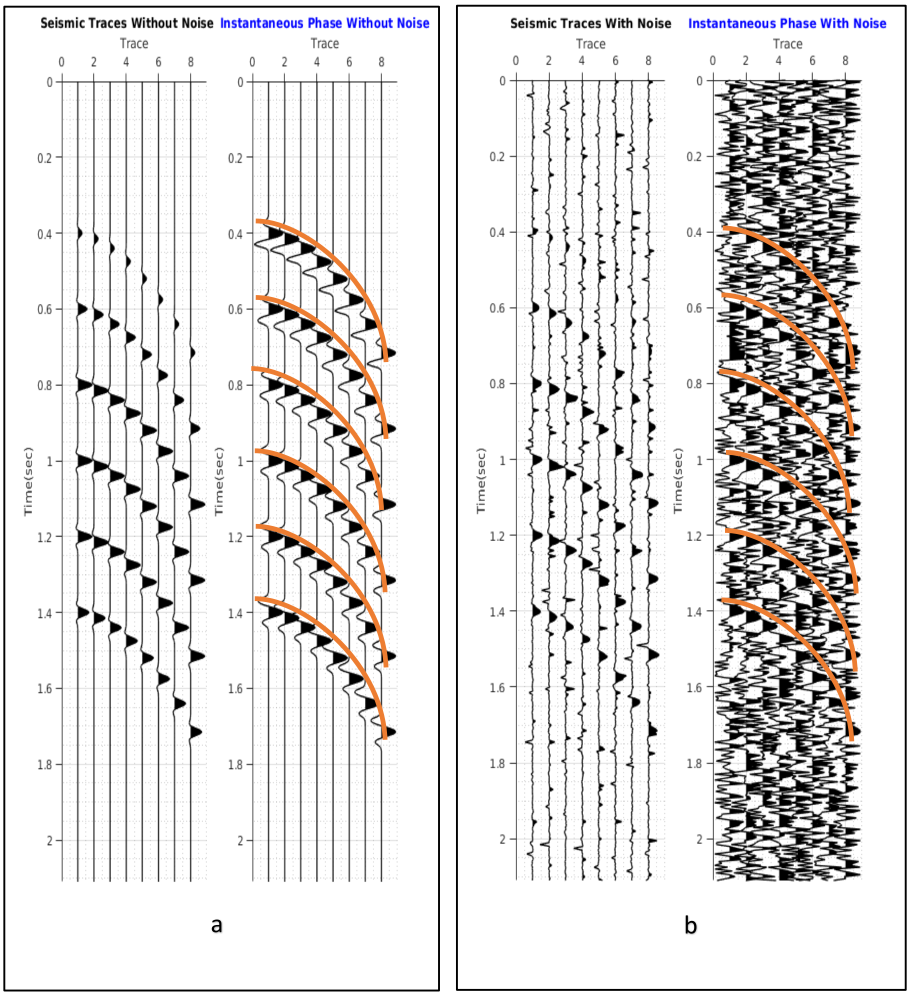

Figure 6. *The synthetic seismic trace with noise on the left side the second seismic attribute (Instantaneous Phase on the right) calculated conventionally [(a) Before adding noise, (b) After adding noise]; orange lines represents the layers*

4. The inversion-based Fourier transform (IRLS-FT) used in robust generation of Hilbert transform

The Fourier transform is a well-known method for creating a link between the time and frequency domain of the signal using the following equations:

$$U(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(t) e^{-jwt} dt \, , \; u(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(\omega) e^{j\omega t} d\omega \qquad (6)$$

The term U(ω) represents the frequency spectrum calculated from the time signal u(t), which is commonly used as a continuous function of the complex value. The spectrum is formulated in terms of a suitably chosen system of basis functions because we used series expansion-based discretization:

$$U(\omega) = \sum_{n=1}^{M} B_n \Psi_n(\omega) \tag{7}$$

The B_n is described as complex expansion coefficients, $\Psi_n(\omega)$ is defined as the n-th known basis function, and M is the total number of the basis function. To define an overdetermined inversion method, we designate the forward problem which is the inverse Fourier transform, specified in the case of the k-th measurement datum as:

$$u^{(calc)}(t_k) = u_k^{(calc)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(\omega) e^{j\omega t_k} d\omega. \tag{8}$$

The following equation contains the expansion coefficients, which can be calculated by minimizing some norm of the deviation vector.

$$e_k = u_k^{(measured)} - u_k^{(calculated)} = u_k^{(measured)} - \sum_{n=1}^{M} B_n G_{kn}. \tag{9}$$

Solving the inverse problem, we can also calculate the spectrum at any frequency via:

$$U^{(estimated)}(\omega) = \sum_{n=1}^{M} B_n^{estimated} \Psi_n(\omega). \tag{10}$$

This inversion-based Fourier transform achieves an order of magnitude better signal-to-noise ratio than the traditional DFT process (Dobróka et al., 2017). The spectrum of the signal U(ω) is important in the Hilbert transform procedure, IRLS-FT method in Fourier transform was used to improve the signal to noise ratio afterwards the spectrum is multiplied by -j sgn(ω). For the return to the time domain, we use inverse Fourier transform, and it's applicable by using IDFT and inversion-based inverse Fourier transform.

$$U_k(\omega_k) = \sum_{n=1}^{M} B_n G_{kn.} \tag{11}$$

where:

$$G_{kn} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi_n(t) e^{-j\omega_k t} dt = \mathcal{F}_k\{\Psi_n(t)\}\,. \tag{12}$$

The G_{kn} term represents the Jacobi matrix, which contains the elements of inverse Fourier transforms of the basis function system.

The Iteratively Reweighted Least Squares (Szegedi and Dobróka, 2014) method using w_k Cauchy-Steiner weights (Steiner, 1997) to make the Fourier transform more robust was implemented by minimizing the weighted norm

$$E_w = \sum_{k=1}^{N} w_k e_k^2\ . \tag{13}$$

In this way, even the existence of the outliers in the data set appears, and the weighted norm shows good results for the iteratively reweighted inverse problem with the normal equation

$$\mathbf{G}^T \mathbf{W}^{(j-1)} \mathbf{G}\, B^{(j)} = \mathbf{G}^T \mathbf{W}^{(j-1)} u^{measured}. \tag{14}$$

5. Integrating the inversion-based Fourier transform with Improved Wavelet Block Thresholding

The implementation of the seismic denoising method which is based on continuous wavelet transform with hybrid block thresholding (Mousavi and Langston, 2016) in the present method gives a new ability to enhance the attributes output and remove the noise in a very effective way. The idea had used a multi-layer technic in the process to obtain the optimum noise reduction value for the input seismic trace.

In the first step, we applied the Inversion-Based Fourier Transform to achieve high noise resistance to excessively noisy data by applying IRLS-FT to the Fourier transform and to produce the spectrum of Hilbert transform. The inverse Fourier transform is also calculated using an inversion-based resistant process *(Figure 10)*, after that, we used block thresholding to enhance the efficiency of the signal on the time-frequency plane, which makes denoising dynamically adjusted to signal shape *(Figure 11)*, the last step applied again the Inversion-Based Fourier Transform on the result data without any changes in the parameters.

In the inversion-based Fourier transform method, we split the seismic traces into windows during the processing as *Figure 10* shows. This technique helps us to apply this method to any time frame of the seismic trace without facing any issues in the procedure. The width of windows is not static it's related to the distance in data space and it's generated depending on the output of the compressed signal results *(Figure 10)*.

To do this we performed wavelet decomposition using MATLAB which is based on three main important steps: the first, enter the function gets filters, the second, filter function sequence and filter coefficient convolution, final (third) is obtaining the coefficient of decomposition downward. *Figure 7* shows the full process (Zhao and Yan, 2010).

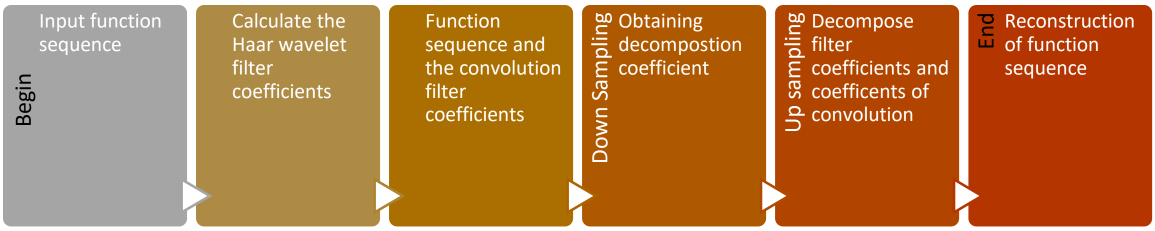

Figure 7. *Program flow chart of the decomposition process*

The decomposition process was made to each trace on the synthetic seismic traces, *Figure 8* shows the first one and the output of the decomposition at level 3.

Every wavelet packet can be characterized by frequency, scale, and position in time, those functions create a set of decomposition bases for the analyzed each synthetic seismic trace in this study.

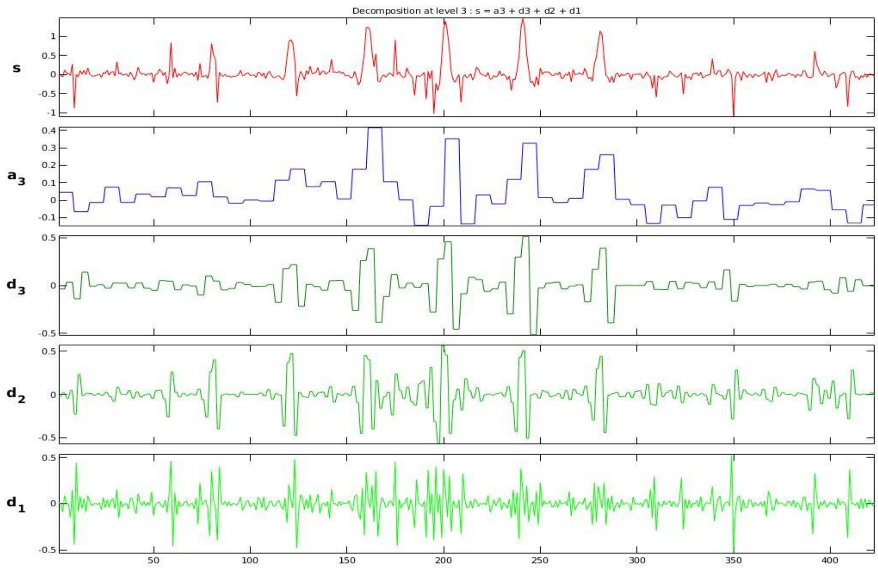

Figure 8. *The output of decomposition of the first trace (s represents the signal, a3 to d_1 represent the levels of detail coefficients)*

A colored table of coefficients can show the wavelet packet decomposition, the row index is related to the scale factor, meanwhile, the column index represents the frequency and time position for each row. To reduce the volume of transmitted or processed data without losing significant information data compression has been applied, the concept can be accurately approximated using the elements that are a small number of approximation coefficients (at a suitably chosen level "Global Threshold") and the detail coefficients with a proper threshold which is in this study 0.5 that gives 79.28% returned energy, the selected threshold able us to define the proper number of windows that in each one will apply the FT inversion-based method without losing important information. A Global Threshold value can be set based on the environment and how much noise is present in the data *(Figure 9)*.

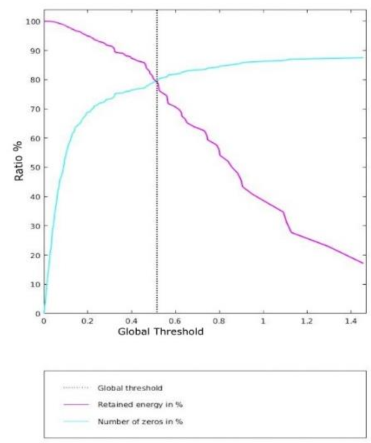

Figure 9. *The relation between returned energy and the number of zeros based on the Global Threshold value in the first synthetic seismic trace*

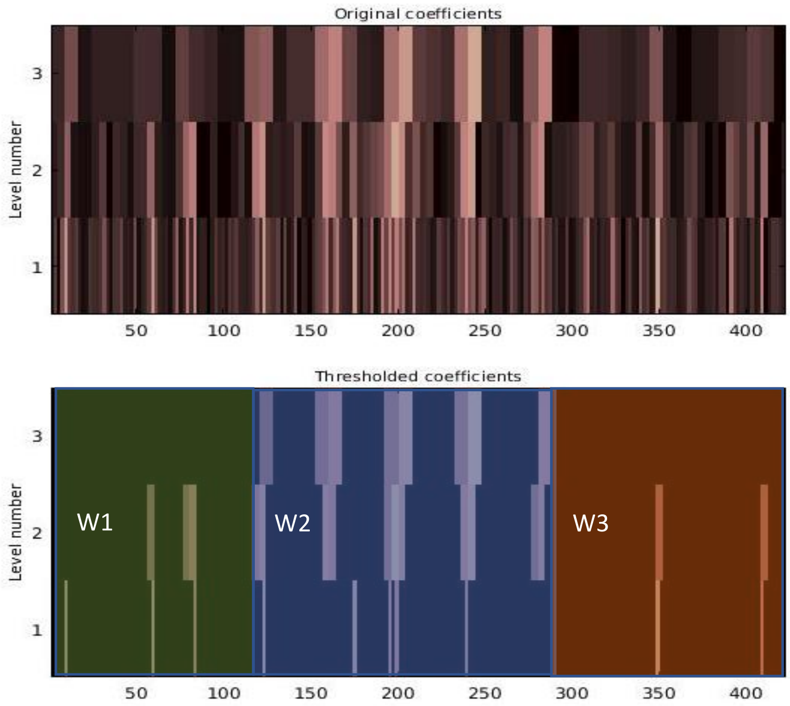

Figure 10. *Using the Thresholded coefficients to determine the number of windows (W)*

We can see *(Figure 10)* that we determine 3 main windows for the first synthetic seismic trace. The other traces will be treated in the same way to assign the right value for each one. In *Figure 11* we can notice the residuals of the compression signal process output.

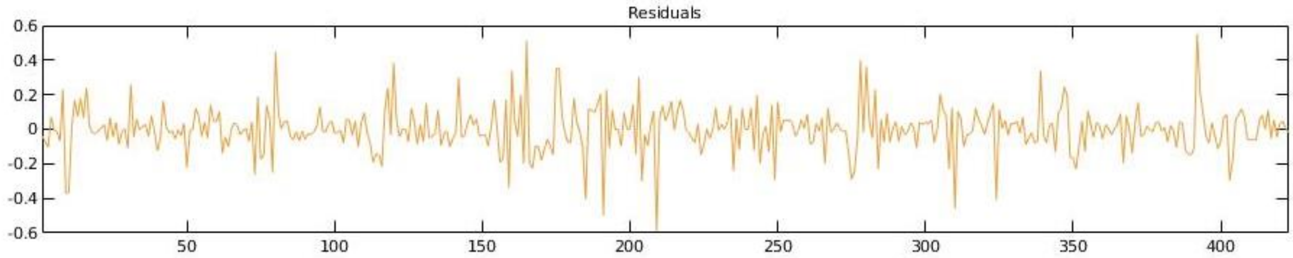

Figure 11. *Residual signal after compression process for the first seismic synthetic trace*

As a second step, we used hybrid Seismic Denoising to enhance the signal to noise ratio. The main reason behind adding this method is related to the sensitivity of the second attribute which is the instantaneous phase. Hybrid Seismic Denoising is considered a useful tool in the seismic denoising area which is based on preprocessing step using HOS (higher-order statistics) criteria, wavelet denoising based on the BT (Block Threshold), and a postprocessing step using Wiener filtering *(Figure 12)*. In addition that this method improves wavelet thresholding by removing high-power noise components *(Figure 13)*. Also, BT improves the efficiency of wavelet thresholding by considering the information of neighboring coefficients on the time-frequency plane (Mousavi and Langston, 2016).

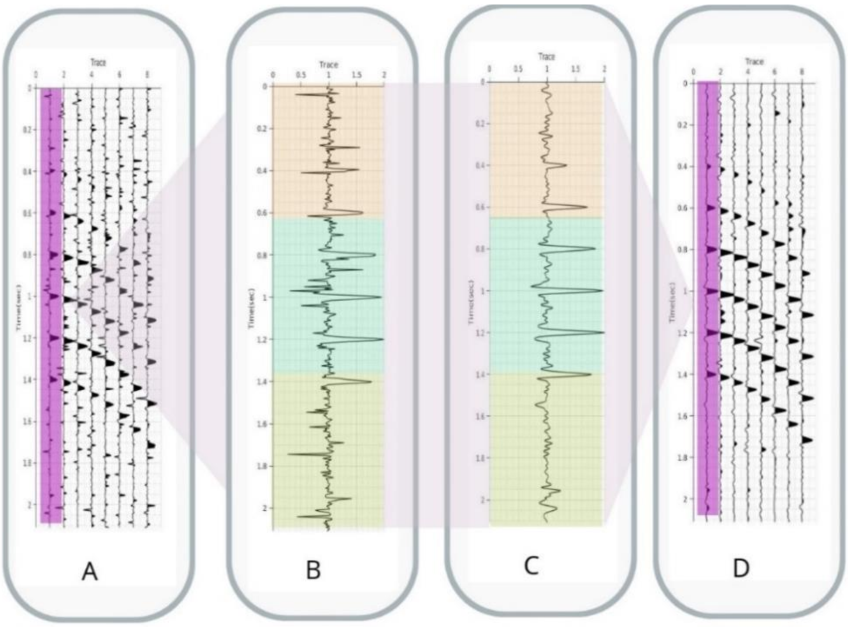

Figure 12. *(a) Synthetic seismic trace with noise, (b) First trace with the noise, (c) First trace after applying an inversion-based Fourier transform, (d) Synthetic seismic trace after applying the new method*

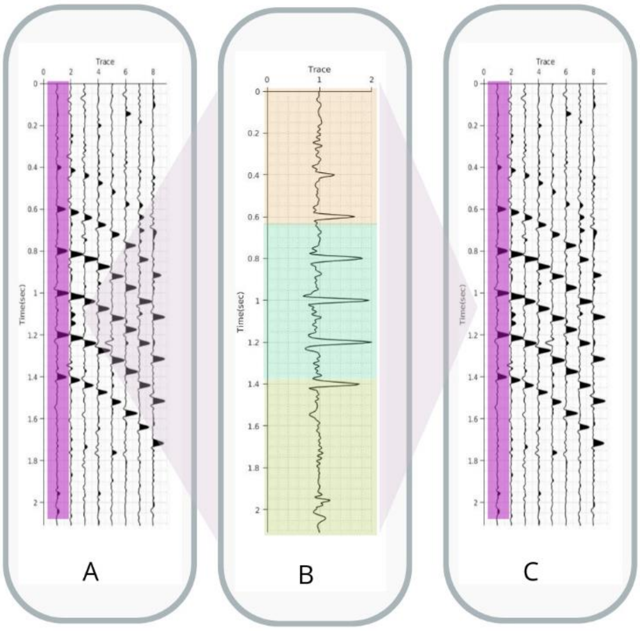

Figure 13. *(a) Synthetic seismic trace after applying the inversion-based Fourier transform method, (b) First trace after applying hybrid Seismic Denoising, (c) Synthetic seismic trace after applying hybrid Seismic Denoising*

6. Numerical tests

In *Figures 5* and *6*, examples were shown of noise and noise-free for two main important attributes that are used in seismic processing using the DFT method. In *Figures 14* and *15*, the two seismic attributes (the Reflection Strength, and the Instantaneous Phase) are generated by the robust inversion-based FT. The new method for noise-free input gives the same result as the Fourier transform using the traditional DFT process and we added to the Instantaneous Phase a second layer of processing using hybrid Seismic Denoising.

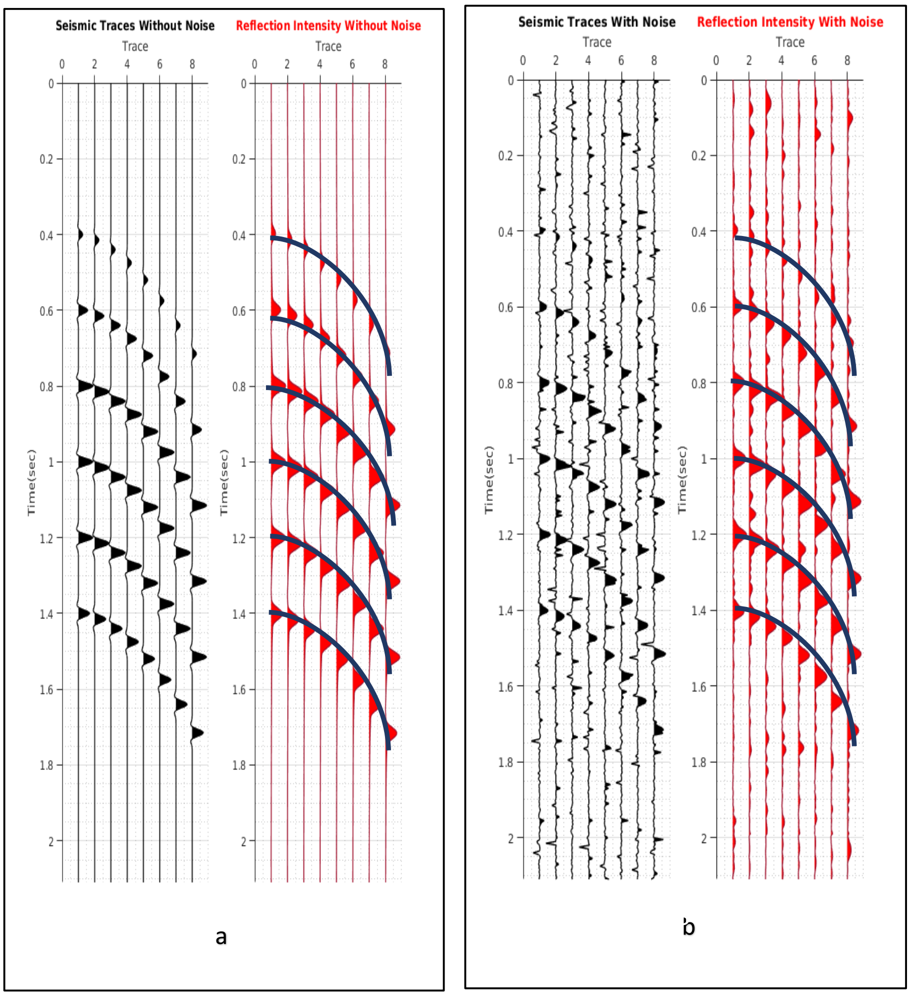

Figure 14. *(a) First seismic attribute (Reflection Intensity) calculated in the new method without noise. (b) First seismic attribute (Reflection Intensity) calculated in the new method with noise*

The results presented in *Figures 14* and *15* confirm a significant improvement in the relation signal/noise when using a robust inversion-based Fourier transform and an inverse Fourier transform built on a robust inversion basis to produce the Hilbert transform.

To characterize the quality of the results, we introduce distance in the data space (15)

$$d = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(Ust^{(noisy)}(t_k) - Ust^{(Noise\_free)}(t_k))^2} \quad (15)$$

which is d(1) = 0.2074 for data set using the conventional DFT and d(2) = 0.0872 using the new method for the first attribute (Reflection Coefficient). For the second attribute (Instantaneous Phase) the distance is d(3) = 0.5723 for the data set using the conventional DFT and d(4) = 0.2481 using the new method, which gives us more than 70% enhancement for the comparison to the traditional one.

In *Figures 16* and *17* we can see the enhancement of the attribute results between different discussed methods. Significant improvement is noticeable in figure 16 between B, C, and D for the first attribute, it's clear that the first thin layer appears using the introduced algorithm compared to the DFT method, also in the second attribute in *Figure 17* the noise reduction help to produce better output comparing to the DFT, the middle layers in D are more noticeable after applying hybrid seismic denoising on inversion output data, which give more ability to have much clear seismic section than ever.

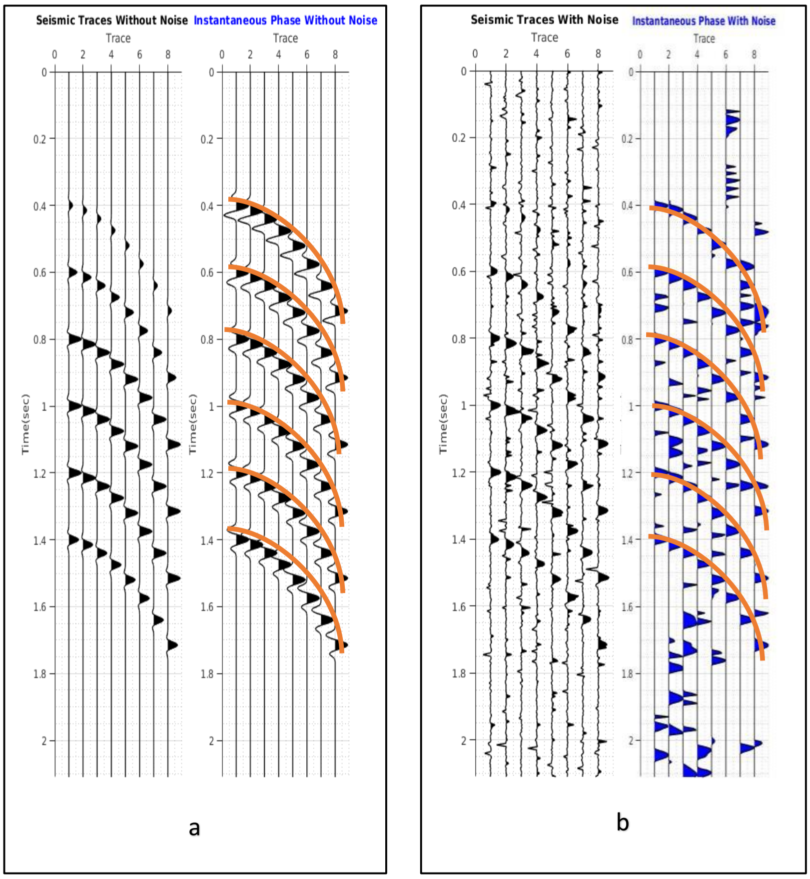

Figure 15. *(a) Second seismic attribute (Instantaneous Phase) calculated in the new method without noise. (b) Second seismic attribute (Instantaneous Phase) calculated in the new method with noise*

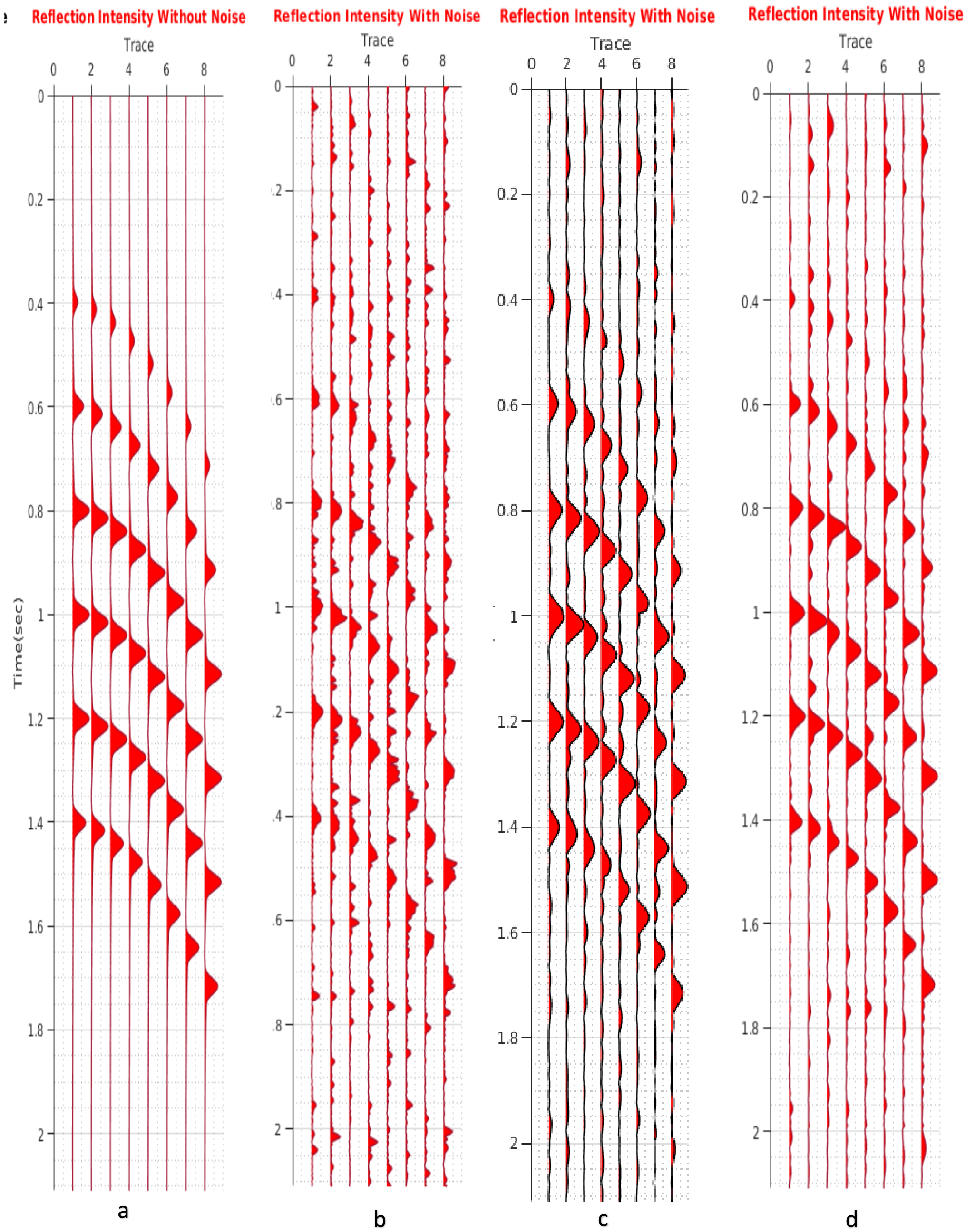

Figure 16. *First seismic attribute with different methods, (a) DFT without noise, (b) DFT with noise, (c) FT inversion-based method, (d) inversion-based method with hybrid seismic denoising*

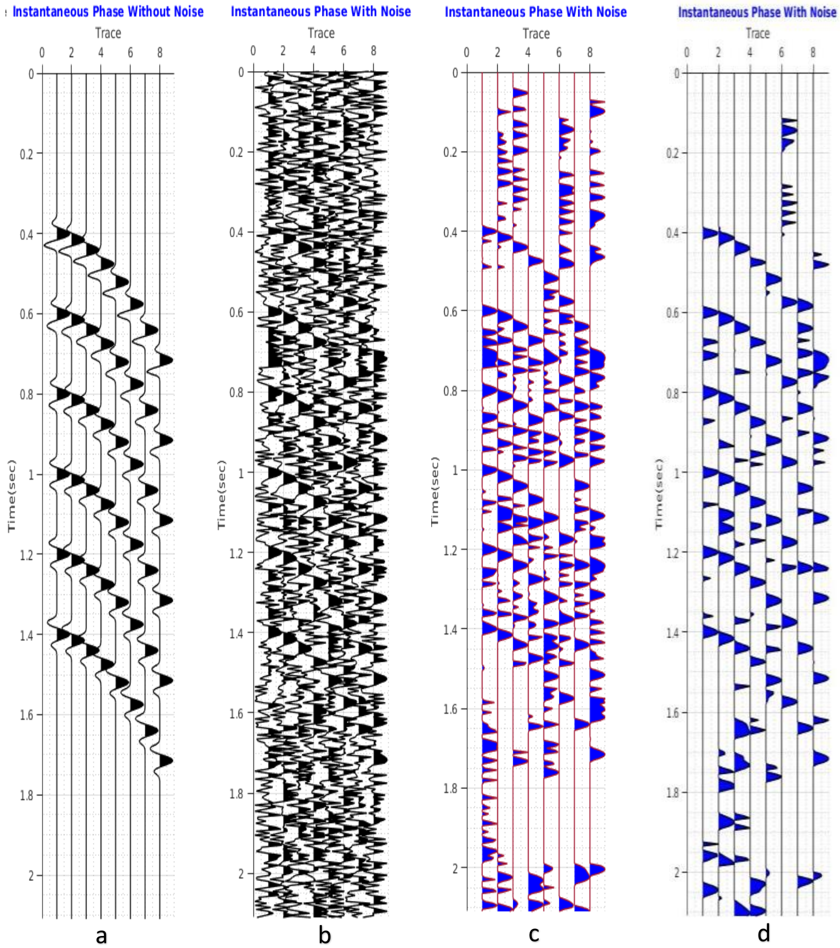

Figure 17. *Second seismic attribute with different methods, (a) DFT without noise, (b) DFT with noise, (c) FT inversion-based method, (d) inversion-based method with hybrid seismic denoising*

7. Conclusions

The empirical evidence garnered from this study suggests that the newly developed IRLS inversion-based Fourier transform method has achieved notable resistance to outliers in the context of noisy data pertaining to seismic attributes. Furthermore, the application of the hybrid seismic denoising method in the second attribute appears to enhance the accuracy of the resultant data as a additional process. The outcomes of this study thus endorse the notion of employing this innovative method in the computation of the Hilbert transform.

This research underscores the significance of utilizing algorithms like IRLS. The first step involves applying the IRLS-FT to the Fourier transform. Subsequently, following the production of the Hilbert transform spectrum, the inverse Fourier transform is also calculated, utilizing an inversion-based robust/resistant process. This methodology was employed across the entire noisy synthetic seismic section, yielding a satisfactory signal-to-noise ratio through the reduction of outliers and noisy data via the inversion -based Hilbert transform process.

The current findings have significant implications, particularly for enhancing the interpretation of seismic data, recommending a shift towards the utilization of more robust computing systems. The demonstrated success of the IRLS inversion-based Fourier transformation, combined with the hybrid seismic denoising method, marks a promising advancement in seismic data interpretation and warrants further exploration and application in the field. The study thus paves the way for further development and refinement of these novel algorithms, with the potential to revolutionize seismic data analysis.

8. Acknowledgements

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