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DIRECT KINEMATICS OF A TILTING TABLE USING DENAVIT – HARTENBERG CONVENTION

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Abstract
The Denavit – Hartenberg convention will be used to determine the kinematic parameters of the analysed structure. This convention uses homogeneous transformation matrices. The 4x4 homogeneous transformation matrices can contain rotation and translation matrices. The common elements formed by the first three rows and the first three columns of the 4x4 matrix define the orientation of the joint, while the common elements formed by the first three rows and the fourth column of the 4x4 matrix defines the position of the joint. The purpose of the Denavit – Hartenberg method is to solve the direct kinematic problem, which is nothing more than determining the coordinates interpreted in the world coordinate system of the TCP point from the joint parameters. During the analysis of the structure, MAPLE and CREO software were used to perform the calculations and check.

Keywords: *direct kinematics, Denavit – Hartenberg, matrix transformation, parallel kinematics, coordinate system*

1. Introduction

The kinematic analysis of the tilting table can be divided into two parts in multibody mechanics, one is the direct kinematic problem, the other is the inverse kinematic problem (Ye et al., 2020; Waldron and Schmiedeler, 2016). In the case of the direct kinematic task, the trajectory of a point of the mechanism (a point of the tilting table) is calculated as a function of the movement of the joints (Gerhard and Kotsireas (Eds.), 2020). The direct kinematics problem is easy to solve in general. In the case of inverse kinematics, the task is reversed, when the trajectory of the end effector is given and it is necessary to calculate the movement of the joints from this. The paper develops the direct kinematics of a structure with parallel kinematics and presents the solution to the problem.

The paper presents homogeneous coordinates in the second chapter, and the third chapter continues with the description of homogeneous transformation matrices. The Denavit – Hartenberg convention uses homogeneous transformation matrices to create the model of the analysed structure, the tilting table. The model of the structure, which is described in the fifth chapter, was created using the Denavit – Hartenberg convention. While the solution to the direct kinematics problem of the examined structure can be found in the sixth chapter. Finally, the seventh chapter summarizes the completed tasks.

2. Homogeneous coordinates

Homogeneous coordinates realize the n+1 dimensional space using n dimensions. In the 2D Cartesian coordinate system, a point can be specified with its two coordinates x and y. It is clear that the point lies in the x – y plane, if we shift this plane along the z axis of a 3D coordinate system with w, then the coordinates of the point can be characterized by the three numbers x, y, w. If the number triple x, y, w satisfies the rule x/w, y/w, 1, then the point x, y of the Cartesian coordinate system is described by the homogeneous coordinates x, y, w (Zhou et al., 2015; Jazar, 2010; Duncan, 2005).

3. Homogeneous transformation matrices

In this chapter, homogeneous transformation matrices will be presented. How the homogeneous transformation matrix is constructed in the case of translation and rotate, and its application will be explained through an example.

3.1. Translation

Shifting a point in space with a vector characterized by three numbers (a, b, c) can be done with the help of the following matrix:

$$T_t = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

3.2. Rotate

In the Cartesian coordinate system, the rotation around the x axis is described by the following matrix:

$$T_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

In the Cartesian coordinate system, the rotation around the y axis is described by the following matrix:

$$T_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

In the Cartesian coordinate system, the rotation around the z axis is described by the following matrix:

$$T_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

3.3. Example in parametric form

A point is first rotated around the y axis with θ_1, then shifted with a vector characterized by the triple number (a, b, c) and finally rotated around the z axis with θ_2, the homogeneous transformation matrix is described by the following expression.

$$HTM = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \tag{5}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & \sin\theta_1 & \cos\theta_1 a + \sin\theta_1 c \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ -\sin\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_2 & \cos\theta_1 & -\sin\theta_1 a + \cos\theta_1 c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

4. The Denavit–Hartenberg convention

The convention assigns a coordinate system to each link of the structure (Zhang et al., 2021; Wang and Chen, 2006).

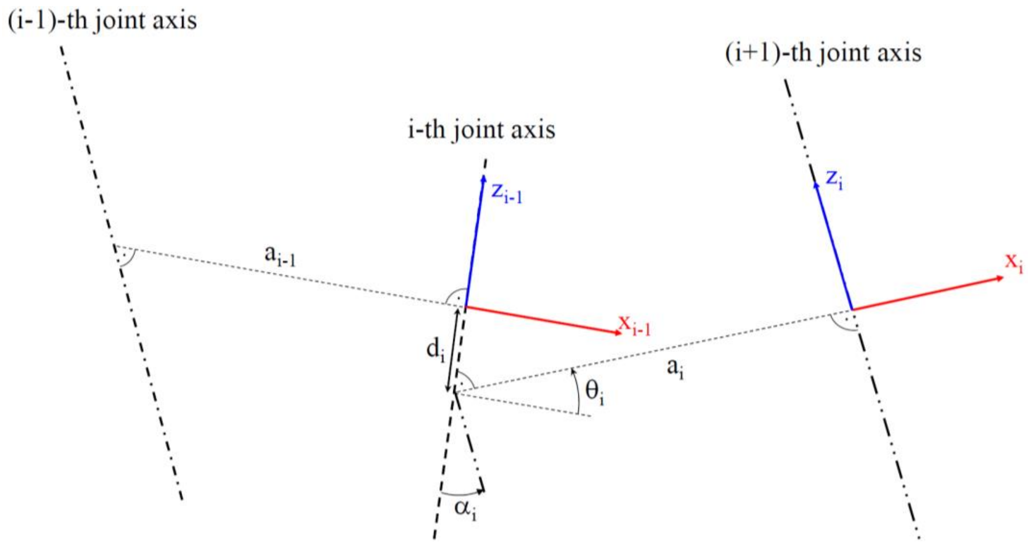

Figure 1. *Length, twist, offset and angle*

Figure 1 illustrates how the Denavit – Hartenberg convention builds the model of the structure. The z-axis of the i-th coordinate system coincides with the (i+1)-th joint axis. The x-axis of the i-th coordinate system coincides with the common normal of the i and (i+1)-th joint axes, it points from the i joint axis in the direction of the i+1 joint axis. It can be seen that the origin of the i-th coordinate system fits on the axis of the (i+1)-th joint, where the common normal between joint axis i and i+1 intersects joint axis i+1. The y-axis is not marked in **Figure 1** for reasons of transparency, but it can be easily determined according to the right-hand rule.

Denavit – Hartenberg parameters are also shown in **Figure 1** The "α" angle indicates the twisting of two successive joint axes in a plane perpendicular to their common normal. The "θ" angle is determined by the angle between two consecutives common normals in a plane which is perpendicular to the z axis. The "a" denotes the common normal of successive joint axes. The "d" denotes the offset distance between successive common normals along the z-axis perpendicular to the common normals.

5. Introduction of the structure

The tilting table with parallel kinematics consists of a platform, three actuators, and a central universal joint (**Table 1**). The platform is connected to the base through the actuators and the central connection (**Hiba! A hivatkozási forrás nem található.**). All three actuators have the same structural design, they contain two universal joints and a prismatic connection (**Hiba! A hivatkozási forrás nem található.**).

Table 1. *Design of structure*

Name	1_actuator	2_actuator	3_actuator	Central connection	Platform
Platform	-	-	-	-	P
Actuators	A1	A2	A3	-	-
Central universal joint	-	-	-	C	-
Upper universal joint	A1UU	A2UU	A3UU	-	-
Prismatic joint	A1P	A2P	A3P	-	-
Lower universal joint	A1UL	A2UL	A3UL	-	-

The universal joint has two degrees of freedom and can rotate around two axes. The prismatic connection has one degree of freedom and can move in a straight line along its axis.

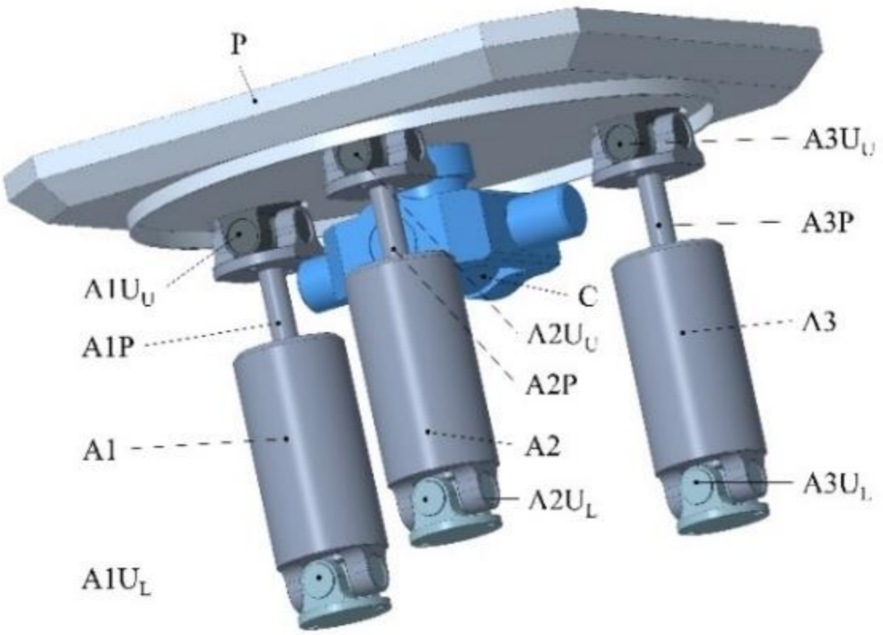

Figure 2. *The structure of the tilting table*

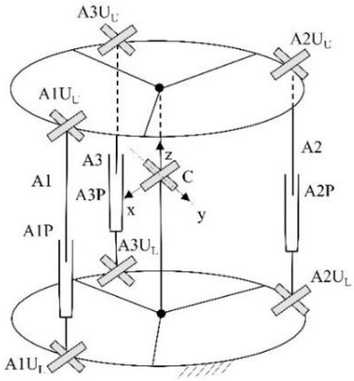

Figure 3. *Schematic representation of the tilting table*

6. Solving the direct kinematic problem of the structure

Given a mechanism with connected links and joints between the connected links, the rotation angle of the joints is known, in the case of a prismatic joint the translation is known, then the specific position and orientation of the end device can be calculated (Lipkin, 2005; Klug et al., 2018).

The structure shown **Hiba! A hivatkozási forrás nem található.** has two degrees of freedom. Although the central universal joint (**Figure 4**) can move the table, the mechanism will be operated by three actuators. Therefore, direct kinematics will also be presented for the central universal joint and the actuators. In the case of the actuators, it is sufficient to present one case, since their structure is the same.

In the case of the central universal joint, the four parameters are listed in **Table 2**.

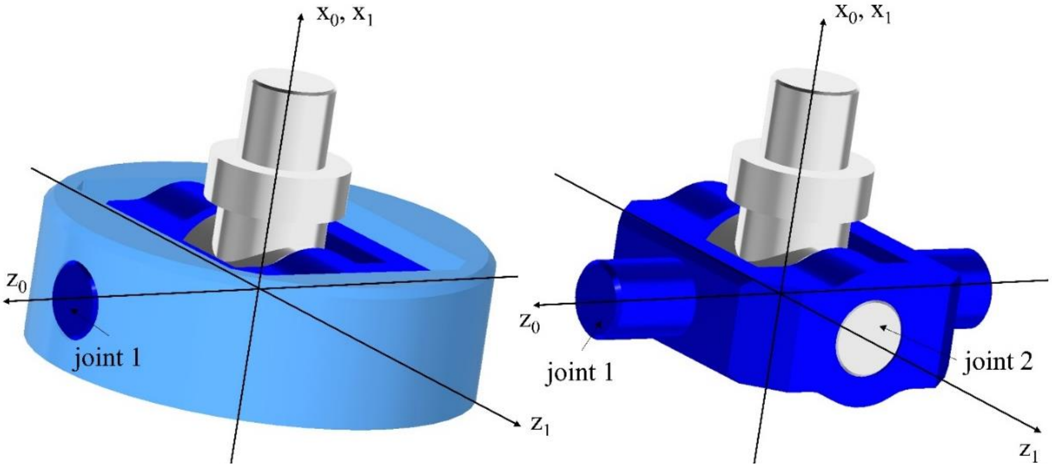

Figure 4. *The central universal joint and coordinate systems*

Table 2. *Parameters of joint variables and links*

Joint	α	a	d	θ
1	α_1	0	0	θ_1
2	0	0	0	θ_2

The transformation matrices can be read from **Table 2**.

$$T_{\theta_1}=\begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0\\ sin\theta_1 & cos\theta_1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{7}$$

$$T_{\alpha_1}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & cos\alpha_1 & -sin\alpha_1 & 0\\ 0 & sin\alpha_1 & cos\alpha_1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{8}$$

$$T_{\theta_2}=\begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 0\\ sin\theta_2 & cos\theta_2 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{9}$$

By multiplying (7) and (8), (9) matrices, we get the direct kinematic function (10).

$$\begin{bmatrix} cos\theta_1 cos\theta_2 - sin\theta_1 cos\alpha_1 sin\theta_2 & -cos\theta_1 sin\theta_2 - sin\theta_1 cos\alpha_1 cos\theta_2 & sin\theta_1 sin\alpha_1 & 0\\ sin\theta_1 cos\theta_2 - cos\theta_1 cos\alpha_1 sin\theta_2 & -sin\theta_1 sin\theta_2 + cos\theta_1 cos\alpha_1 cos\theta_2 & -cos\theta_1 sin\alpha_1 & 0\\ sin\alpha_1 sin\theta_2 & sin\alpha_1 cos\theta_2 & cos\alpha_1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{10}$$

In the case of the actuator (**Figure 5**), the four parameters are listed in **Table 3**.

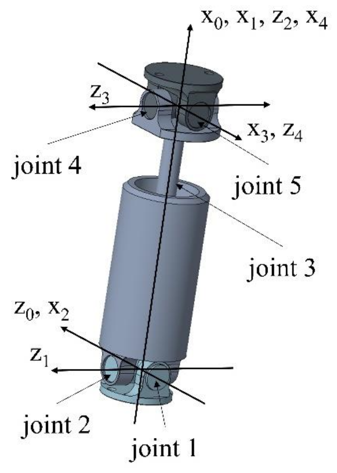

Figure 5. *Actuator joints and coordinate systems*

Table 3. *Parameters of joint variables and links*

Joint	α	a	d	θ
1	α_2	0	0	θ_3
2	0	0	0	θ_4
3	0	0	d_1	0
4	α_3	0	0	θ_5
5	0	0	0	θ_6

The transformation matrices can be read from **Table 3**.

$$T_{\theta_3} = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & 0 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$T_{\alpha_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha_2 & -sin\alpha_2 & 0 \\ 0 & sin\alpha_2 & cos\alpha_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

$$T_{\theta_4} = \begin{bmatrix} cos\theta_4 & -sin\theta_4 & 0 & 0 \\ sin\theta_4 & cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

$$T_{d_1} = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

$$T_{\theta_5} = \begin{bmatrix} cos\theta_5 & -sin\theta_5 & 0 & 0 \\ sin\theta_5 & cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

$$T_{\alpha_3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha_3 & -sin\alpha_3 & 0 \\ 0 & sin\alpha_3 & cos\alpha_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$T_{\theta_6} = \begin{bmatrix} cos\theta_6 & -sin\theta_6 & 0 & 0 \\ sin\theta_6 & cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

By multiplying (11) – (17) matrices, we get the direct kinematic function (18).

$$DKF_A = \begin{bmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} & \mathrm{a}_{13} & \mathrm{a}_{14} \\ \mathrm{a}_{21} & \mathrm{a}_{22} & \mathrm{a}_{23} & \mathrm{a}_{24} \\ \mathrm{a}_{31} & \mathrm{a}_{32} & \mathrm{a}_{33} & \mathrm{a}_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

$$a_{11} = (cos(\theta_3)\,cos(\theta_4)\,cos(\theta_5) - cos(\theta_3)\,sin(\theta_4)\,sin(\theta_5))cos(\theta_6) \; + \; sin(\theta_3)sin(\theta_6) \quad (19)$$

$$\mathrm{a}_{12} = -(cos(\theta_3)cos(\theta_4)cos(\theta_5) \; - \; cos(\theta_3)sin(\theta_4)sin(\theta_5))sin(\theta_6) + \; sin(\theta_3)cos(\theta_6) \quad (20)$$

$$\mathrm{a}_{13} = cos(\theta_3)cos(\theta_4)sin(\theta_5) \; + \; cos(\theta_3)sin(\theta_4)cos(\theta_5) \quad (21)$$

$$\mathrm{a}_{14} = cos(\theta_3)cos(\theta_4)d_1 \quad (22)$$

$$a_{21} = (sin(\theta_3)cos(\theta_4)cos(\theta_5) - sin(\theta_3)sin(\theta_4)sin(\theta_5))cos(\theta_6) - cos(\theta_3)sin(\theta_6) \tag{23}$$

$$a_{22} = -(sin(\theta_3)cos(\theta_4)cos(\theta_5) - sin(\theta_3)sin(\theta_4)sin(\theta_5))sin(\theta_6) - cos(\theta_3)cos(\theta_6) \tag{24}$$

$$a_{23} = sin(\theta_3)cos(\theta_4)sin(\theta_5) + sin(\theta_3)sin(\theta_4)cos(\theta_5) \tag{25}$$

$$a_{24} = sin(\theta_3)cos(\theta_4)d_1 \tag{26}$$

$$a_{31} = (sin(\theta_4)cos(\theta_5) + cos(\theta_4)sin(\theta_5))cos(\theta_6) \tag{27}$$

$$a_{32} = -(sin(\theta_4)cos(\theta_5) + cos(\theta_4)sin(\theta_5))sin(\theta_6) \tag{28}$$

$$a_{33} = sin(\theta_4)sin(\theta_5) - cos(\theta_4)cos(\theta_5) \tag{29}$$

$$a_{34} = sin(\theta_4)d_1 \tag{30}$$

7. Summary

The paper presents the Denavit–Hartenberg convention and the Denavit–Hartenberg parameters. Furthermore, the homogeneous transformation matrices were also defined. Direct kinematic function was presented with homogeneous transformation matrices in parametric form. The Denavit-Hartenberg model of the mechanism was built, describing the investigated structure. The direct kinematic functions were determined using the homogeneous transformation matrices, which was the goal of the study. The result is of great importance, as it can be used to solve the inverse kinematics.

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