CALCULATING THE CONNECTION BETWEEN PRECIPITATION AND SHALLOW GROUNDWATER

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Absztrakt
Vízgazdálkodási szempontból különösen fontos, hogy a sekély vagy akár a mélyebb rétegek utánpótlódását adó csapadék időbeliségét vizsgálni tudjuk. Tanulmányunkban egy olyan módszert használunk, amivel mind a lehullott csapadék, mind a talajvízszintek alakulását együtt tudjuk elemezni. Keresztkorrelációs és keresztspektrális elemzésekkel, a két hidrológiai paraméter közötti kapcsolatot számszerűsíteni tudjuk egy adott mintaterületre, valamint a kapcsolatban fellelhető periódusok is kimutathatóak. Számításaink során egy debreceni mintaterületre sikerült ilyen késleltetési időket meghatározni havi, valamint napi adatsorok felhasználásával, így kimutatásra került egy 8-10 hónapos, valamint egy rövidebb 3-4 napos lineáris kapcsolat a csapadék és talajvíz között. Eredményeink alapján ebben a kapcsolatban számos periodikus komponens is megtalálható.

Kulcsszavak: *csapadék, talajvíz, korreláció, spektrális elemzés, Debrecen*

Abstract
From a water management point of view, it is particularly important to be able to examine the temporality of the precipitation that replenishes the shallow or even deeper groundwater layers. In our study, we use a method that allows us to analyze the evolution of both the precipitation and groundwater levels together. With cross-correlation and cross-spectral analyses, we can quantify the relationship between the two hydrological parameters for a given sample area, and the periods found in the relationship can also be detected. During our calculations, we were able to determine such delay times for a sample area in Debrecen using monthly and daily data series, so an 8-10 month and a shorter 3-4 day linear relationship between precipitation and groundwater was detected. Based on our results, several periodic components can also be found in this relation.

Keywords: *precipitation, shallow groundwater, correlation, spectral analysis, Debrecen*

1. Introduction

The ever-changing climate, as well as increased agricultural or drinking water needs, affect the entire Carpathian basin, but it particularly affects the regions of Eastern Hungary. The ever-increasing meteorological extremes and irrigation water demands affect the development of water stored in shallow or deeper layers.

In the course of our previous investigations, examining the temporality of precipitation, we defined several deterministic periods for the area around Debrecen (Ilyés 2017, 2018). In the following, the specific relationship between rainfall and groundwater will be quantified using statistical and spectral methods.

The aim of our research was to determine the quantifiable linear relationship between the amount of precipitation, which is the main source of replenishment of these layers, and the registered water levels.

The relationship between rainfall amounts and groundwater levels was also investigated in several sample areas in Hungary, mainly between the Danube and Tisza region (Szalai, 2004; Szalai et al., 2011), and the relationship between groundwater and deeper layers was also investigated in the Sendai Plain (Gunawardhana and Kazama, 2012). Similar methods were used to investigate the relationship between rainfall and karst waters (Padilla and Pulido-Bosch, 1995; Pulido-Bosch et al., 1995; Jukic and Denic-Jukic, 2015), where a time shift of a few days was observed in the rainfall-water level relationship, behind which local hydrogeological reasons were found.

2. Methods and Materials

To understand the relationship, we chose the methodology of cross-correlation and cross-spectral analysis. For our calculation, we used data sets from the OMSZ (Hungarian Meteorological Service) meteorological network, namely the measured monthly and daily precipitation values, measured at the Debrecen station, and monthly and daily groundwater level time series measured in shallow groundwater monitoring wells previously purchased from various water management directorates. The examined time frame is between August 1935 and December 2010 in case of monthly data, and between April and December in 2019 in case of daily data.

The presented method was previously successfully used to analyze karst water levels and their high-resolution relationship with precipitation (Padilla & Pulido-Bosch, 1995) (Darabos, 2017):

When comparing two time series, in the correlation calculation, we assume that the time series x_i affects the time series y_i, where i = 1,...,n, is the series of measured data.

The cross correlation examination:

$$r_{+k} = r_{xy}(k) = \frac{C_{xy}(k)}{\sqrt{C_x^2(0)C_y^2(0)}} \quad (1)$$

$$r_{-k} = r_{yx}(k) = \frac{C_{yx}(k)}{\sqrt{C_x^2(0)C_y^2(0)}} \quad (2)$$

where:

$$C_{xy}(k) = \frac{1}{n}\sum_{t=1}^{n-k}(x_t - \bar{x})(y_{t+k} - \bar{y}) \quad (3)$$

$$C_{yx}(k) = \frac{1}{n}\sum_{t=1}^{n-k}(y_t - \bar{y})(x_{t+k} - \bar{x}) \quad (4)$$

$$C_x(0) = \frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2 \quad (5)$$

$$C_y(0) = \frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y})^2 \quad (6)$$

where $\bar{x}$ *and* $\bar{y}$ are the two averages of the time series.

The cross spectral analysis:

Due to the asymmetry of the cross-correlation function, the interpretation of the spectral density function on the set of complex numbers is essential:

$$\Gamma_{xy}(f) = |\alpha_{xy}(f)| exp[-i\Phi_{xy}(f)] \quad (7)$$

where i is √(–1), the α_{xy} (f), Φ_{xy} (f) the values of the cross amplitude and phase functions, in detail:

$$\alpha_{xy}(f) = \sqrt{\Psi_{xy}^2(f) + \Lambda_{xy}^2(f)} \quad (8)$$

$$\phi_{xy}(f) = arctan\frac{\Lambda_{xy}(f)}{\Psi_{xy}(f)} \quad (9)$$

where, he cross-spectrum, $\Psi_{xy}(f)$ and the square spectrum, $\Lambda_{xy}(f)$ are the following:

$$\Psi_{xy}(f) = 2\{r_{xy}(0) + \sum_{k=1}^{m}[r_{xy}(k) + r_{yx}(k)]D_k \cos(2\pi f k)\} \quad (10)$$

$$\Lambda_{xy}(f) = 2\{\sum_{k=1}^{m}[r_{xy}(k) - r_{yx}(k)]D_k \sin(2\pi f k)\} \quad (11)$$

where D_k is a weighting function that serves to eliminate the distortion appearing in the coefficients $\Psi_{xy}(f)$ and $\Lambda_{xy}(f)$.

The location of the examined wells can be seen in *Figure 1*.

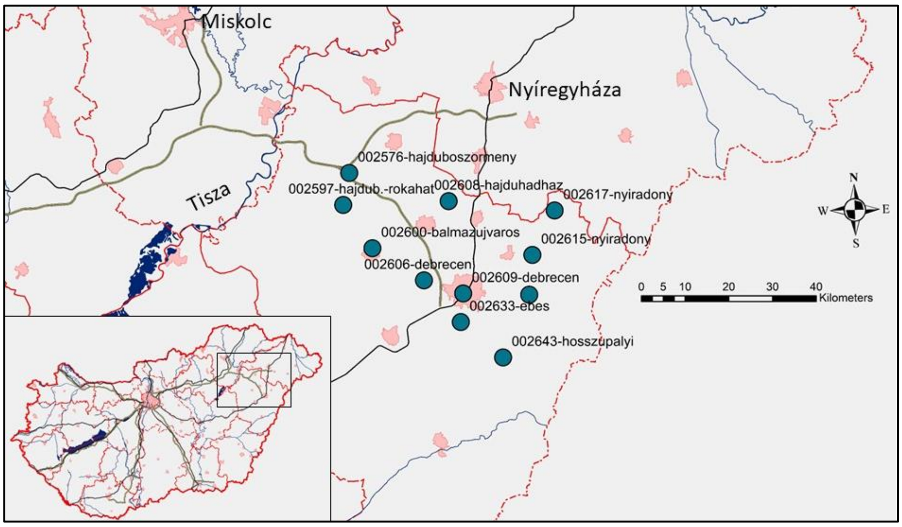

Figure 1. *The location of the examined wells in Eastern-Hungary*

3. Results

3.1. Monthly data

In the first step of the investigation, cross-correlation tests were performed. The asymmetry of the values shown in *Figure 2* in the direction of the x axis shows the extent to which the input parameter (precipitation) influences the output parameter (groundwater level). In the figure, the maximum distances of the cross-correlation values from the origin are the same as the delay times.

The very large co-movement shown in the figure can also be attributed to the resolution of the examination. Since we used monthly values for this part, I did not examine daily or even weekly fluctuations, but only the changes beyond the monthly time step.

It can be seen from the figure that the delay time is around 7-10 months, which is most influenced by the altitude of the well. It is also noticeable that already in the cross-correlation calculations, a strong annual period is visible in the temporal dynamics of the wells.

The examined wells belong to two different areas, Hajdúság and Nyírség, which can be easily separated due to the different geological structure, but this did not affect the results.

After carrying out the cross-amplitude tests, period times can be determined for the different measurement locations *(Figure 3)*.

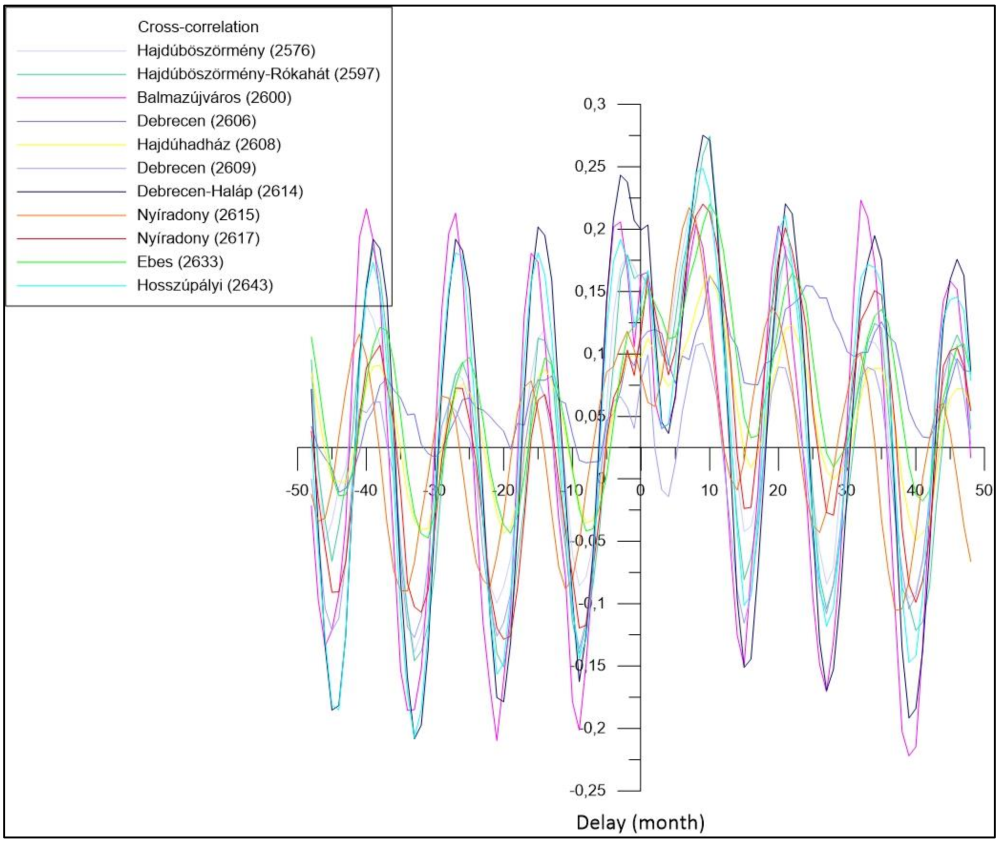

Figure 2. *The results of the cross-correlation calculation*

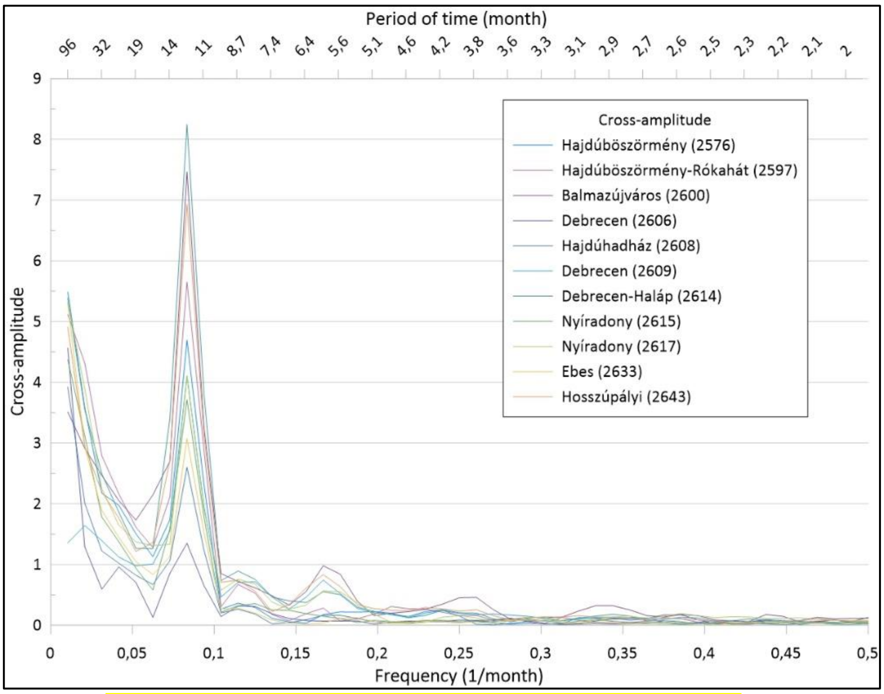

Figure 3. *The results of the cross-spectral calculation*

When examining the data, several cycles were detected. Cycles of one year, 8-9 and 5-6 months were detected at all measuring points, while cycles of 3-4 months were only detected in half of the measuring points. At the two stations in Debrecen, we calculated a period of 24 and 48 months.

All the calculated cycles can be seen on *Table 1*.

Table 1. *The calculated periods, based on the water level-precipitation cross amplitude functions*

Well No.	Cycles (month)				
Hosszúpályi (2643)		12	8.72	6	4,3
Ebes (2633)		12	8.72	5.33	
Nyíradony (2617)		12	8.72	6	4
Nyíradony (2615)		12	8	5.05	
Debrecen-Haláp (2614)		12	8.72	6	
Debrecen (2609)	48	12	8	6	4.18
Hajdúhadház (2608)		12	8.72	5.64	
Debrecen (2606)	24	12	8.72		
Balmazújváros (2600		12		6	3.84
Hajdúböszörmény-Rókahát (2597)		12	8.72	6	4.8
Hajdúböszörmény (2576)		12	8.72	5.05	4.18

3.2. Daily data

The next step of the examination was to collect more densely spaced data, such as daily measurements, to define a closer connection between these parameters.

For this calculation datasets from Debrecen were acquired from several monitoring points within city limits. The collected date represents the year 2019.

The location of the examined wells can be seen on *Figure 4*.

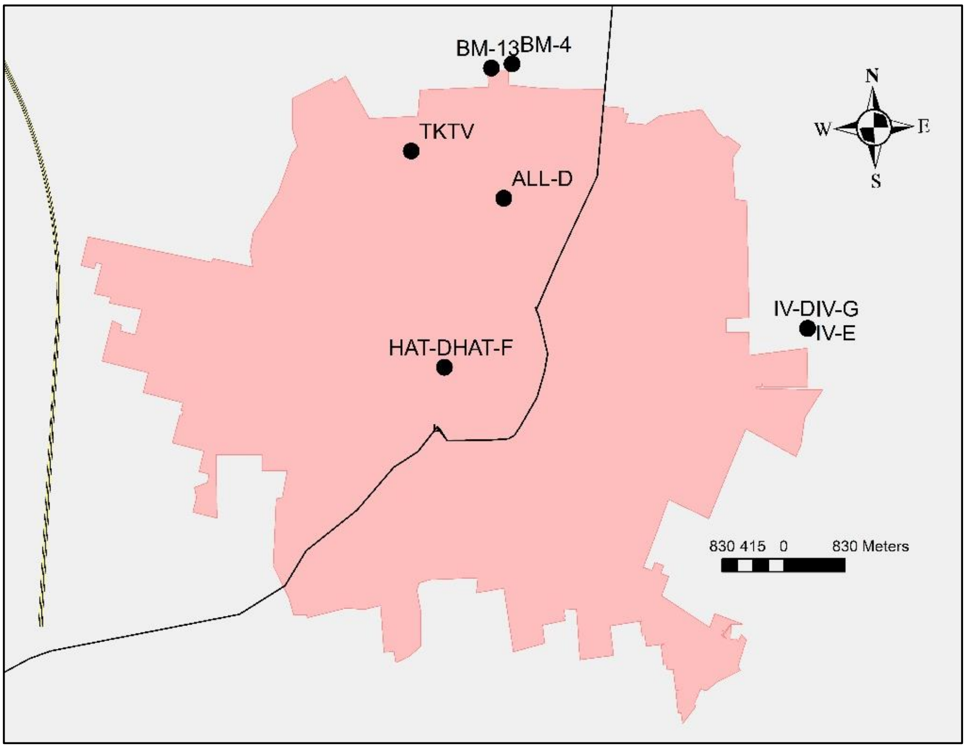

Figure 4*. The location of the examined wells within the city limits of Debrecen*

Figure 5 shows that when examining the relationship between the time series of the water level measured in the wells and the rainfall, a clear result cannot be shown in many cases. The main reason behind it, that within the city limits large number of different groundwater usage can be found, all of them influencing the natural state and level of these shallow groundwaters.

In the case of those wells where a clear maximum was detected during the examination of the relationship, a difference of 3-4 days could be calculated. Those wells either located on a remote area of the town, or not close to large groundwater using facilities.

The results of the cross-spectral analysis showed that the 7-day, i.e. weekly, and a 4.5-day cycle can most clearly be seen in the rainfall and groundwater time series, followed by several other local maxima.

Most of the wells have a periodicity of around 5.2-4.6 days, some around 2.5-3.5 days. The 4.5-day cycle was also detected in two thirds of the wells.

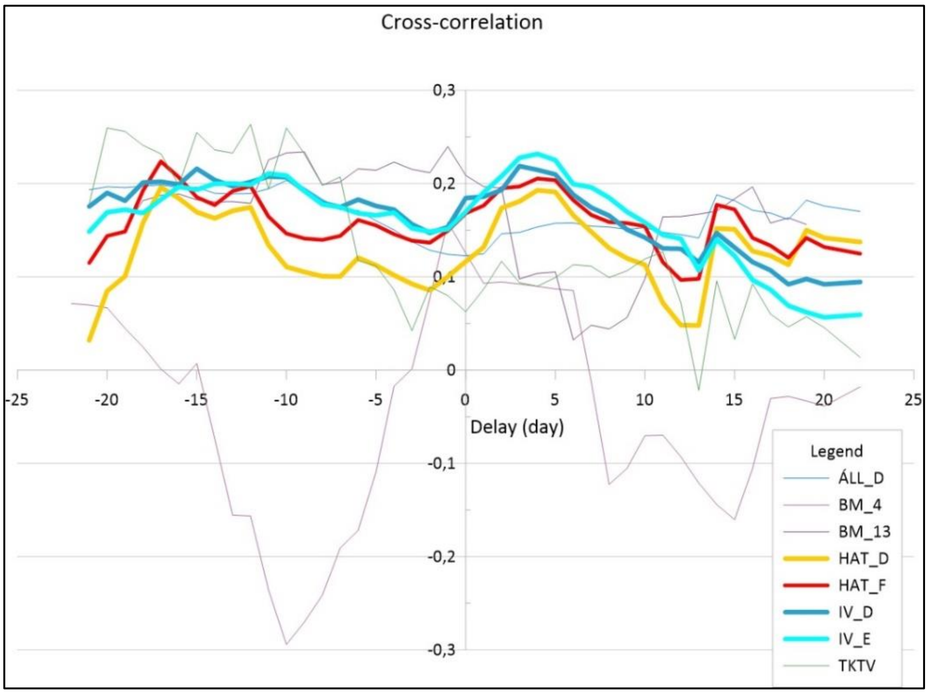

Figure 5. *The results of the cross-correlation calculations*

4. Conclusion

Overall, it can be said that the amount of precipitation is clearly related to groundwater, however, quantifying the relationship and demonstrating the delayed effect is a new result.

In the first case of the calculations, we succeeded in demonstrating a larger, longer-term relationship. During the interpretation of the results, we are looking for an answer to which parameters can explain this 8-month delay in the case of groundwater wells, and we will also examine time series with higher resolution and denser sampling.

The delay is affected by the altitude of the well. The stratigraphic differences between Hajdúság and Nyírség, which are known to have different geological structures, do not affect the delay time. The strong periodicity can also be noticed in the cross-correlation, the annual cycle can also be detected here.

The results are similar to the findings of Ubell (1953). When examining the fluctuations of groundwater levels, he observed how the maximum of the 1-year cycle shifts with depth. In the depth I examined, I successfully detected a shift of around 8 months.

In case of daily sampled data, a more physical connection is defined in the region of Debrecen.

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