PURE BENDING OF CURVED BEAM WITH NON-UNIFORM CROSS SECTION

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Abstract

The purpose of this paper is to study the state of stress and displacement field of a curved beam with non-uniform cross section. The material of the beam is non-homogeneous isotropic linearly elastic. The curved beam is subjected to couple moment at the ends of geometry. Utilizing analytical closed form solutions are obtained for the radial and circumferential stresses and for the displacements and cross sectional rotation. A numerical example illustrates the applications of the derived formulae.

Keywords: *curved beam, non-homogeneous, pure bending, non-uniform, normal stresses, radial displacement*

1. Introduction, governing equations

Structural curved elements such as bars, beams and curved panels are often used in different engineering branches. These type of curved elements are frequently encountered in defense industry, aviation or as basic structural components of bridges. The aim of this paper is to give an analytical method to the analysis of mechanical properties of curved beam made of non-homogeneous isotropic and linearly elastic materials. The cross section of the curved beam is a rectangle. The geometry of the considered curved beam with the loads acting on it is shown in Fig. 1. The thickness of the curved beam is denoted by b. The computations are made in cylindrical coordinate system $Or\varphi$, r means the radial coordinate and the polar angle is φ. The unit vectors of the cylindrical coordinate system $Or\varphi$ are $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ as shown in Fig. 2.

The solution of the pure bending problem is based on the following displacement field (Ecsed and Dluhi, 2005)

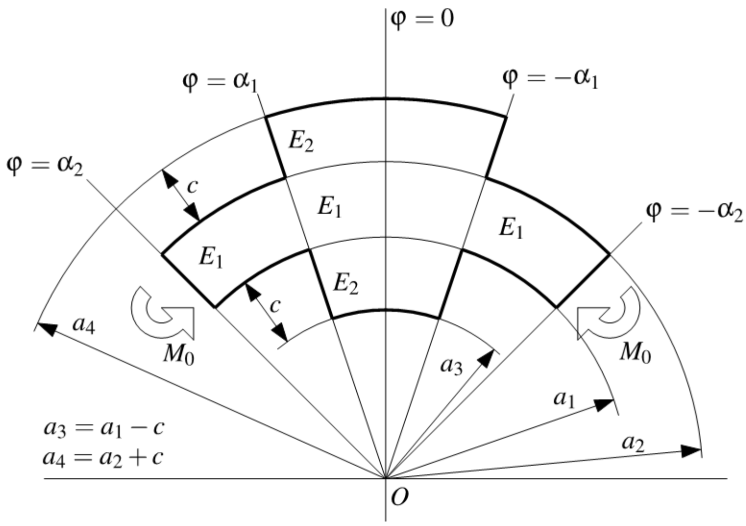

Figure 1. *The geometry of the curved beam with non-uniform cross section*

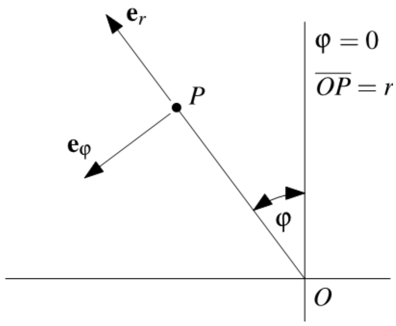

Figure 2. *The unit vectors of the cylindrical coordinate system*

$$\mathbf{u} = u\mathbf{e}_r + v\mathbf{e}_\varphi + w\mathbf{e}_z, \tag{1}$$

$$u = U(\varphi), \quad v = r\phi(\varphi) + V(\varphi), \quad V(\varphi) = \frac{\mathrm{d}U}{\mathrm{d}\varphi}, \quad w = 0. \tag{2}$$

The corresponding strains are (Sokolnikoff, 1956; Slaughter, 2002)

$$\varepsilon_r = \varepsilon_z = 0, \quad \gamma_{r\varphi} = \gamma_{rz} = \gamma_{\varphi z} = 0, \tag{3}$$

$$\varepsilon_\varphi = \frac{1}{r}\left(\frac{\mathrm{d}^2U}{\mathrm{d}\varphi^2} + U\right) + \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}. \tag{4}$$

This type of strain field satisfies the requirements of the Euler-Bernoulli beam theory (Barber, 2011). All the shearing strains $\gamma_{r\varphi}$, γ_{rz}, $\gamma_{\varphi z}$ and the normal strains ε_r, ε_z vanish, only the normal strain ε_φ is different from zero. Application of the strain-displacement relations gives

$$\sigma_\varphi(r,\varphi) = E\varepsilon_\varphi = E\left(\frac{W(\varphi)}{r} + \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}\right), \tag{5}$$

where

$$W(\varphi) = \frac{\mathrm{d}^2 U}{\mathrm{d}\varphi^2} + U. \tag{6}$$

In formula (5) E is the modulus of elasticity and σ_φ denotes the circumferential normal stress. The domain B occupying the composite curved beam consists of three parts $B = B_1 \cup B_2 \cup B_3$

$$B_1 = \{(r,\varphi) \,|\, a_1 \le r \le a_2,\ -\alpha_2 \le \varphi \le \alpha_2\}, \tag{7}$$

$$B_2 = \{(r,\varphi) \,|\, a_1 - c \le r \le a_1,\ -\alpha_1 \le \varphi \le \alpha_1\}, \tag{8}$$

$$B_3 = \{(r,\varphi) \,|\, a_2 \le r \le a_2 + c,\ -\alpha_1 \le \varphi \le \alpha_1\}. \tag{9}$$

The modulus of elasticity in the beam component B_1 is E_1 and the value of modulus of elasticity in the beam components B_2 and B_3 is E_2 (Fig. 1). The following cross sectional properties are introduced (Ecsedi and Dluhi, 2005)

$$A_1 = (a_2 - a_1)b, \quad A_2 = bc, \quad A_3 = bc, \tag{10}$$

$$\rho_1 = \frac{a_1 + a_2}{2}, \quad \rho_2 = a_1 - \frac{c}{2}, \quad \rho_3 = a_2 + \frac{c}{2}, \tag{11}$$

$$R_1 = \frac{a_2 - a_1}{\ln\left(\dfrac{a_2}{a_1}\right)}, \quad R_2 = \frac{c}{\ln\left(\dfrac{a_1}{a_1 - c}\right)}, \quad R_3 = \frac{c}{\ln\left(\dfrac{a_2 + c}{a_2}\right)}. \tag{12}$$

The circumferential normal stress in B_1 is

$$\sigma_{1\varphi}(r,\varphi) = E_1\left(\frac{W(\varphi)}{r} + \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}\right) \tag{13}$$

furthermore $\sigma_{2\varphi}$ and $\sigma_{3\varphi}$ denote the circumferential normal stress in B_2 and B_3. The formulae of those stresses are

$$\sigma_{2\varphi}(r,\varphi) = E_2\left(\frac{W(\varphi)}{r} + \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}\right), \tag{14}$$

$$\sigma_{3\varphi}(r,\varphi) = E_2\left(\frac{W(\varphi)}{r} + \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}\right). \tag{15}$$

In the beam component B_1 the normal stress resultant N_1 is

$$N_1(\varphi) = \int_{A_1} \sigma_{1\varphi} \mathrm{d}A = E_1 A_1 \left(\frac{W(\varphi)}{R_1} + \Theta(\varphi) \right), \tag{16}$$

where

$$\Theta(\varphi) = \frac{\mathrm{d}\phi}{\mathrm{d}\varphi}. \tag{17}$$

The couple stress resultant M_1 in the beam component B_1 is obtained as

$$M_1(\varphi) = \int_{A_1} r\sigma_{1\varphi} \mathrm{d}A = E_1 A_1 \left(W(\varphi) + \rho_1 \Theta(\varphi) \right). \tag{18}$$

In the beam components B_2 and B_3 the normal stress resultants N_2 and N_3 are as follows

$$N_2(\varphi) = \int_{A_2} \sigma_{2\varphi} \mathrm{d}A = E_2 A_2 \left(\frac{W(\varphi)}{R_2} + \Theta(\varphi) \right), \tag{19}$$

$$N_3(\varphi) = \int_{A_3} \sigma_{3\varphi} \mathrm{d}A = E_2 A_3 \left(\frac{W(\varphi)}{R_3} + \Theta(\varphi) \right). \tag{20}$$

The formulae of the couple stress resultants in the beam components B_2 and B_3 are

$$M_2(\varphi) = \int_{A_2} r\sigma_{2\varphi} \mathrm{d}A = E_2 A_2 \left(W(\varphi) + \rho_2 \Theta(\varphi) \right), \tag{21}$$

$$M_3(\varphi) = \int_{A_3} r\sigma_{3\varphi} \mathrm{d}A = E_2 A_3 \left(W(\varphi) + \rho_3 \Theta(\varphi) \right), \tag{22}$$

According to the definitions of function $W(\varphi)$ and $\Theta(\varphi)$ which are

$$W(\varphi) = \frac{\mathrm{d}^2 U}{\mathrm{d}\varphi^2} + U, \quad \Theta(\varphi) = \frac{\mathrm{d}\phi}{\mathrm{d}\varphi} \tag{23}$$

the values of function $U = U(\varphi)$ and $\Theta = \Theta(\varphi)$ are depend on the polar angle φ as

$$U = U_1(\varphi), \quad \Theta = \Theta_1(\varphi), \quad \phi = \phi_1(\varphi) \quad \text{for} \quad -\alpha_2 \le \varphi \le -\alpha_1, \tag{24}$$

$$U = U_2(\varphi), \quad \Theta = \Theta_2(\varphi), \quad \phi = \phi_2(\varphi) \quad \text{for} \quad -\alpha_1 \le \varphi \le \alpha_1, \tag{25}$$

$$U = U_3(\varphi), \quad \Theta = \Theta_3(\varphi), \quad \phi = \phi_3(\varphi) \quad \text{for} \quad \alpha_1 \le \varphi \le \alpha_2. \tag{26}$$

According to equations (24), (25) and (26) the function $W_i = W_i(\varphi)$ $(i = 1,2,3)$ is defined as (Fig. 1)

$$W_1(\varphi) = \frac{\mathrm{d}^2 U_1}{\mathrm{d}\varphi^2} + U_1(\varphi), \quad -\alpha_2 \le \varphi \le -\alpha_1, \tag{27}$$

$$W_2(\varphi)=\frac{\mathrm{d}^2U_2}{\mathrm{d}\varphi^2}+U_2(\varphi), \quad -\alpha_1\le\varphi\le\alpha_1, \tag{28}$$

$$W_3(\varphi)=\frac{\mathrm{d}^2U_3}{\mathrm{d}\varphi^2}+U_3(\varphi), \quad \alpha_1\le\varphi\le\alpha_2, \tag{29}$$

The normal force stress resultant $N=N(\varphi)$ vanishes in all cross section and the bending moment is constant that is $M(\varphi)=M_0$. From the statements mentioned above it follows that

$$N(\varphi)=A_1E_1\left(\frac{W_1(\varphi)}{R_1}+\Theta_1(\varphi)\right)=0, \quad -\alpha_2\le\varphi\le-\alpha_1, \tag{30}$$

$$M(\varphi)=M_0=A_1E_1\left(W_1(\varphi)+\rho_1\Theta_1(\varphi)\right), \quad -\alpha_2\le\varphi\le-\alpha_1, \tag{31}$$

$$\begin{gathered}N(\varphi)=A_1E_1\left(\frac{W_2(\varphi)}{R_1}+\Theta_2(\varphi)\right)+A_2E_2\left(\frac{W_2(\varphi)}{R_2}+\Theta_2(\varphi)\right)+\\ +A_3E_3\left(\frac{W_2(\varphi)}{R_3}+\Theta_2(\varphi)\right)=0, \quad -\alpha_1\le\varphi\le\alpha_1,\end{gathered} \tag{32}$$

$$\begin{gathered}M(\varphi)=M_0=A_1E_1\left(W_2(\varphi)+\rho_1\Theta_2(\varphi)\right)+A_2E_2\left(W_2(\varphi)+\rho_2\Theta_2(\varphi)\right)+\\ +A_3E_3\left(W_2(\varphi)+\rho_3\Theta_2(\varphi)\right), \quad -\alpha_1\le\varphi\le\alpha_1,\end{gathered} \tag{33}$$

$$N(\varphi)=A_1E_1\left(\frac{W_3(\varphi)}{R_1}+\Theta_3(\varphi)\right)=0, \quad \alpha_1\le\varphi\le\alpha_2, \tag{34}$$

$$M(\varphi)=M_0=A_1E_1\left(W_3(\varphi)+\rho_1\Theta_3(\varphi)\right), \quad \alpha_1\le\varphi\le\alpha_2. \tag{35}$$

The solution of the system of equations (30–35) for $W_i(\varphi)$ and $\Theta_i(\varphi)$ $(i=1,2,3)$ is as follows

$$W_1=-\frac{M_0R_1}{A_1E_1(\rho_1-R_1)}, \quad \Theta_1=\frac{M_0}{A_1E_1(\rho_1-R_1)}, \quad -\alpha_2\le\varphi\le-\alpha_1, \tag{36}$$

$$W_2=-\frac{E_1A_1+E_2A_2+E_3A_3}{Q}M_0, \quad -\alpha_1\le\varphi\le\alpha_1, \tag{37}$$

$$Q=\left(\frac{E_1A_1}{R_1}+\frac{E_2A_2}{R_2}+\frac{E_3A_3}{R_3}\right)\left(E_1A_1\rho_1+E_2A_2\rho_2+E_3A_3\rho_3-\frac{\left(E_1A_1+E_2A_2+E_3A_3\right)^2}{\frac{E_1A_1}{R_1}+\frac{E_2A_2}{R_2}+\frac{E_3A_3}{R_3}}\right), \tag{38}$$

$$\Theta_2 = \frac{M_0}{E_1A_1\rho_1 + E_2A_2\rho_2 + E_3A_3\rho_3 - \dfrac{\left(E_1A_1 + E_2A_2 + E_3A_3\right)^2}{\dfrac{E_1A_1}{R_1} + \dfrac{E_2A_2}{R_2} + \dfrac{E_3A_3}{R_3}}}, \quad -\alpha_1 \le \varphi \le \alpha_1, \tag{39}$$

$$W_3 = -\frac{M_0R_1}{A_1E_1\left(\rho_1 - R_1\right)}, \quad \alpha_1 \le \varphi \le \alpha_2, \tag{40}$$

$$\Theta_3 = \frac{M_0}{A_1E_1\left(\rho_1 - R_1\right)}, \quad \alpha_1 \le \varphi \le \alpha_2. \tag{41}$$

It must be remarked that W_i, Θ_i $(i = 1,2,3)$ do not depend on the polar angle.

2. Determination of circumferential and radial normal stresses

2.1. Circumferential normal stress

The circumferential normal stress σ_φ can be computed in the domain $-\alpha_2 \le \varphi \le -\alpha_1$, $a_1 \le r \le a_2$ as

$$\sigma_{1\varphi}(r) = E_1\left(\frac{W_1}{r} + \Theta_1\right), \tag{42}$$

and in the domain $\alpha_1 \le \varphi \le \alpha_2$, $a_1 \le r \le a_2$ the expression of the circumferential normal stress is as follows

$$\sigma_{3\varphi}(r) = E_1\left(\frac{W_3}{r} + \Theta_3\right). \tag{43}$$

The $\sigma_{2\varphi}(r)$ can be obtained in $-\alpha_1 \le \varphi \le \alpha_1$ from the following formula

$$\begin{aligned}\sigma_{2\varphi}(r) = {} & \left(H\left(r - a_3\right) - H\left(r - a_1\right)\right)E_2\left(\frac{W_2}{r} + \Theta_2\right) + \left(H\left(r - a_1\right) - H\left(r - a_2\right)\right)E_1\left(\frac{W_2}{r} + \Theta_2\right) + \\ & + H\left(r - a_2\right)E_2\left(\frac{W_2}{r} + \Theta_2\right), \quad a_3 \le r \le a_4.\end{aligned} \tag{44}$$

Here the function $H(x)$ is the Heaviside function.

2.2. Radial normal stress

The radial normal stress σ_r is computed by the application of stress equilibrium equation

$$\frac{d}{dr}\left(r\sigma_r\right) - \sigma_\varphi = 0. \tag{45}$$

Combination of equations (42), (43) and (44) with equation (45) yields

$$\sigma_{1r}(r) = E_1\left(\frac{W_1}{r}\ln\frac{r}{a_1} + \Theta_1\left(1-\frac{a_1}{r}\right)\right), \quad -\alpha_2 \le \varphi \le -\alpha_1, \tag{46}$$

$$\sigma_{3r}\left(r\right) = E_1\left(\frac{W_3}{r}\ln\frac{r}{a_1} + \Theta_3\left(1-\frac{a_1}{r}\right)\right), \quad \alpha_1 \le \varphi \le \alpha_2. \tag{47}$$

In the three layered beam component the following formula can be used to evaluate the radial normal stress $\sigma_{2r}(r)$

$$\begin{gathered}\sigma_{2r}(r) = \left(H\left(r-a_3\right) - H\left(r-a_1\right)\right)\sigma_{1br}(r) + \left(H\left(r-a_1\right) - H\left(r-a_2\right)\right)\sigma_{2br}(r) + \\ +H\left(r-a_2\right)\sigma_{3br}(r), \quad -\alpha_1 \le \varphi \le \alpha_1,\end{gathered} \tag{48}$$

where

$$\sigma_{1br}(r) = E_2\left(\frac{W_2}{r}\ln\left|\frac{r}{a_1-c}\right| + \Theta_2\left(1-\frac{a_1}{r}+\frac{c}{r}\right)\right), \tag{49}$$

$$\sigma_{2br}(r) = E_1\left(\frac{W_2}{r}\ln\frac{r}{a_1} + \Theta_2\left(1-\frac{a_1}{r}\right)\right) + \frac{a_1}{r}\sigma_{1br}(a_1), \tag{50}$$

$$\sigma_{3br}(r) = E_2\left(\frac{W_2}{r}\ln\frac{r}{a_2} + \Theta_2\left(1-\frac{a_2}{r}\right)\right) + \frac{a_2}{r}\sigma_{2br}(a_2). \tag{51}$$

In the presented numerical example the numerical value of Von-Mises stress is computed by the use of following formula

$$\sigma_0(r) = \sqrt{\sigma_r^2 - \sigma_r\sigma_\varphi + \sigma_\varphi^2}. \tag{52}$$

3. Formulae of the radial and circumferetial displacements

To obtain the radial displacement the undermentioned ordinary differential equation will be used

$$\frac{\mathrm{d}^2U_1}{\mathrm{d}\varphi^2} + U_1(\varphi) = W_1, \quad -\alpha_2 \le \varphi \le -\alpha_1, \tag{53}$$

$$\frac{\mathrm{d}^2U_2}{\mathrm{d}\varphi^2} + U_2(\varphi) = W_2, \quad -\alpha_1 \le \varphi \le \alpha_1, \tag{54}$$

$$\frac{\mathrm{d}^2U_3}{\mathrm{d}\varphi^2} + U_3(\varphi) = W_3, \quad \alpha_1 \le \varphi \le \alpha_2, \tag{55}$$

with the following boundary conditions

$$U_2(0)=0,\ \left(\frac{\mathrm{d}U_2}{\mathrm{d}\varphi}\right)_{\varphi=0}=0. \tag{56}$$

It is evident that

$$U_1(-\alpha_1)=U_2(-\alpha_1), \tag{57}$$

$$U_3(\alpha_1)=U_2(\alpha_1), \tag{58}$$

$$V_1(-\alpha_1)=V_2(-\alpha_1), \tag{59}$$

$$V_3(\alpha_1)=V_2(\alpha_1), \tag{60}$$

since the function $U(\varphi)$ and $V(\varphi)=\dfrac{\mathrm{d}U}{\mathrm{d}\varphi}$ are continuous on the whole interval of φ $\left(-\alpha_2\le\varphi\le\alpha_2\right)$. Simple computation gives

$$U_1(\varphi)=W_1+c_1\cos\varphi+c_2\sin\varphi,\quad -\alpha_2\le\varphi\le-\alpha_1, \tag{61}$$

$$V_1(\varphi)=-c_1\sin\varphi+c_2\cos\varphi,\quad -\alpha_2\le\varphi\le-\alpha_1, \tag{62}$$

$$U_2(\varphi)=W_2\left(1-\cos\varphi\right),\quad -\alpha_1\le\varphi\le\alpha_1, \tag{63}$$

$$V_2(\varphi)=W_2\sin\varphi,\quad -\alpha_1\le\varphi\le\alpha_1, \tag{64}$$

$$U_3(\varphi)=W_3+c_3\cos\varphi+c_4\sin\varphi,\quad \alpha_1\le\varphi\le\alpha_2, \tag{65}$$

$$V_3(\varphi)=-c_3\sin\varphi+c_4\cos\varphi,\quad \alpha_1\le\varphi\le\alpha_2. \tag{66}$$

In equations (61–66)

$$c_1=\left(W_2-W_1\right)\cos\alpha_1-W_2, \tag{67}$$

$$c_2=\left(W_1-W_2\right)\sin\alpha_1, \tag{68}$$

$$c_3=\left(W_2-W_3\right)\cos\alpha_1-W_2, \tag{69}$$

$$c_4=\left(W_2-W_3\right)\sin\alpha_1. \tag{70}$$

The expression of the radial displacement $U=U(\varphi)$ is as follows

$$U(\varphi)=\left(H\left(\varphi+\alpha_2\right)-H\left(\varphi+\alpha_1\right)\right)U_1(\varphi)+\left(H\left(\varphi+\alpha_1\right)-H\left(\varphi-\alpha_1\right)\right)U_2(\varphi)+ \\ +H\left(\varphi-\alpha_1\right)U_3(\varphi),\quad -\alpha_2\le\varphi\le\alpha_2. \tag{71}$$

The cross sectional rotation is obtained from the following equations

$$\phi_1(\varphi)=\frac{M_0}{A_1E_1(\rho_1-R_1)}\left(\varphi+\alpha_1\right)-\Theta_2\alpha_1,\quad -\alpha_2\le\varphi\le-\alpha_1, \tag{72}$$

$$\phi_2(\varphi) = \Theta_2 \varphi, \quad -\alpha_1 \le \varphi \le \alpha_1, \tag{73}$$

$$\phi_3(\varphi) = \frac{M_0}{A_1 E_1 (\rho_1 - R_1)} (\varphi - \alpha_1) + \Theta_2 \alpha_1, \quad \alpha_1 \le \varphi \le \alpha_2. \tag{74}$$

4. Numerical example

The following data are used in the numerical example: $a_1 = 0.05$ m, $a_2 = 0.1$ m, $c = 0.025$ m, $a_3 = a_1 - c = 0.025$ m, $a_4 = a_2 + c = 0.125$ m, $b = 0.05$ m, $\alpha_1 = \pi / 6$, $\alpha_2 = \pi / 3$, $E_1 = 1.45 \times 10^{11}$ Pa, $E_2 = 2 \times 10^{11}$ Pa, $E_3 = 2 \times 10^{11}$ Pa, $M_0 = 3000$ Nm.

The plot of $\sigma_{1\varphi}(r)$ as a function of the radial coordinate r is shown in Fig. 2. The radial normal stress $\sigma_{1r}(r)$ as a function of r is presented in Fig. 3. The Von-Mises stress $\sigma_1(r)$ is given in Fig. 4 as a function of radial coordinate r. The graph of the circumferential normal stress $\sigma_{2\varphi}(r)$ is shown in Fig. 5 for $a_3 \le r \le a_4$, $-\alpha_1 \le \varphi \le \alpha_1$. The plot of the radial normal stress $\sigma_{2r}(r)$ is given in the Fig. 6 for $a_3 \le r \le a_4$, $-\alpha_1 \le \varphi \le \alpha_1$. The Von-Mises stress $\sigma_2(r)$ is presented in Fig. 7 for $a_3 \le r \le a_4$, $-\alpha_1 \le \varphi \le \alpha_1$. The graphs of the radial displacement and cross sectional rotation are shown in Fig. 8 and Fig. 9.

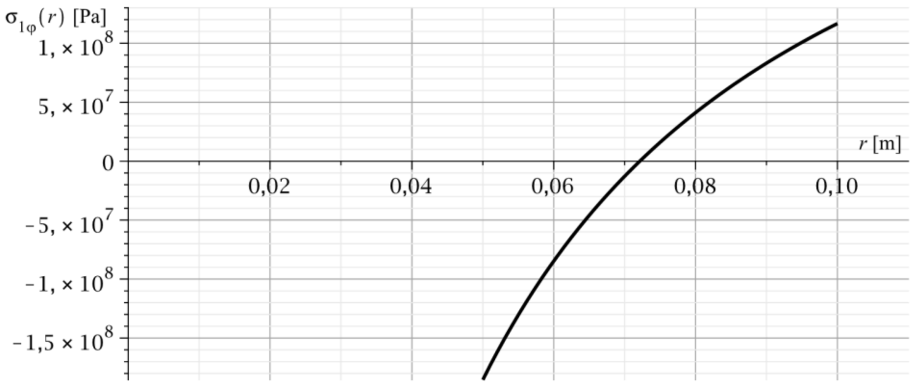

Figure 3. *The plot of the $\sigma_{1\varphi}(r)$ function*

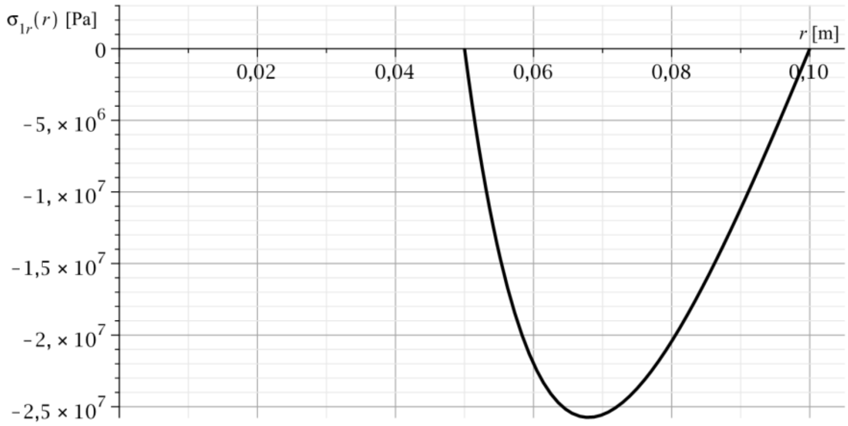

Figure 4. *The plot of the* $\sigma_{1r}(r)$ *function*

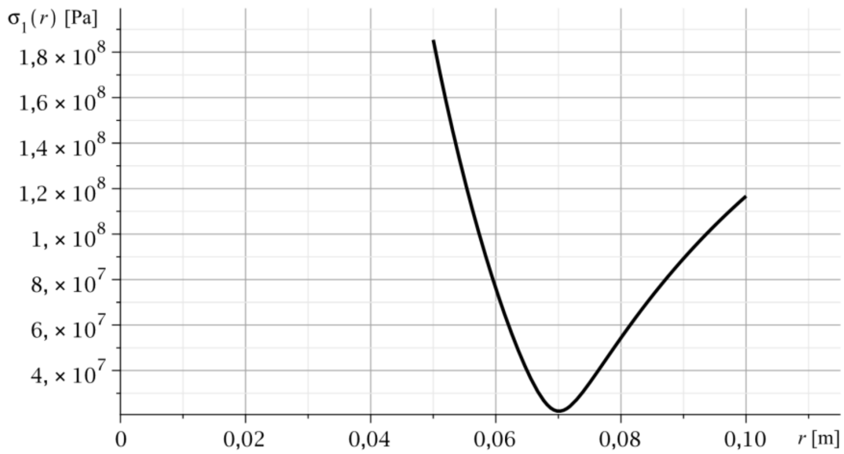

Figure 5. *The Von-Mises stress function* $\sigma_1(r)$

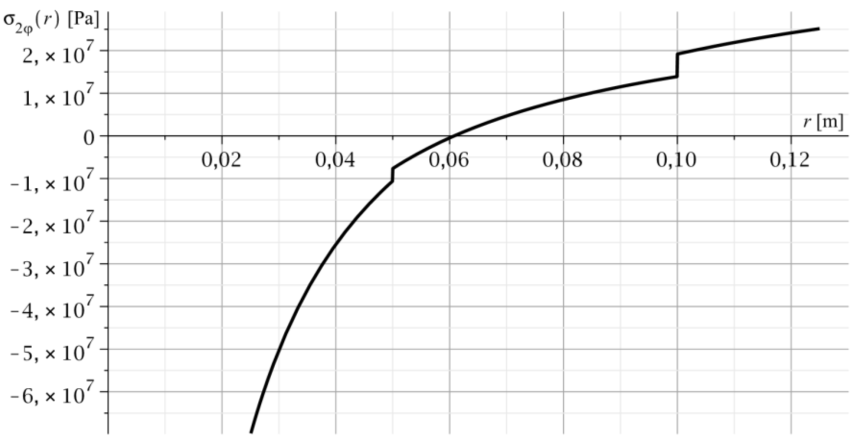

Figure 6. *The circumferential normal stress* $\sigma_{2\varphi}(r)$ *in the domain* $a_3 \le r \le a_4,\ -\alpha_1 \le \varphi \le \alpha_1$

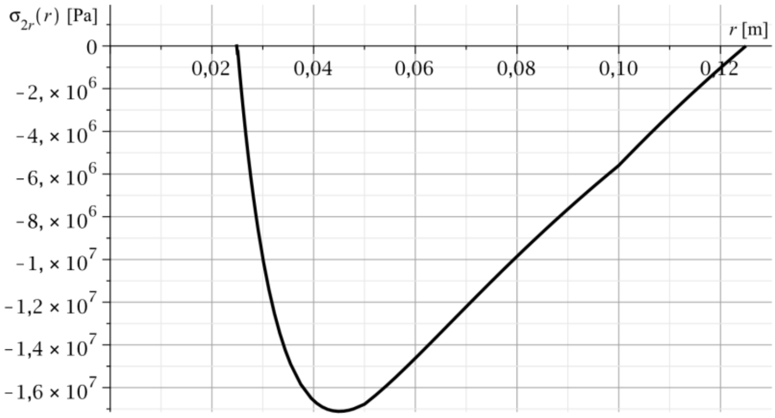

Figure 7. *The radial normal stress* $\sigma_{2r}(r)$ *in the domain* $a_3 \le r \le a_4,\ -\alpha_1 \le \varphi \le \alpha_1$

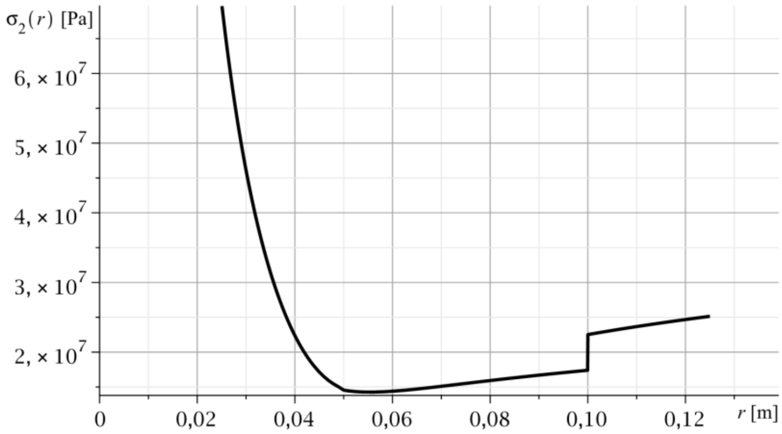

Figure 8. *The Von-Mises stress* $\sigma_2(r)$ *in the domain* $a_3 \le r \le a_4,\ -\alpha_1 \le \varphi \le \alpha_1$

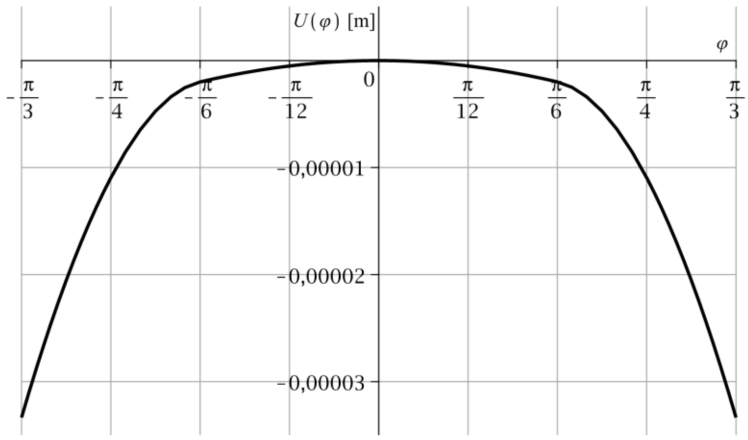

Figure 9. *The plot of the* $U(\varphi)$ *radial displacement function*

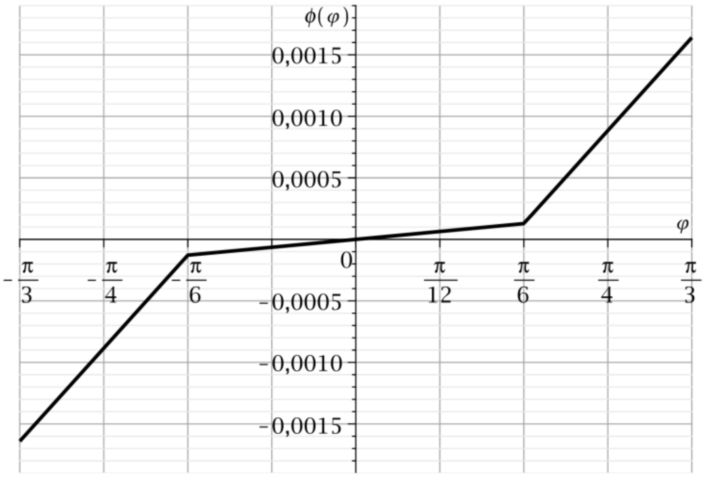

Figure 10. *The plot of the* $\phi(\varphi)$ *cross sectional rotation function*

5. Conclusions

Paper deals with the static bending problem of isotropic non-homogeneous curved beam whose cross section is non-uniform. The curved beam is loaded by bending moments at its end cross section. An analytical model is developed to compute the stresses and displacements. The obtained result can be used as benchmark solution to check the solution come from different numerical methods such as finite differences, finite element method, etc.

References

[1] Ecsedi, I., Dluhi, K. (2005). A linear model for the static and dynamic analysis of non-homogeneous curved beams. *Applied Mathematical Modelling*, 29(12), 1211–1231. **https://doi.org/10.1016/j.apm.2005.03.006**

[2] Sokolnikoff, I. S. (1956). *Mathematical Theory of Elasticity*. McGraw-Hill, New York.

[3] Barber, J. R. (2011). *Intermediate Mechanics of Materials.* Springer, New York. **https://doi.org/10.1007/978-94-007-0295-0**

[4] Slaughter, W. S. (2002). *The Linearized Theory of Elasticity*. Birkhäuser, Basel. **https://doi.org/10.1007/978-1-4612-0093-2**