MATHEMATICAL INVESTIGATION OF SEALING INEFFICIENCIES OF HOT AIR ENGINES AND OTHER PISTON MACHINES

Gyula Krámer
PhD student, University of Miskolc, Institute of Energy and Chemical Machinery,
Department of Chemical Machinery,
3515 Miskolc-Egyetemváros, e-mail: gyula.kramer@uni-miskolc.hu

Zoltan Siménfalvi
full professor, University of Miskolc, Institute of Energy and Chemical Machinery,
Department of Chemical Machinery,
3515 Miskolc-Egyetemváros, e-mail: zoltan.simenfalvi@uni-miskolc.hu

Gábor L. Szepesi
assoc. professor, University of Miskolc, Institute of Energy and Chemical Machinery,
Department of Chemical Machinery,
3515 Miskolc-Egyetemváros, e-mail: gabor.szepesi@uni-miskolc.hu

Abstract
Reciprocating machines, compressors and engines are the most commonly used machines in our technical environment. It is impossible to create a perfect seal between moving parts, therefore the phenomenon of leakage occurs in all such machines, however as we will see, it mainly affects hot air engines, which operates without charge exchange. This article attempts to describe this phenomenon in a mathematical form, and assist helping to understand the difference between calculated and measured values.

Keywords: *piston, pressure loss, leakage*

1. Introduction

Volumetric displacement machines are used in a wide range of applications. They can be for example prime movers, such as diesel, petrol and gas engines, or in the past, steam engines and Stirling engines. However the piston is also a main component of countless other equipment, in rarely used thermal and pressure power machines. The other extremely important machine family, consist of pumps and compressors also mostly work on the principle of piston displacement. This category includes all the hydraulic and pneumatic cylinders, the gas spring cylinders and moreover the shock absorbers.

In the case of a turbine or a centrifugal pump, we calculate with the so called gap loss or leakage loss, but the piston machines are mostly considered to be perfectly sealed. That's not a case. The gap loss is considered irrelevant, hence that is a simplification.

2. Beginning

A new type of hot-air engine was built in the workshop of our department a few years ago, followed by the mathematical model of its operation (Krámer et al., 2019) *(Figure 1)*, but its verification by measurement was only completed much later, where the time course of the internal pressure of the running machine was examined. During the long time that passed until the measurement, the machine operated for hundreds of hours, which caused its parts to become let to worn out parts and age its material due to the elapsed years.

The result of the measurement was the following: although the measured pressure curve was almost perfectly matched to the calculated curve, its values were much lower. Moreover, the values were much lower than the atmospheric base pressure *(Figure 2)*. The measurement seemed to be incorrect. Therefore, the experiment was repeated with a much more accurate sensor, however the second time we obtained similar results.

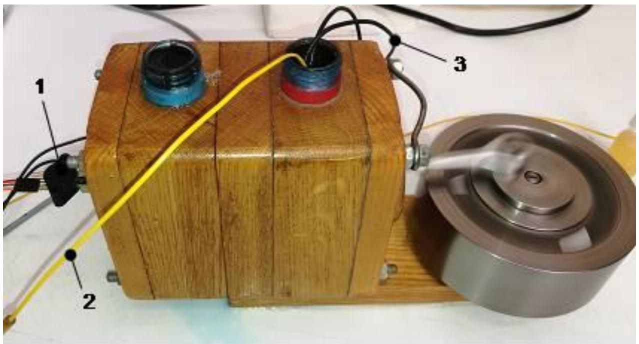

Figure 1. *The running Stirling engine under test.*
1. pressure sensor, 2. thermocouple, 3. heater solenoid

It could be explained that the amplitude of the pressure curve was smaller than the calculated value, but it was not understood how the minimum value of the pressure could fall below the value of the base pressure. *(Figure 2)*

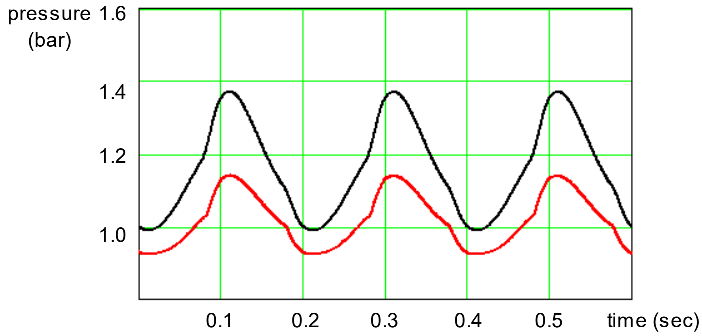

Figure 2. *The calculated (black) and the measured (red) curve of internal pressure at the running engine*

3. Research for the missing pressure

A piston engine always has certain amount of leakage and pressure loss (Li et al., 2016). This can only be avoided by using a diaphragm (Bégot et al., 2016), but in most applications this cannot be done. It is possible to increase the degree of sealing, but this will lead to an increase in friction, which in some cases is not acceptable (Bedajangam et al., 2013). This loss is negligible for a hydraulic power cylinder, but in the area of the internal combustion engines, serious efforts are made to reduce this phenomenon of leak and of friction, as it can consume up to 15% of the power (Furuhama et al., 1981) (Kennedy et al., 2014). The heat engine is considered to be a special case due to its low power density, as there is too little power to cover the friction losses.

4. Simplified model of the phenomenon

For the easier understanding of the phenomenon let us assume that there is a perfect moving piston in a closed cylinder filled with gas. The piston only compresses and expands the gas, without any work or heat transfer. Let us model with a controllable valve not taking into consideration where the gap is *(Figure 3)*. If the valve is closed, there is no gas leak, if the valve is completely open, there is no compression, all the gas is blown out or sucked back through it, by the pressure difference, created by the movement of the piston. The piston performs a pure sinusoidal motion for generality.

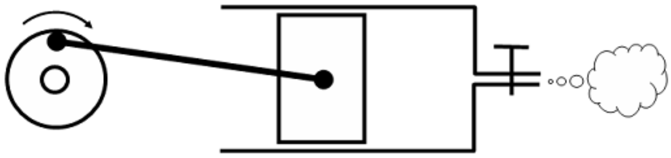

Figure 3. *The simplified test model with an adjustable valve*

5. Reasons of the leakage loss

The leakage loss is a stream of gas or liquid indeed, which is driven by the pressure difference between the sides of the piston. Thus, the starting point is the Bernoulli equation. Logically there is no height difference, so the pressure on one side balances the flow speed on the other side. Therefore, the speed of stream is defined.

$$\frac{\rho}{2} \cdot v^2 + \rho \cdot g \cdot h + p = Const. \tag{1}$$

$$p - p_{gnd} = \frac{\rho}{2} \cdot v^2 \tag{2}$$

$$v = \sqrt{\frac{2}{\rho} \cdot (p - p_{gnd})} \tag{3}$$

The pressure depends on the mass of the gas charge of the piston. This is expressed by the following equation, where the density assumed as constant.

$$\frac{dm}{dt} = A \cdot \rho \cdot \sqrt{\frac{2}{\rho} \cdot (p - p_{gnd})} \tag{4}$$

$$\frac{dm}{dt} = A \cdot \sqrt{2\rho} \cdot \sqrt{p - p_{gnd}} \tag{5}$$

This equation cannot be solved algebraically just numerically, only if the pressure function is known.

$$m(t+\Delta t)=m_0-\Delta t\cdot A\cdot\sqrt{2\rho}\cdot\sqrt{p-p_{gnd}} \tag{6}$$

6. Effect of the loss of working medium

The pressures can be calculated from the gas law. The pressure of the bottom and top dead center are given from their corresponding volumes.

$$p_{BDC}=\frac{m_{gnd}\cdot R\cdot T}{V_{BDC}} \quad \text{and} \quad p_{TDC}=\frac{m_{gnd}\cdot R\cdot T}{V_{TDC}} \tag{7}$$

The function of pressure over time can also be expressed from these pressures for the sake of clarity, using the frequency of rotation of the machine. According to the equation below, the pressure alternates between the value of the bottom dead center and the top dead center. This is a general calculation, which is adoptable to a given situation, so now, the piston motion is assumed pure sinusoidal. Cosine function is applied simply to start the pressure curve from zero – but that does not change the final result of the calculation.

$$p_{ideal}(t)=p_{BDC}+p_\delta\cdot\left[0.5\cdot\left(1-cos\left(\frac{2\pi\cdot N}{60}\cdot t\right)\right)\right], \tag{8}$$

where:

$$p_\delta=p_{TDC}-p_{BDC} \tag{9}$$

This equation describes only the pressure function of the ideal case. That is, when there is no leakage at all, because the value of the real pressure depends on the mass of the gas charge. These two parameters depend on each other recursively.

$$p(t)=\frac{m(t)}{m_{gnd}}\cdot p_{ideal}(t) \tag{10}$$

7. The common solution of equations

As shown above, the situation arose that the mass of the gas charge depends on the value of the pressure, and the pressure depends on the mass of the gas charge. Given a differential equation and given an algebraic equation. To solve this problem a certain algorithm is applied. The algorithm *(Figure 4)* used in Mathcad environment.

The algorithm must generate 3 results simultaneously. The first is the time distribution, the second is mass of gas charge, and the third one is the pressure function.

The algorithm has 4 main sections.

Section N1:

Input of the base parameters of the calculation: the length of the time step, the number of time steps, and the initial values must be determined.

Section N2:

The first “for” cycle generates the time net. At the end of it is the collection of time values are under the first “for” cycle. This is just an auxiliary result indeed that makes possible to represent the two main results in a graph.

Section N3:

Inputs of the main values of the calculation happens here.

Section N4:

The second “for” cycle gives the mean results. That is divided into two parts.

Section N4/1:

This generates the change of mass of the charge from the pressure difference during a time step given above in section N1. It contains the *equation (6)* indeed. This is divided into to parts as well to avoid the negative result under the root. The first equation is used when the inner pressure is higher than the ambient one, so the charge leaks out of the cylinder. The second one is used when it is lower, so the charge leaks in the cylinder.

Section N4/2:

This is the *equation (8)* where the pressure is calculated form the position of the piston and the mass of charge during a time step given in the section N1.

Outputs:

Two technical expressions close the calculation. The first one in double brackets is the two main outputs of the section N4, finally in triple brackets is the output of the entire calculation.

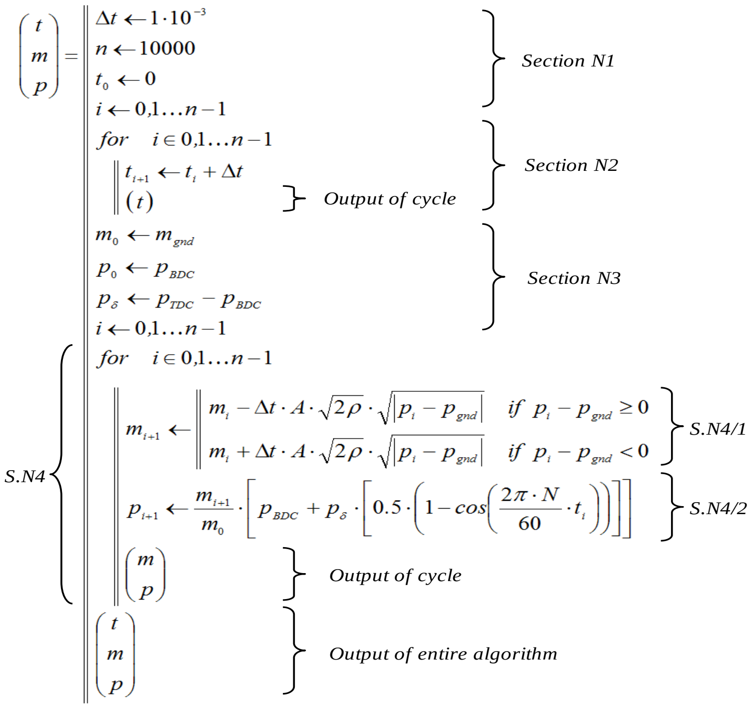

Figure 4. *The algorithm of leakage calculation*

8. Results

Let us test the completed mathematical model. The pressure of BDC should be the ambient pressure, therefore 1 bar, and the TDC pressure for example 2 bars, and the frequency of rotation should be 3000 rotation per minute, at the beginning, than 300, 30, finally 3 rpm *(Figure 5)*. The lower the speed of rotation, the greater the pressure loss. In extreme case there is no pressure rise at all.

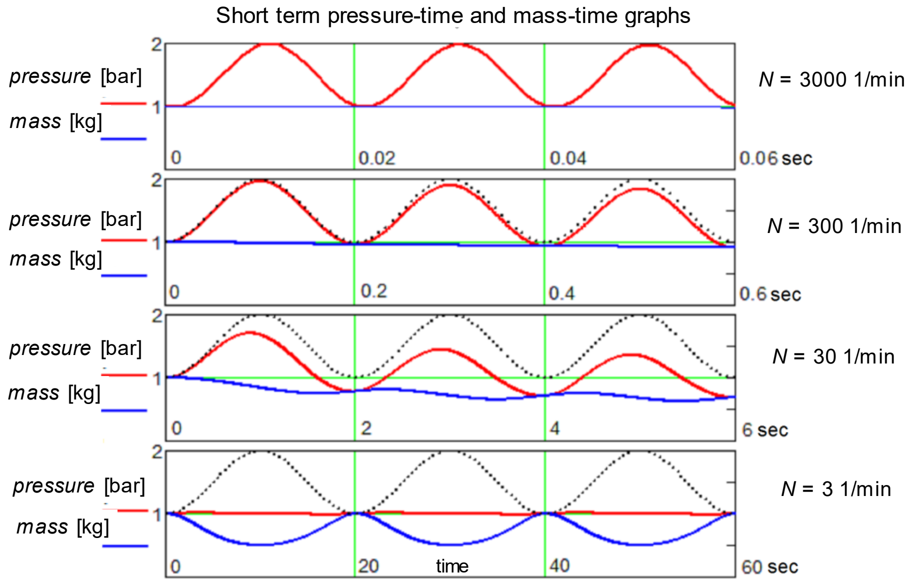

Figure 5. *Time diagrams of pressure and mass of charge at different speeds (pressure: red, mass: blue, ideal pressure: black dot line)*

Now, the speed of rotation will be constant (125 1/min), but the orifice of the gap will be changed, growing bigger and bigger. As it can be seen, the more the gap size increases, the faster the TDC pressure curve decreases, and the medium pressure decreases as well, which asymptotically approaches the ground pressure. Meanwhile, the amplitude of is getting smaller. Its limit is zero at the infinitely large gap *(Figure 6)*.

After this calculation, it is already understandable, why there was a remarkable difference between the calculated and the measured pressure on *Figure 2*. The pressure curve was measured in a stabilized working state, and that time the minimum pressure was already far below the ground pressure.

Of course, such a striking phenomenon can only occur in the case of a hot air engine, because it works continuously with the same gas charge (Karabulut et al., 2010), but for example an internal combustion engine change the charge after each work cycle, thus restarting the leakage process from the beginning. In this case the mean pressure of compression will be weaker, and the crank house gas will be stronger.

The decrease of the average pressure makes the performance of both types of engine worse. In order to reduce this harmful effect, the crank case of more serious hot air engines is closed and the outlet of the crankshaft is sealed (Cinar, 2014), (Carlsen, 1994).

It should be noted that the compression ring of the piston does not allow the stream to flow to both direction the same way (Delprete et al., 2019). Its design created to keep the gas in. In this case, we must be calculated with two different gap orifice, so it must be written A_{out} to the upper line and A_{in} to the lower line of mass equation in the section N4/1 of the algorithm instead of pure A.

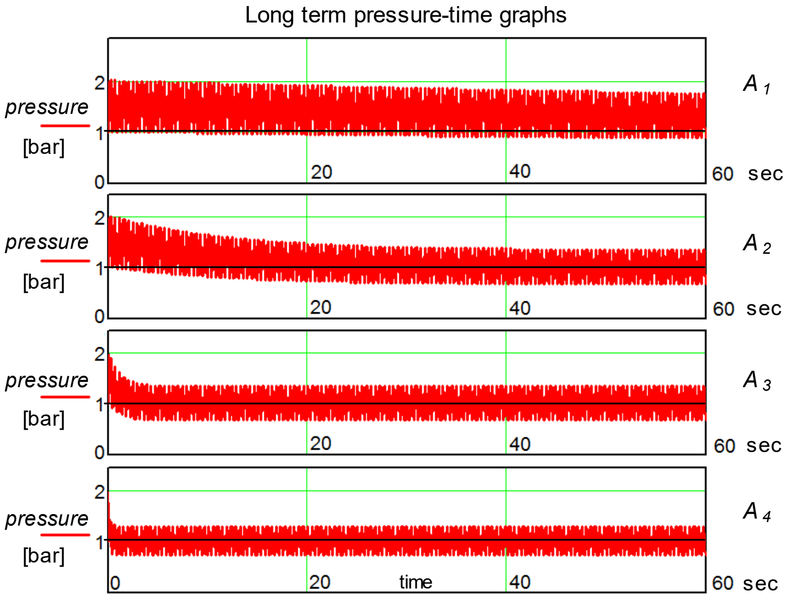

Figure 6. *The long term pressure time graphs at different orifice of gap (size of gap: $A_1 < A_2 < A_3 < A_4$)*

9. Conclusions

In this article we have introduced the phenomenon of leak of working fluid in piston machines. This problem always occurs in all such machines. Despite the damage caused, it is mostly ignored although it is a very familiar phenomenon. The leak most noticeable at the hot air engines, since there is no change of charge. A simple mathematical model has been developed in order to help to understand the phenomenon, its influence of the work and to estimate the resulting loss. Finally, the behaviour of this model has been presented.

Nomenclature

A	size of orifice of valve	[mm^2]
A_{in}	size of orifice for inward flow	[mm^2]
A_{out}	size of orifice for outward flow	[mm^2]
g	gravitational acceleration	[m/s^2]
h	height	[m]
m	mass of charge	[kg]
m_0	initial mass	[kg]
m_{gnd}	ground mass	[kg]
n	number of steps	[–]
N	rotational frequency	[1/min]
p	pressure	[bar]

p_0	initial pressure	[bar]
p_{BDC}	pressure at bottom dead center	[bar]
p_{gnd}	ground pressure or ambient pressure,	[bar]
p_{ideal}	ideal, no leakage pressure	[bar]
p_{TDC}	pressure at top dead center	[bar]
R	specific gas constant	[J/kgK]
rpm	rotation per minute	[1/min]
T	temperature	[K]
t	time	[s]
t_0	initial point of time	[s]
v	velocity of stream	[m/s]
V	volume	[m^3]
V_{BDC}	volume of bottom dead center	[m^3]
V_{TDC}	volume of top dead center	[m^3]
Δt	time step size	[s]
ρ	density	[kg/m^3]

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