COMPARATIVE EXERGY ANALYSIS OF A THERMOELECTRIC HEAT PUMP AND A VAPOUR COMPRESSION HEAT PUMP

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Abstract
The article is performing a comparative exergy analysis of a thermoelectric heat pump and a vapour compression heat pump, operating in the same conditions, from heat source temperature point of view. The COP of vapour compression heat pump was found to be 3.02 compared to 1.349 for the thermoelectric heat pump for 60 °C temperature lift. For the same temperature lift, corresponding to the increase in the temperature of the heat source from 5 °C to 65 °C required for the delivery of heat for household purposes, exergy efficiency of vapour compression heat pump is 59.44% compared to 23.90% for the thermoelectric heat pump, while the energy needed to operate the thermoelectrical heat pump is 20.76 kW compared to 9.29 kW for vapour compression heat pump.

Keywords: vapour compression heat pump, thermoelectrical heat pump, coefficient of performance, exergy efficiency

1. Introduction
A major concern in all countries around the world is energy efficiency as energy crisis it’s a topic that’s always up to date, and there are also difficulties linked to finding a secure energy supply.

The importance of energy efficiency in the EU is highlighted by Directive 2012/27/EU on energy efficiency. First adopted in 2012, the directive was updated in 2018 and 2023, setting rules and obligations for achieving the EU’s ambitious energy efficiency targets, emphasizing the need to increase energy efficiency in order to achieve the 2030 climate target and ensure energy security within the EU [1].

One aspect of efficient energy use is to employ heat pumps for heat usage in households and in industry. Also, the recovery secondary energy resources represent an ongoing concern of governments around the world. Various guidelines were developed and examples of good practices were presented in support of this initiative by governmental and non-governmental organizations [2].
There are a wide variety of heat pumps and hence the difficulty in choosing them, taking into account some objective criteria. Unless they have been designed for specific uses, choosing a particular type of heat pump can be a difficult task.

The difficulty of choosing a type of heat pump can also be shown by simply listing the main types of heat pumps, classified based on their mode of operation, vapour compression, gas compression, absorption, resorption, ejection and thermoelectric heat pumps.

2. Schematics of the vapour compression and thermoelectric heat pumps

Schematic of the mechanically driven vapour compression heat pump (a) and temperature–entropy diagram of the vapour-compression cycle (b) is presented in Figure 1.

The mathematical model of the vapour compression heat pump is well known and it is widely presented in several sources [3] [4] [5] [6].

![Figure 1. Vapour compression heat pump (a) and temperature–entropy diagram of the vapour-compression cycle (b)](image)

Heat is delivered to the heat-pump evaporator in which the heat-pump working fluid is vaporized. In case of water source heat pumps, the heat for evaporator is drawn from surface water or pond using loops.

The schematic of a thermoelectric heat pump assembly (a) and a thermoelectric element (b) is presented in Figure 2.

These devices work based on the thermoelectric phenomenon. The Seebeck, Peltier and Thomson effects fall into the category of thermoelectric phenomena. The phenomenon in which a temperature difference between two dissimilar electrical conductors or semiconductors produces a voltage difference between the two substances is known as the Seebeck. Applying heat to one of the two conductors or semiconductors, heated electrons will flow toward the cooler one.
The reverse phenomenon of the Seebeck effect is called the Peltier; the electrical current flowing through the junction connecting two materials will emit or absorb heat per unit time at the junction to balance the difference in the chemical potential of the two materials.

The evolution or absorption of heat when electric current passes through a circuit composed of a single material that has a temperature difference along its length is known as the Thomson effect. This transfer of heat is superimposed on the common production of heat associated with the electrical resistance to currents in conductors. It should be noted that Thomson and Peltier effects refer to reversible processes.

When designing thermoelectric systems, the influence of the Thomson effect is usually neglected.

![Diagram of thermoelectric heat pump](image)

**Figure 2.** Thermoelectric heat pump assembly (b) and thermoelectric element (b)

3. Mathematical model of thermoelectrical heat pump

A working mathematical model of the vapour compression heat pump was already developed and presented in paper [7].

For the thermoelectrical heat pump, based on schematic of a thermoelectric element, depicted in Figure 2 b), the heat flux through the cold end of the element can be calculated using [4]:

\[
Q_r = \alpha \cdot I \cdot T_r - 0.5 \cdot R \cdot I^2 - \Lambda \cdot \Delta T, \text{ W} \tag{1}
\]

where: \(\alpha\) is the overall Seebeck coefficient in V·K\(^{-1}\); \(R\) is the overall electric resistance, in Ω; \(\Lambda\) the overall conductivity of the thermoelectric element, in W·K\(^{-1}\).
The equations for calculating these elements are:

\[ \alpha = |\alpha_A| + |\alpha_B| \frac{V}{K} \]  \hspace{1cm} (2)

where: \( \alpha_{A,B} \) – the Seebeck coefficient of component A and B of the thermoelectric element in V·K\(^{-1}\).

\[ R = \rho_A \cdot \frac{l_A}{A_A} + \rho_B \cdot \frac{l_B}{A_B} \cdot \Omega \]  \hspace{1cm} (3)

where: \( \rho_{A,B} \) – is the resistivity of component A and B of the thermoelectric element in \( \Omega \cdot m \); \( l_{A,B} \) – length of component A and B of the thermoelectric element in m; \( A_{A,B} \) – area of component A and B of the thermoelectric element in m\(^2\).

Finally:

\[ A = \lambda_A \cdot \frac{A_A}{l_A} + \lambda_B \cdot \frac{A_B}{l_B} \cdot \frac{W}{K} \]  \hspace{1cm} (4)

where: \( \lambda_{A,B} \) – is the thermal conductivity of component A and B of the thermoelectric element in W·m\(^{-1}\)·K\(^{-1}\).

In order to establish the thermal characteristics of the thermoelectric system, the outlet heat at the warm end of the thermoelectric element needs to be calculated, starting with its energy balance equation [4]:

\[ Q_c = Q_r + P_i \cdot W \]  \hspace{1cm} (5)

where: \( P_i \) – electrical power required to operation, in W, \( Q_r \) - the heat flux through the cold end of the element, in W.

As thermal characteristics of vapour compression and thermoelectric heat pump will be compared the main indicators can be computed using the following equations [4].

Maximal cooling efficiency of the system:

\[ \varepsilon_{f,\text{max}} = \frac{Q_r}{P_i} \]  \hspace{1cm} (6)

Maximal heat pump efficiency also known as coefficient of performance (COP):

\[ \mu_{\text{max}} = \frac{Q_r}{P_i} \]  \hspace{1cm} (7)

Cooling efficiency of the equivalent Carnot cycle:

\[ \varepsilon_c = \frac{T_r}{\Delta T} \]  \hspace{1cm} (8)

where: \( T_r \) – is the temperature of the cool end of the thermoelectric element, in K; \( \Delta T \) – is the temperature difference between the cool end and the warm end of the thermoelectrical element, in K.

Coefficient of performance (COP) of the equivalent Carnot cycle:

\[ \mu_c = \frac{T_r}{\Delta T} \]  \hspace{1cm} (9)

where: \( T_c \) – is the temperature of the warm end of the thermoelectric element, in K; \( \Delta T \) – is the temperature difference between the cool end and the warm end of the thermoelectrical element, in K.

Exergy efficiency of cooling operation:

\[ \eta_{E_f} = \frac{m \cdot \frac{T_c}{T_r}}{m + 1} \]  \hspace{1cm} (10)
In which \( m \) is a parameter that can be calculated using:

\[
m = \sqrt{1 + Z \cdot \frac{T_c + T_r}{2}}
\]  

(11)

Where \( Z \) is the efficiency of the thermoelectric element, given by equation:

\[
Z = \frac{\alpha^2}{\Phi_{min}}
\]  

(12)

While \( \Phi_{min} \) is given by equation:

\[
\Phi_{min} = (\sqrt{\lambda_A \cdot \rho_A} + \sqrt{\lambda_B \cdot \rho_B})^2
\]  

(13)

The exergy efficiency of the thermoelectric heat pump can be calculated using:

\[
\eta_{Et} = \frac{m \cdot \frac{T_r}{T_c}}{m + 1}
\]  

(14)

4. Results

As literature points to the fact that the coefficient of performance of a heat pump increases with the heat source temperature, low heat source temperatures were selected to perform calculations in order to study the worst-case scenario for the operation of heat pumps.

If we consider that the heat source of the vapour compression heat pump waste water, the starting ambient temperature will be the average temperature in the evaporator, taking into account that at outlet of evaporator water temperature must be 5 °C to prevent freezing. Then temperature increments of 5 °C can be used, resulting the following heat source temperatures: 5 °C, 10 °C and 15 °C.

As the usual hot water temperature is around 65 °C, temperature lifts of 60, 55 and 50 °C will occur.

Data for calculation are: \( Q_i = 28 \text{ kW} \) – required heat delivery, \( T_i = 65 \text{ °C} \) – temperature of delivered hot water, \( T_a = 5, 10, 15 \text{ °C} \) – ambient temperature, \( \Delta T_c = 5 \text{ °C} \) – temperature difference required for heat transfer in condenser (heat delivery), \( \Delta T_0 = 5 \text{ °C} \) – temperature difference required for heat transfer in evaporator, \( \Delta T_{sr} = 5 \text{ °C} \) – temperature difference for sub-cooling and \( \eta_{em} = 0.9 \) mechanical efficiency.

The refrigerant used is ammonia (R717).

In Table 1 results for calculation of vapour compression heat pump are presented, the main thermal characteristics, used for comparison will be selected after checking results obtained from calculations for the thermoelectrical heat pump.

**Table 1. Results for vapour compression heat pump**

<table>
<thead>
<tr>
<th>Nom.</th>
<th>Temperature lift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60 °C</td>
</tr>
<tr>
<td>Refrigerant flow rate, kg·s⁻¹ x 10⁻³</td>
<td>20.732</td>
</tr>
<tr>
<td>Work supplied to compressor ( P_e ), kW</td>
<td>9.286</td>
</tr>
<tr>
<td>Evaporator heat transferred ( q_0 ), kJ·kg⁻¹</td>
<td>947.41</td>
</tr>
</tbody>
</table>
Comparative exergy analysis

<table>
<thead>
<tr>
<th>Nom.</th>
<th>Temperature lift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60 °C</td>
</tr>
<tr>
<td>Condenser heat transferred $q_c$, kJ·kg$^{-1}$</td>
<td>1,350.54</td>
</tr>
<tr>
<td>Ideal Carnot COP $\mu_C$</td>
<td>5.64</td>
</tr>
<tr>
<td>Theoretical COP $\mu$</td>
<td>3.35</td>
</tr>
<tr>
<td>Practical COP $\mu_e$</td>
<td>3.02</td>
</tr>
</tbody>
</table>

Losses:

<table>
<thead>
<tr>
<th>Nom.</th>
<th>Material A</th>
<th>Material B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression $\pi_{ic}$, kJ·kg$^{-1}$</td>
<td>50.730</td>
<td>43.20</td>
</tr>
<tr>
<td>Expansion $\pi_{el}$, kJ·kg$^{-1}$</td>
<td>33.690</td>
<td>28.695</td>
</tr>
<tr>
<td>Heat transfer in evaporator $\pi_{e0}$, kJ·kg$^{-1}$</td>
<td>17.342</td>
<td>17.129</td>
</tr>
<tr>
<td>Heat transfer in condenser $\pi_{ATe}$, kJ·kg$^{-1}$</td>
<td>62.035</td>
<td>54.001</td>
</tr>
<tr>
<td>Work of Ideal Carnot cycle $l_{max}$, kJ·kg$^{-1}$</td>
<td>239.630</td>
<td>212.783</td>
</tr>
<tr>
<td>Exergy efficiency $\eta_e$, %</td>
<td>59.44</td>
<td>59.75</td>
</tr>
</tbody>
</table>

The calculations for the thermoelectric heat pump were performed considering the same heat source temperatures, temperature lifts and required heat delivery of 28 kW, which represents the average required heat for a household.

In addition, the semiconductor materials that were used to build the thermoelectric elements need to be selected, in this case telluride (A) and bismuth (B).

Physical characteristics of the materials are presented in Table 2.

**Table 2. Physical characteristics of semiconductors used for the thermoelectric element**

<table>
<thead>
<tr>
<th>Nom.</th>
<th>Material A</th>
<th>Material B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter $d$, mm</td>
<td>(will be computed)</td>
<td>7</td>
</tr>
<tr>
<td>Length $l$, mm</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Seebeck coefficient $\alpha$, V·K$^{-1}$</td>
<td>0.23·10$^{-5}$</td>
<td>−0.21·10$^{-3}$</td>
</tr>
<tr>
<td>Electrical conductivity $\rho$, Ω·m</td>
<td>10$^{-5}$</td>
<td>10$^{-5}$</td>
</tr>
<tr>
<td>Thermal conductivity $\lambda$, W·m$^{-1}$·K$^{-1}$</td>
<td>1.7</td>
<td>1.45</td>
</tr>
</tbody>
</table>
Results obtained for the thermoelectric heat pump are presented in Table 3.

Table 3. Results for the thermoelectric heat pump

<table>
<thead>
<tr>
<th>Nom.</th>
<th>Temperature lift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60 °C</td>
</tr>
<tr>
<td>Optimal current intensity $I_{\text{opt}}$, A</td>
<td>38.51</td>
</tr>
<tr>
<td>Voltage (for one thermoelectric unit) $U$, V</td>
<td>0.093</td>
</tr>
<tr>
<td>Thermal power for one thermoelement unit $Q_{\text{cu}}$, W</td>
<td>4.834</td>
</tr>
<tr>
<td>Number of required thermoelectric units</td>
<td>5792</td>
</tr>
<tr>
<td>Overall electric power required $P_i$, kW</td>
<td>20.76</td>
</tr>
<tr>
<td>Overall voltage $U_i$, V</td>
<td>539.17</td>
</tr>
<tr>
<td>Maximal cooling efficiency of the assembly</td>
<td>0.349</td>
</tr>
<tr>
<td>Maximal heat pump efficiency (theoretical COP)</td>
<td>1.349</td>
</tr>
<tr>
<td>Cooling efficiency of the equivalent Carnot cycle</td>
<td>4.636</td>
</tr>
<tr>
<td>Thermal efficiency of the equivalent Carnot cycle</td>
<td>5.636</td>
</tr>
<tr>
<td>Exergy efficiency of cooling for the assembly, %</td>
<td>7.5</td>
</tr>
<tr>
<td>Exergy efficiency of the heat pump, %</td>
<td>23.9</td>
</tr>
</tbody>
</table>

5. Summary

Analyzing data in Table 1 and Table 3 the following thermal characteristics of the studied heat pumps were selected in order to compare the different types of heat pumps: work supplied to compressor for vapour compression heat pump versus overall electric power required for thermoelectric heat pump; the coefficient of performance COP and the exergy efficiency.

In Figure 3 work supplied to compressor for vapour compression heat pump versus overall electric power required for thermoelectric heat pump is presented. As expected, for low heat source temperatures (high temperature lifts) the energy needed to be supplied in order to deliver the required amount of heat is increasing.
**Figure 3.** Power required for the operation of the heat pumps

**Figure 4.** Coefficient of performance (COP) comparison
Comparative exergy analysis

The energy provided to operate the thermoelectrical heat pump is nearly double than that for the vapour compression heat pump, 9.29 kW for vapour compression heat pump compared to 20.76 kW for thermoelectrical heat pump for 60 °C temperature lift.

In Figure 4 the COP of heat pumps is compared. Again, the vapour compression heat pump performs better as the COP of vapour compression heat pump is 3.02 compared to 1.349 for the thermoelectrical heat pump for 60 °C temperature lift.

In Figure 4 the exergy efficiency of heat pumps is compared. The vapour compression heat pump has a better exergy efficiency as expected given the results presented above. Exergy efficiency of vapour compression heat pump is 59.44% compared to 23.90% for the thermoelectric heat pump for 60 °C temperature lift.

Conclusively, the overall performance of vapour compression heat pump is much bigger than the performance of thermoelectric heat pump, not to mention the energy consumption needed to operate the equipment.

Data found in literature is consistent with results obtained and presented above [8] [9]. Viral K. Patel et al. in paper [8] computed the COP of a thermoelectric heat pump resulting a COP of 1.29 for 31.41 ΔT.

Similar results obtained by G. N. Kozhemyakin et al., are showing that for increased electrical power (to 40 W) and ΔT = 30 K, the studied thermoelectrical heat pump’s COP was 1.8.

Conclusively if the only criterion for the choice of a heat pump are better thermal characteristics, vapour compression heat pumps perform better in every aspect.

In future, research for finding new materials will increase the efficiency of thermoelectrical heat pumps.

Another important aspect for results obtained in this work is the fact that calculations were made for low temperatures of the heat source, which negatively influences the coefficient of performance.
References


