# FIRST RESULTS OF THE WAVELET-ANALYSIS OF KARST WATER LEVELS FROM THE BÜKK MNTS.

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***Abstract***
*During our research, we used long-term data measured by the Bükk Karst Water Level Monitoring System to subject them to a periodicity test based on statistical methods. The goal was to search for cyclic components in the data of karst water levels, water temperature and conductivity measurements, using the methodology of wavelet analysis to supplement traditional deterministic procedures. Based on the calculations, it was possible to detect several periods in the data series, we also detected several properties that are only characteristic of hot or cold karst wells. Cycles with a period of 1 year, 3 years, or even longer were detected at several measurement sites, and with the help of the method, we were also able to determine which cycles appeared in the time series, and whether there was a period when it was not detectable in the data series. During the wavelet coherence tests, we determined to what extent these cycles move together in the data series of the measuring points of the Bükk Mountains, and whether there is a case where one measuring point "leads" the other time series.*

***Keywords:*** *periodicity analysis, water levels, wavelet, Bükk*

## 1. Introduction

The Bükk Mountains plays a significant role fulfilling the different water demands of Northern-Hungary. The karst waters produced from the aquifer give the drinking water of the nearby cities, the balneological waters of the famous thermal baths of the area and deep hot karst waters are used for heating whole neighborhoods in Miskolc (Miklós et al., 2020).

Examining the behavior of the quantity and quality of the stored water is researched for more than a century, with the current Bükk Karst Water Monitoring System operational since 1992 (Lénárt, 2023). With the current monitoring network large numbers of measurements were taken in the Mountains, now (2024) more than 23 million data are available for research.

Previously several mathematical and statistical methods were implemented to examine the water bodies of the cold and warm karst systems, such as determining the available water resource, and defining the effects of the different water usages (Darabos et al., 2022). With the time series of the monitoring evolving into a large database more advanced mathematical methods can be used for number of goals. After clearing the datasets, the longer time series of the monitoring network is used for wavelet and bi-wavelet coherence calculations to determine the periodicity of different measurements of cold and hot water wells and addressing statistical significance values to the defined cycles (Ilyés et al., 2018).

During our investigation, we used daily, monthly and annual data (the length of the data series was typically 8–11 years, but there are also time series used that are more than 30 years long), so that we can carry out our investigations in the widest possible spectrum.

Through the first results of our investigations, the temporality of the karst water aquifer of the Bükk Mountains becomes better known, with subsequent investigations we can also map the exact cause-and-effect relationships.

## 2. Methods and Materials

### *2.1. Wavelet transform*

To achieve the goals, We considered the wavelet spectrum estimation to be the most suitable, since the method is localized in time and frequency, i.e. it enables a long time-frequency resolution. The periodic characteristics of the investigated signal can be determined.

The wavelet transform (WT) is a decomposition procedure based on the Fourier transform, which divides the examined time series into sine-cosine waves. The basic function of WT is as follows:

$$W_n(s) = \sum_{n'-1}^{n} X_{n'} \, \Psi^* \left[ \frac{(n' - n)\delta t}{s} \right] \quad (1)$$

where Ψ denotes the wavelet, the star its complex conjugate, $X_n$ is the original time series, and the scale, $\delta_t$ is the degree of resolution. The wavelet functions are consisting of so-called main wavelets:

$$\Psi_{t,s}(t) = s^{-0,5} \Psi \left[ \frac{(t - \tau)}{s} \right] \quad (2)$$

The value of the scale parameter *s* determines how wide the examination window is. We used the Morlet-wavelet for the tests, which is most often used in the natural sciences. The boundary condition of the WSA analysis is that the time series taken with equidistant sampling.

The result of the test is the wavelet spectrum, which is a function of two independent variables, time and frequency. There are many visual techniques for displaying it, such as isoline diagram, the color scale of which indicates the probability of the existence of periodicity. Towards warm colors, the probability of the existence of the given period increases, and the area delimited by a thick black line can be accepted at the 5% significance level. The figure shows a hatched part, the COI (cone of influence), the area influenced by the confidence interval (Garamhegyi et al., 2018).

### *2.2. Wavelet-coherence*

The WSA is basis of the wavelet coherence (WTC, wavelet transform coherence), which can show co-movement between two time series, in addition to the time dimension, in the frequency space as well.

$$R_{x,s}(\tau,s) = \frac{\left|S\left(W_{x,y}(\tau,s)\right|\right.}{\left\{S[|W_x(\tau,s)|^2]S\left[\left|W_y(\tau,s)\right|^2\right]\right\}^{0,5}} \tag{3}$$

Where Wx,y (τ, s) and Rx,y (τ, s) are cross-wavelet and wavelet coherence transformed at time τ, in addition to the scale parameter between the time series x and y, with the smoothing function S(-). There is a similarity between traditional correlation procedures and WTC, so the solution can also be interpreted as a determination coefficient localized on a time scale. At the same time, although the WTC shows strong coherence, the value of the correlation coefficient may be low, since the periodic components must be present in both time series. Also, between two opposite processes, the coherence will be high, while the correlation will be low. Like the WSA, the COI cannot be neglected here either. A confidence level can also be assigned to the results. For the sake of simplicity, we chose a level of 95% here as well. The method can be supplemented by the delay structure between individual frequencies, also by examining the so-called phase difference. The value of the phase difference can fall into the interval between (–π,π). If its value is equal to the extreme value, – π or + π, it indicates perfect antiphase, while if the phase difference is 0, then by definition the two frequencies are in perfect phase with each other. The different cases are shown in the figure below.

Among the extreme values, one of the time series "leads" the process, which, with a suitable professional interpretation, can also help establish a background process.

These methods were used previously on several karst water data (Charlier et al., 2015; Xing et al., 2018; Schuler et al., 2020).

### *2.3. Input data*

For the tests, we used the available data of the Bükk Karst Water Level Monitoring System. The *Table 1* below shows the input data in detail *(Figure 1).*

***Table 1***

*Main properties of examined datasets*

| *Well/Spring ID* | *Interval* | *Parameter* | *Additional info* |
|---|---|---|---|
| Bel_III | 08. 07. 2008 – 27. 12. 2019 | water level | |
| Fe_Sz5 | 22. 03. 2010 – 27. 12. 2019 | water level | |
| Mi_Selyemrét2 | 01. 01. 2014 – 01. 04. 2023 | water level | |
| Mi_T3 | 01. 01. 2014 – 01. 04. 2023 | water level, temperature, conductivity | |
| NV_17 | 11. 10. 1992 – 04. 05. 2023 | water level | |
| Mi_Kertészet | 01. 01. 2014 – 01. 04. 2023 | water level, temperature | |
| Mi_Parki | 27. 12. 2015 – 01. 04. 2023 | water level, temperature, conductivity | 2015–2017; 2018–2023 |
| Mi_Selyemrét1 | 2013–2023 | water level | 2013–2015; 2016–2023 |
| Mi_2f | 01. 01. 2014 – 04. 03. 2023 | water level | |

| | | | |
|---|---|---|---|
| Mi_T10 | 01. 04. 2011 – 31. 03. 2023 | water level | 2011–2022; 2022–2023 |
| Mi_Szinva | 12. 05. 1994 – 09. 01. 2022 | water level | 1994–1999; 2002–2022 |
| KGy_F1 | 06. 10. 2006 – 31. 12. 2019 | water level | |

Daily, monthly and annual data were available for the tests. Monthly and annual average values were created from the daily measured data, which were further analyzed. The results are presented in the next chapter.

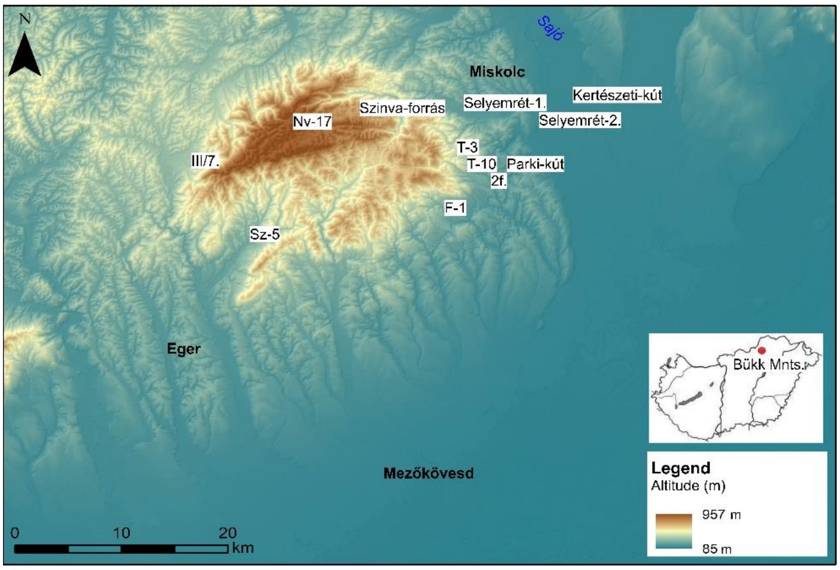


***Figure 1.*** *The location of the examined monitoring points*

## 3. Results

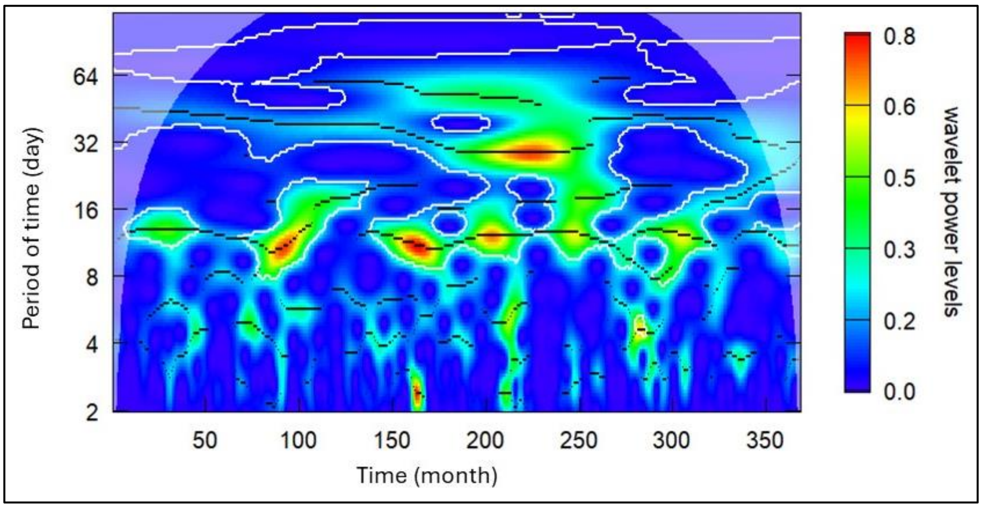


***Figure 2.*** *Wavelet from monthly NV-17 karst water level data*

In case of BEL-III a significant 2.8 and 4 year long cycles were defined, in most of the examined period, in F-SZ5 a 1 and a 1.5 year long, with a 7–8 month long cycle were determined, along with a not-significant 2 year long one.

Mi-Kertészet's temperature data, no significant cycle is shown, possibly because of a small examined interval, but 0.17, 0.7, 1 and a 2 year long cycle were calculated. In case of water levels, a significant 0.2, 0.7, 1 and a 2.5 year long cycle were calculated.

In Mi-Park well data because of an interval with missing data 2 periods were used for the calculations. A 1 month long and several other significant cycles were defined, such as the 1 and a half year long one. In case of temperature no significant cycles were calculated. Mi-Park wells conductivity measurements showed a 0.3 and 0.2 year long periodicity along with a 1.5 year long one.

In the Selyemrét 1 and 2 wells a 0.4, 0.5, 0.9 and a 1 year long cycles are present. In most cases these are significant, or have a strong wavelet power.

The Mi-Szinva datasets have a long time interval, although with missing data, the first results showed a 3, 4–5, 7–8 and a 12 month long cycle, when using annual data a strong 3 and 5 year long period were calculated *(Figure 3)*.

In the T-10 well a 0.3, 0.7, 1, 1.5 and a 2.5–3 year long cycle is present, although significantly, most of the were not definable in the whole examined time intervals.

In the Mi-2f well data number of significant cycles were calculated with smaller wavelet power levels. The calculated cycles are the 1.3, 2.5, 6 and 12 month long ones ,when using annual data, only the 1 year long cycle was significant.

The NV-17, which is the backbone of the karst monitoring system, a large interval was available for the calculations. The half, 1 and 3 year long cycles were present in the daily datasets, in the monthly a 2.5 year long was defined along with a period at around 3 years *(Figure 2)*.

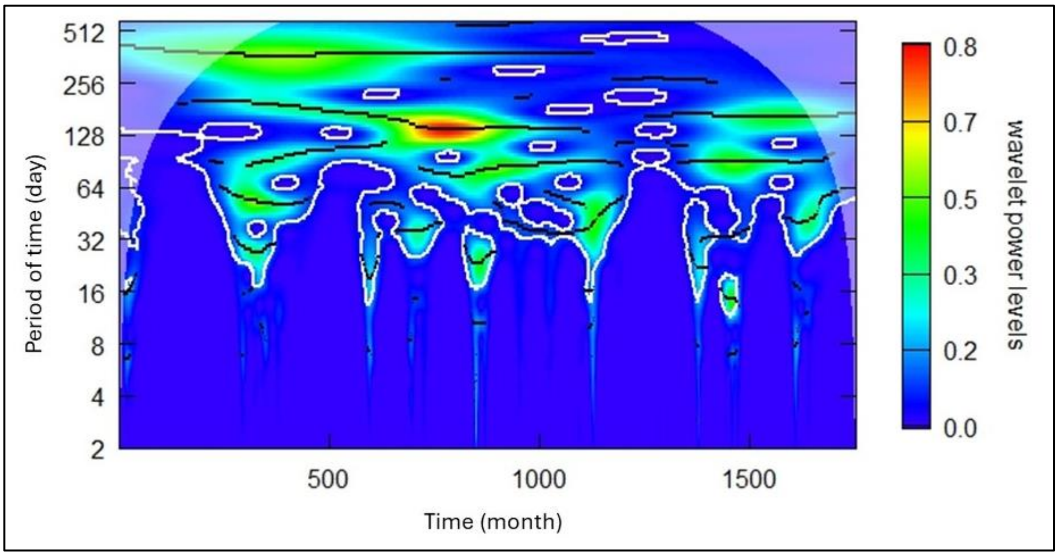


***Figure 3.*** *Wavelet from daily Mi_Szinva karst water level data (1994–1999)*

Mi-T3 temperature data showed a 0.3 and a 1 year long period, but the annual graph showed that even the 1 year long period was not present in the whole examined time interval. The conductivity wavelets showed a 0.7, 1, 1.5 and a 2.6–3 year long periodicity but only in parts of the time interval also. The water levels showed only a 2.3–2.5 and a 3 year long periods, with number of less than 1 year long periods.

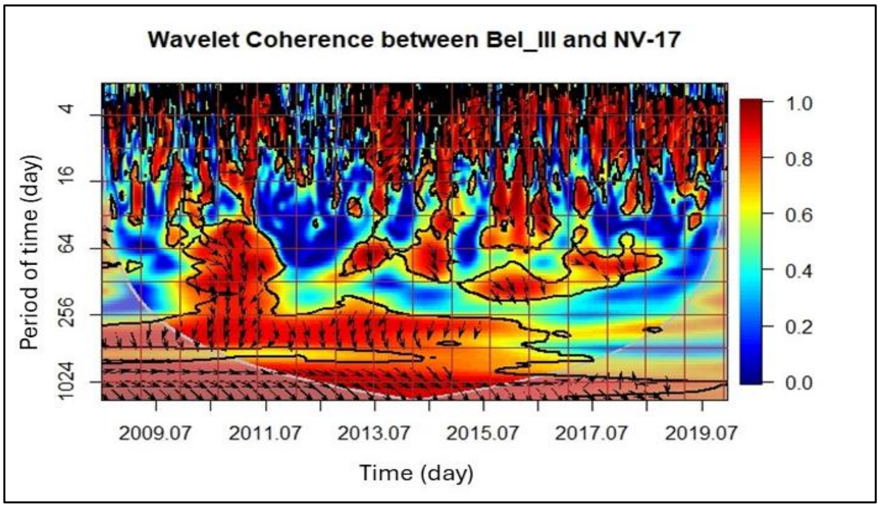


***Figure 4.*** *Wavelet-coherence between BEL_III and NV-17*

The results of the bi-wavelet coherence tests clearly show when the periods were when the data series moved together, and when the given period appeared sooner or later in terms of periodicity *(Figure 4)*.

Based on the first results, the smallest co-movement can be demonstrated with the data series of Selyemrét 1-2 and the Park well, while in the case of the other test points, in the case of several long-period cycles, they are either in phase, or one leads the other time series *(Figure 5)*. Antiphase could not be detected.

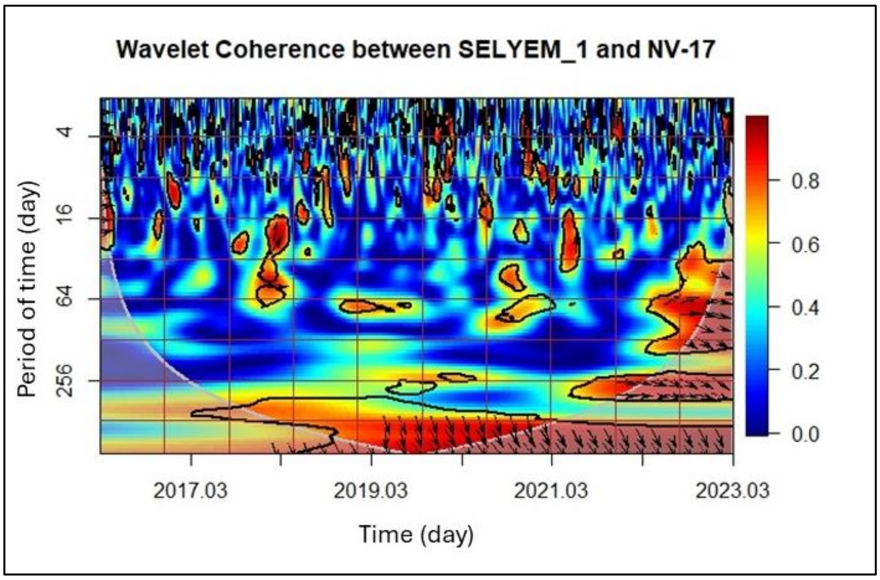


***Figure 5.*** *Wavelet-coherence between Selyem-1 and NV-17*

In the future, for a better understanding of extreme events, we will supplement these basic studies with analyzes operating on a similar principle, so that we can filter out natural and artificial effects, in order to promote climate adaptation.

## 4. Summary

The Bükk Mountains serve a vital role in fulfilling the area's different water needs, so examining the behavior of the stored karst water is essential for sustainable ground water management.

During our research, we used long-term data measured by the Bükk Karst Water Level Monitoring System to subject them to a periodicity test based on statistical methods. The goal was to search for cyclic components in the data of karst water levels, water temperature and conductivity measurements, using the methodology of wavelet analysis in addition to traditional deterministic procedures.

The results showed several significant cycles in different water level, temperature, and conductivity measurements, proving that these parameters have a strong periodicity in them. These first results need to be examined in the future to determine the causes behind these cycles, and what are the main factors that define the nature of the karst water levels, as well as searching for the effects of the changing climate and define its effect on the groundwaters of the Bükk Mountains.

Through the first results of our investigations, the temporality of the karst water aquifer of the Bükk Mountains becomes better known, with subsequent investigations we can also map the exact cause-and-effect relationships.

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## References

[1] Charlier, J-B., Ladouche, B., Maréchal, J-C. (2015). Identifying the impact of climate and anthropic pressures on karst aquifers using wavelet analysis. *Journal of Hydrology*, 523, 610–623. **https://doi.org/10.1016/j.jhydrol.2015.02.003**

[2] Darabos E., Tóth M. (2022). Karsztvízkészlet meghatározási módszer fejlesztései a Bükkben. *Hidrológiai Közlöny*, 102 (4), 37–43.

[3] Garamhegyi, T., Kovács, J., Pongrácz, R., Tanos, P., Hatvani, I.G. (2018). Investigation of the climate-driven periodicity of shallow groundwater level fluctuations in a Central-Eastern European agricultural region. *Hydrogeology Journal*, 26, 677–688. **https://doi.org/10.1007/s10040-017-1665-2**

[4] Ilyés, C., Turai, E., Szűcs, P. (2018). Examination of rainfall data for 110 years using spectral and wavelet analysis. *Central European Geology*, 61 (1), 1–15. **https://doi.org/10.1556/24.61.2018.01**

[5] Lénárt L. (2023). A Bükki Karsztvízszint Észlelő Rendszer (BKÉR) történetének, felépítésének, műszerezettségének, adatmennyiségének áttekintő összefoglalása. *Magyar Hidrológiai Társaság XL. Országos Vándorgyűlése* 2023. 07. 05. Győr, MHT.

[6] Miklós, R., Lénárt, L., Darabos, E., Kovács, A., Pelczéder, Á., Szabó, N. P., Szűcs, P. (2020). Karst water resources and their complex utilization in the Bükk Mountains, northeast Hungary: an assessment from a regional hydrogeological perspective. *Hydrogeology Journal*, 28, 2159–2172. **https://doi.org/10.1007/s10040-020-02168-0**

[7] Schuler, P., Cantoni, É., Duran, L., Johnston, P., Gill, L. (2020): Using Wavelet Coherence to Characterize Surface Water Infiltration into a Low-Lying Karst Aquifer. *Groundwater*, 59 (1), 71–79. **https://doi.org/10.1111/gwat.13012**

[8] Xing, L., Huang, L., Chi, G., Yang, L., Li, C., Hou, X. (2018): A Dynamic Study of a Karst Spring Based on Wavelet Analysis and the Mann-Kendall Trend Test. *Water*, 10 (6), 698. **https://doi.org/10.3390/w10060698**