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EXPLORING STRESS-DEPENDENT ACOUSTIC VELOCITY HYSTERESIS IN ROCKS – A PRELIMINARY STUDY

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Abstract

Knowledge of acoustic wave velocity in rocks is essential for geophysical studies, as it helps identify geological structures that may contain hydrocarbon accumulations. Acoustic velocity is strongly influenced by pressure and the presence of pore volume. This paper integrates experimental data with theoretical modeling to investigate stress-dependent acoustic velocity hysteresis in rocks and to analyze how stress affects acoustic velocity during loading and unloading. The study focuses on quasi-static elastic properties and introduces a new petrophysical model that explains acoustic hysteresis through pore closure and opening, thereby affecting P-wave propagation. The model is consistent with the observed data and provides a coherent framework for understanding the interactions among stress, pressure, and acoustic behavior. To validate the model, a sandstone sample was tested under incremental pressures, and the model parameters were determined using a linearized inversion method. The calculated and measured velocities showed close agreement, confirming the model's applicability.

Keywords: *Acoustic velocity, Stress-dependent, Hysteresis, Petrophysical model, Rock pressure*

1. Introduction

The mechanical behavior of rocks under stress is a central topic in several fields, including rock mechanics, geotechnical engineering, and geophysical exploration. Understanding how acoustic wave velocity changes under different stress conditions is significant for seismic interpretation and borehole logging data, as these properties directly influence the evaluation of subsurface stability and the identification of potential hydrocarbon-bearing formations [1–4]

Acoustic velocity in rocks is highly sensitive to pressure, and the relationship between the two is highly nonlinear. Many rocks exhibit quasi-static elastic behavior with noticeable hysteresis, meaning that the velocity measured during loading differs from that during unloading. This effect, called acoustic hysteresis, results from several physical processes. These include the irreversible closing of microcracks under increasing stress, the gradual compaction of pore spaces, and changes in grain-to-grain contact conditions that affect the elastic stiffness of the rock structure [7]

The closure of microcracks during pressurization (loading) was first proposed by Walsh and Brace [5], while Birch [6] demonstrated how the collapse of pores contributes to irreversible deformation. Hashin and Shtrikman [7] later explained how the development of ductile grain-contact zones further enhances nonlinear elastic behavior. These processes collectively produce the path-dependence pressure-velocity relationship observed in laboratory data.

Although several empirical mathematical models describe acoustic velocity as an exponential function of pressure, these models often lack a clear physical interpretation or explicitly account for hysteresis [8]. To address this gap, the present study investigates stress-dependent acoustic velocity hysteresis by integrating experimental data with theoretical modelling. Our aim is twofold: 1) To explore how stress influences acoustic velocity during loading and unloading, and 2) to introduce and validate a physically interpretable petrophysical model based on pore-volume changes.

We propose a simplified yet meaningful model that directly links pore closure and opening to acoustic wave propagation. The model is tested on laboratory P-wave velocity measurements performed on a sandstone sample subjected to cyclic uniaxial loading. A linearized inversion method is used to estimate the model parameters, and the results are compared with those of the well-known Preisach hysteresis model, a mathematical framework initially developed for ferromagnetic materials but increasingly applied to geoscience problems [9–11].

Comparing the classical linearized inversion with the Preisach-based inversion provides insight into the strengths and limitations of both approaches in capturing acoustic hysteresis. The results presented here serve as a preliminary step toward a more robust description of stress-dependent acoustic properties in rocks.

1.1. Overview of hysteresis phenomena in rocks

Hysteresis in rocks refers to the nonlinear, irreversible behavior of rock properties under varying external conditions, such as stress, pressure, or temperature. In simple terms, hysteresis means that the present state of a rock depends not only on the current applied load but also on its loading history. This behavior becomes evident when a rock exhibits different responses during loading and unloading cycles, as shown in *Figure 1*. Such hysteresis effects are frequently observed in acoustic velocity, permeability, porosity, and electrical conductivity [12–16].

The mechanisms underlying hysteresis in rocks is caused by irreversible microstructural changes during cyclic loading. Microcracks close under increasing stress but only partly reopen during unloading, producing a persistent reduction in pore volume and a hysteretic velocity-pressure trend [5]. Pore collapse under high pressure adds further irreversibility [6], while stress-induced changes in grain contacts also prevent the rock from returning to its initial elastic state [7]. These combined effects make the loading and unloading velocity curves follow different paths, as confirmed by experimental studies [14–16].

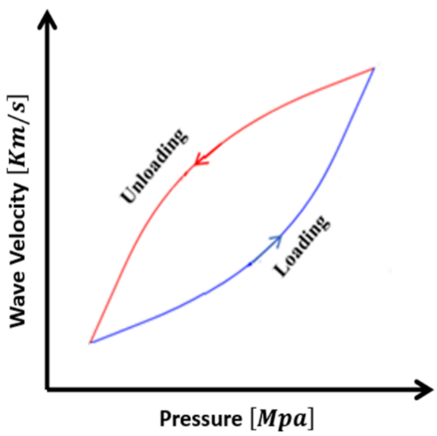

Figure 1: *Schematic view of wave velocity as a Function of Pressure in the Case of Loading and Unloading (Hysteresis Curve)*

2. Materials and Methods

To carry out this study, we begin with a literature review of international papers on the topic. Representative rock samples were selected and prepared from relevant geological formations, and the samples were subjected to varying levels of uniaxial stress using an electromechanical pressing device. The corresponding changes in acoustic velocity with pressure throughout the loading and unloading cycles for each sample were measured. The datasets obtained are analyzed to identify hysteresis patterns and quantify the extent of velocity changes as a function of applied stress levels. Finally, the Preisach inversion implemented in MATLAB was used to compare experimental results with predictions from the physical models to assess agreement and validate the proposed theoretical framework.

3. Data Analysis and Results

3.1. Describing the acoustic Hysteresis of P wave velocity

In physics, a model represents a simplified version of a real system, designed to facilitate study. It includes the aspects relevant to describing the system, while irrelevant features are neglected [14]. In our model, we assume that pore opening or closure is the main factor determining the pressure dependence of propagation velocity. We considered a rock with a pore volume (V), if this rock is placed under stress (σ), the pore volume begins to close from the largest to the smallest. For this reason, we assume that the change in pore volume (dV) is directly proportional to the stress ($d\sigma$) applied and the pore volume (V) as well. The following differential equation can represent this assumption.

$$dV = -\lambda_V V d\sigma, \quad (1)$$

Solving equation (1) from V_0, pore volume at stress-free ($\sigma = 0$) to V, we found the following solution.

$$V = V_0 e^{-\lambda_V \sigma}, \quad (2)$$

Where λ_V is a new material-quality-dependent petrophysical parameter [12]. The negative signal means that pore volume decreases ($\lambda_V > 0$) with increasing stress.

As we have already mentioned, if stress increases, pore volume decreases, and this will affect the propagation of acoustic waves, i.e., the velocity will increase because attenuation and energy scattering will be lower. Therefore, we assume a linear relationship between the infinitesimal change of the P wave propagation velocity $d\alpha$, due to stress increase dV.

$$d\alpha = -k_p dV, \quad (3)$$

where k_p represent a positive proportionality factor, a new material characteristic. Combining equations (2) and (3) and solve the differential equation (2) we can have.

$$d\alpha = \lambda_V V_0 k_p e^{-\lambda_V \sigma} d\sigma \rightarrow \alpha = K - k_p V_0 e^{-\lambda_V \sigma} \quad (4)$$

Where K is an integration constant. With the assumption of a stress-free state ($\sigma = 0$), equation (4) can be written as follows.

$$\alpha_0 = K - k_p V_0 \rightarrow K = \alpha_0 + k_p V_0 \quad (5)$$

With equation (5) and introducing the notation $\Delta\alpha_0 = k_p V_0$, equation (4) can be written in the following form.

$$\alpha = \alpha_0 + \Delta\alpha_0 - \Delta\alpha_0 e^{-\lambda_V \sigma} \rightarrow \alpha = \alpha_0 + \Delta\alpha_0 (1 - e^{-\lambda_V \sigma}) \quad (6)$$

Equation (6) provides a theoretical connection between the propagation velocity and rock pressure in the case of pressurization (loading).

Now, let us characterize the depressurization (unloading) phase. And, to do this, we introduce the closed pore volume as $v = V_0 - V$. We assume that all the pores closed at the maximum pressure value (σ_m) start to open again if we decrease the pressure from σ_m. For this reason, we assume that a change in the closed pore volume (dv) is directly proportional to the decreased stress ($d\sigma$) and the instantaneously closed pore volume (v)

$$dv = \lambda'_V v d\sigma, \quad (7)$$

Solving equation (7) from v_m, pore volume at maximum stress ($\sigma = \sigma_m$) to v, we found the following solution.

$$v = v_m e^{\lambda'_V (\sigma - \sigma_m)} \rightarrow v = v_m e^{-\lambda'_V (\sigma_m - \sigma)}, \quad (8)$$

Where λ'_V is another material-quality-dependent petrophysical parameter. During the depressurization phase, there is an amount of closed pore that does not reopen completely [12], i.e., there is a certain amount of irreversibility, which is characterized by two different parameters λ_V and λ'_V in our model. On the other hand, during depressurization, the closed pore volume decreases, while the open pore volume increases. This assumption can be written mathematically as follows.

$$dv = -dV \quad (9)$$

Combining equations (3) and (9) and solving the differential equation (8), we will have.

$$d\alpha = k_p V_m \lambda'_V e^{-\lambda'_V (\sigma_m - \sigma)} d\sigma \rightarrow \alpha = K + k_p V_m e^{-\lambda'_V (\sigma_m - \sigma)} \quad (10)$$

With the assumption of the maximum stress state ($\sigma = \sigma_m$), equation (10) can be written as follows.

$$\alpha_m = K + k_p V_m \rightarrow K = \alpha_m - k_p V_m \quad (11)$$

With equation (11) and introducing the notation $\Delta\alpha_m = k_p V_m$, equation (10) can be written in the following form.

$$\alpha = \alpha_m - k_p V_m + k_p V_m e^{-\lambda_V'(\sigma_m-\sigma)} \rightarrow \alpha = \alpha_m - \Delta\alpha_m(1 - e^{-\lambda_V'(\sigma_m-\sigma)}) \quad (12)$$

Equation (12) shows the theoretical connection between the propagation velocity and rock pressure in case of depressurization (unloading). In the two limiting cases (at a pressure value $\sigma = \sigma_m$ and $\sigma = 0$) and by introducing the notation $\alpha(0) = \alpha_1$ and $\Delta\alpha_1 = -\alpha V_m e^{-\lambda_V'\sigma_m}$, equation (12) can be written in the following form

$$\alpha = \alpha_1 + \Delta\alpha_1(1 - e^{-\lambda_V'\sigma}) \quad (13)$$

3.2. Case Study

The pressure-dependent velocity model was tested using longitudinal wave velocity datasets. The pulse transmission technique was used to measure P-wave velocity [12]. We measured wave velocities on a fine- to medium-grained sandstone sample from an oil-drilling well. The specimen was subjected to uniaxial stress up to 20 MPa using an electromechanical pressing device, and wave velocities were measured at adjoining pressures during loading and unloading cycles. Our measurements showed that the longitudinal velocity is directly proportional to pressure. Moreover, a significant difference in the curves' characteristics was observed, which can be explained by acoustic hysteresis. The measured data were inverted using the well-known Gaussian Least Squares method. The result is shown in *Figure 2*, where the solid line shows the calculated (by LSQ inversion) velocity–pressure function for the velocity model above. At the same time, the asterisks represent the measured data.

The quality of the inversion result was characterized using the relative misfit between the measured and calculated data [17].

$$\mathrm{RMS} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{d_k^{(m)} - d_k^{(c)}}{d_k^{(c)}}\right)^2} \cdot 100\,[\%], \quad (14)$$

where $d_k^{(m)}$ is the measured velocity at the *k*-th pressure and $d_k^{(c)}$ is the *k*-th calculated data, computed using the proper forward modeling equations, and *N* is the total number of data. The reliability of the inversion estimates can be described by the mean spread (*S*)

$$S = \sqrt{\frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j=1}^{M}\left(\mathrm{corr}(m)_{ij} - \delta_{ij}\right)^2}, \quad (15)$$

Where δ is a Kronecker-delta symbol ($\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$), *M* is the number of model parameters, and $\mathrm{corr}(m)$ is the correlation matrix of model parameters, which characterizes the strength of the relationship between the model parameters. The predicted model parameters, RMS values, and mean spread values are given in *Table 1*.

Table 1
Inversion results: model parameters, root mean squared error, and mean spread of loading and unloading phases estimated by the joint inversion method

Sample	**P wave**							
	Loading			Unloading			RMS (%)	S
	α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ_V (1/MPa)	α_1 (km/s)	$\Delta\alpha_1$ (km/s)	λ'_V (1/MPa)		
A	3,56	1,06	0,0212	3,55	0,94	0,0401	0,39	0,57

It was observed that the RMS values are small-scale, which confirms the goodness of the inversion results and the feasibility of the suggested petrophysical model. The moderate mean spread values also confirm that the inversion results are reliable.

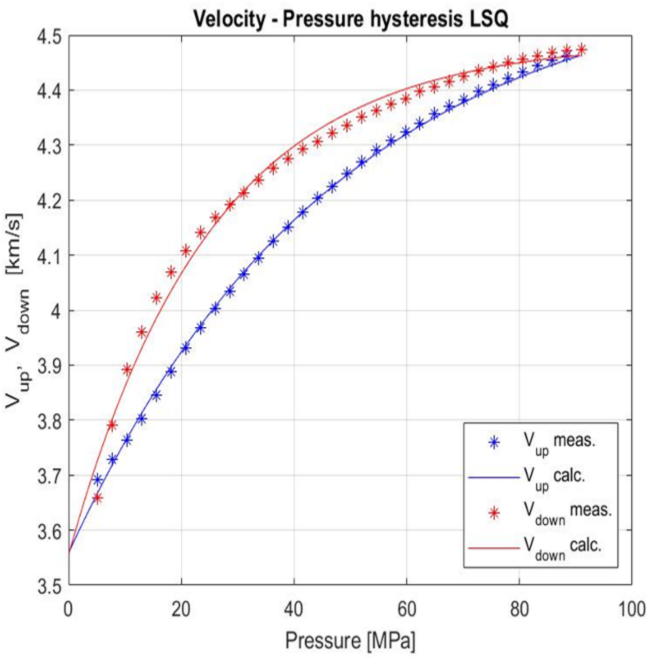

Figure 2: *Velocity vs. pressure curves on the uploading and downloading phase given by LSQ inversion*

The phenomenon of acoustic velocity hysteresis can also be investigated using the Preisach model, a mathematical framework originally developed to describe magnetic hysteresis in ferromagnetic materials. First introduced by Preisach in 1935 [9], the model has since been widely applied to systems exhibiting nonlinear, path-dependent behavior, including ferroelectric materials, mechanical systems with friction, and more recently, selected geophysical applications [10–11].

In the Preisach model, the hysteresis behavior of a material is characterized by the distribution and properties of elementary hysterons, each of which corresponds to a discrete unit of magnetic dipoles within the material. These elementary units (Hysteresis operators), which are denoted by α (transition from upper to lower values) and β (transition from lower to upper values) exhibit hysteresis loops individually *(Figure 3)*, and the overall response of the material is obtained by summation of the contribution from all hysterons [10], suggested a mathematical formula for the Preisach model, which can be written as

$$f(t) = \iint_{\alpha \geq \beta} \mu(\alpha,\beta)\hat{\gamma}_{\alpha\beta}u(t)d\alpha d\beta$$

Where the time signals $u(t)$ and $f(t)$ denote the input and output. The weight function $\mu(\alpha,\beta)$ defines the shape of the hysteresis curves. The operators $\hat{\gamma}_{\alpha\beta}$ can only have the discrete values 0 and 1.

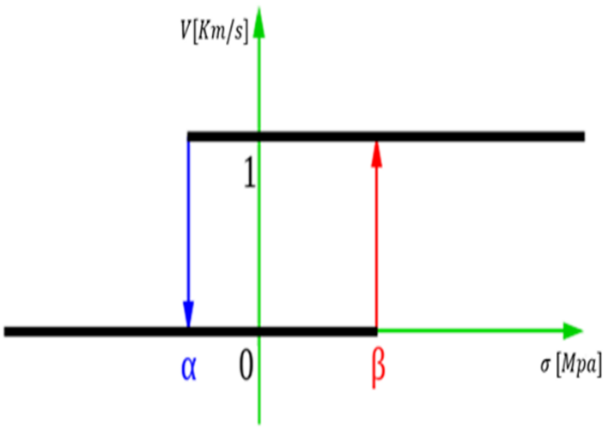

Figure 3: *The simple "hysteron" the building block of Preisach Hysteresis model,* adapted from Janicic et al., 2014

As the Preisach model for characterizing hysteresis phenomena is well established across various fields, its application to stress-dependent acoustic-velocity hysteresis in rocks is a relatively new research avenue. And our preliminary study using the Preisach approach yielded the results shown in *Figure 4*. As can be seen, the fit between the measured and calculated data is not satisfactory, particularly in the lower portion of the downloading curve. The reason is probably that, while uploading, some new cracks are also produced. This effect is not considered in the present stage of our research. In future work, we plan to address this problem to improve the fit between measured and calculated data.

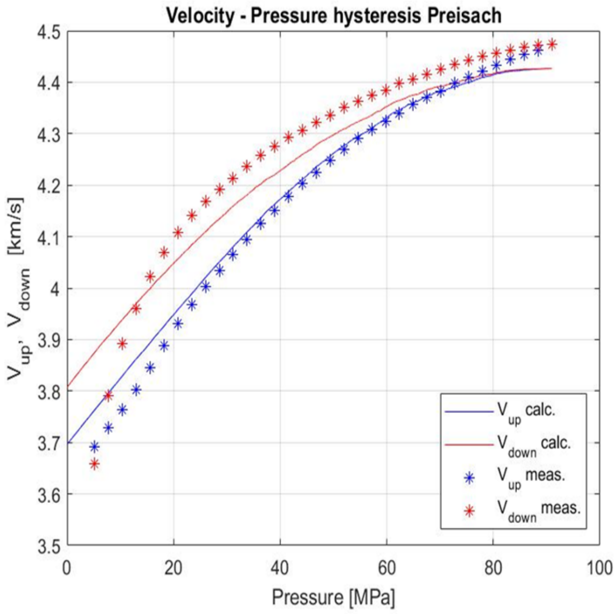

Figure 4: *Velocity-pressure hysteresis using the Preisach Model*

4. Discussion

The phenomenon of stress-dependent acoustic velocity hysteresis in rocks has been a central topic in several studies conducted at the University of Miskolc. In this preliminary work, laboratory measurements were combined with theoretical modeling to understand better how cyclic stress influences acoustic wave propagation in porous and microcracked rocks.

The new petrophysical model developed in this study provides a physically meaningful explanation for the velocity-pressure relationship during loading and unloading. By linking acoustic velocity directly to the stress-induced evolution of pore volume, the model captures the asymmetric behavior of the two branches of the hysteresis loop. Parameter estimation using LSQ inversion resulted in velocity-pressure curves that closely matched the measurements, confirming that irreversible pore closure and incomplete reopening are the dominant mechanisms controlling acoustic hysteresis. The overall RMS misfit between measured and modeled velocities is very small, at about 0.39%, indicating that the physical model performs consistently well throughout the complete stress cycle.

The Preisach model, implemented in MATLAB, provided a different perspective by emphasizing memory and path dependence. While it successfully reproduced the general shape of the hysteresis loop, noticeable discrepancies appeared in the lower part of the unloading branch. This is reflected in a significantly larger RMS misfit of about 6.18% compared to the LSQ result, as well as a relatively high mean spread (0.66) in the parameter correlation matrix. These indicators suggest that the classical Preisach formulation, with fixed hysteron thresholds, cannot fully represent the irreversible microstructural changes that occur during high-stress loading, particularly the formation of new microcracks.

These results suggest that a combination of reversible and irreversible mechanisms governs the acoustic behavior of rocks under cyclic loading. The LSQ-based petrophysical model excels in capturing the smooth stress dependence of velocity. In contrast, the Preisach model naturally incorporates memory effects but requires further refinement to account for evolving microstructural states. Future work will focus on improving this formulation and extending the analysis to additional rock types.

5. Conclusions

This study provided a detailed investigation of stress-dependent acoustic-velocity hysteresis in rocks by integrating laboratory measurements with both physical and mathematical modeling. A new petrophysical model was introduced that describes acoustic velocity in terms of stress-induced pore closure and reopening. The model successfully reproduced the observed loading and unloading branches of the velocity-pressure curves, demonstrating its ability to capture the essential physical mechanisms responsible for hysteresis.

The Preisach model was also applied to the same dataset to assess its ability to describe path-dependent behavior. While the overall hysteresis loop was captured, discrepancies appeared in the lower portion of the unloading branch, suggesting that additional irreversible processes, such as the generation of new microcracks during loading, are not fully accounted for in the classical Preisach formulation.

These preliminary findings highlight the complementary strengths of the two approaches: the physically based petrophysical model provides accurate velocity predictions, whereas the Preisach model captures memory effects and nonlinear behavior. A quantitative evaluation based on RMS misfit values will further clarify the comparative performance of these models and guide future improvements. Future work will focus on extending the Preisach-based interpretation to include evolving microstructural parameters, redefining hysteron thresholds under increasing stress, and conducting additional laboratory measurements on other rock types to validate the approach's generality.

6. Acknowledgments

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References

[1] Mitra, S. (2019). Fundamentals of Rock Mechanics. John Wiley & Sons.

[2] Zhao, J., Zhang, J., Wang, M., & Wang, H. (2020). Experimental study on the dynamic mechanical properties of sandstone under different strain rates. *Rock Mechanics and Rock Engineering*, 53 (5), 2325–2337.

[3] Mavko, G., Mukerji, T., & Dvorkin, J. (2009). *The Rock Physics Handbook: Tools for Seismic Analysis of Porous Media*. 2nd ed. Cambridge University Press. **https://doi.org/10.1017/CBO9780511626753**

[4] Johnson, D. L., Strykowski, G., Niesen, K., & Davis, D. (2014). *Rock Mechanics: Meeting Society's Challenges and Demands*. CRC Press.

[5] Walsh, J. B., & Brace, W. F. (1964). Friction, stick-slip, and the mechanics of faulting. *Journal of Geophysical Research*, 69 (20), 4513–4527.

[6] Birch, F. (1960). The velocity of compressional waves in rocks to 10 kilobars, Part 2. *Journal of Geophysical Research*, 65 (4), 1083–1102. **https://doi.org/10.1029/JZ065i004p01083**

[7] Hashin, Z., & Shtrikman, S. (1963). A variational approach to the theory of the elastic behavior of multiphase materials. *Journal of the Mechanics and Physics of Solids*, 11 (2), 127–140. **https://doi.org/10.1016/0022-5096(63)90060-7**

[8] Ji, W., Liu, H., & Zhang, J. (2007). The application of stress-dependent acoustic velocity in rocks to seismic exploration. *Petroleum Exploration and Development*, 34 (3), 301–305.

[9] Preisach, F. (1935). Über die magnetische Nachwirkung. *Zeitschrift für Physik*, 94 (5–6), 277–302. **https://doi.org/10.1007/BF01349418**

[10] Mayergoyz, I. D. (1991). *The Classical Preisach Model of Hysteresis*. Springer, New York. **https://doi.org/10.1007/978-1-4612-3028-1_1**

[11] Janicic, V., Ilic, V., Pjevalica, N., & Nikolic, M. (2014). An approach to modeling the hysteresis in ferromagnetic materials by adaptation of the Preisach model. *International Journal of Rock Mechanics and Mining Sciences*, 95, 45–54.

[12] Molnár, J., & Dobróka, M. (2012). New petrophysical model describing the pressure dependence of seismic velocity. *Acta Geodaetica et Geophysica*, 47 (4), 425–438. **https://doi.org/10.1556/AGeod.47.2012.2.4**

[13] Molnár, J., Dobróka, M., & Szabó, P. N. (2013). New results in modeling the phenomenon of acoustic hysteresis. In: *75th EAGE Conference & Exhibition*, London, UK.

[14] Molnár, J., Kiss, A., & Dobróka, M. (2014). Petrophysical models to describe the pressure dependence of acoustic wave propagation characteristics. *Acta Geodaetica et Geophysica*, 49, 293–304.

[15] Molnár, J., Dobróka, M., & Kiss, A. (2014). Proving the applicability of the petrophysical model describing acoustic hysteresis of P and S wave velocities. In: *Near Surface Geoscience 2014*.

[16] Wetzel, M., Kempka, T., & Kühn, M. (2022). Hysteresis in permeability evolution simulated for a sandstone by mineral precipitation and dissolution. *Advances in Geosciences*, 56, 45–56. **https://doi.org/10.5194/adgeo-58-1-2022**

[17] Menke, W. (1984). *Geophysical data analysis – Discrete inverse theory*. Academic Press, London. **https://doi.org/10.1016/B978-0-12-490920-5.50014-4**