ELECTROMECHANICAL MODEL OF A HEDGE TRIMMER MECHANISM

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Abstract
Hedge trimmers are used for various applications, from lighter vegetations to tree-like plants. A key component of the hedge trimmer is the eccentric disk within its drive train, that can be looked at as a cam-follower mechanism. The eccentric is responsible for transforming the rotating motion into an alternating one. Another important thing about the eccentric is the acceleration it generates during the operation for the blades, this acceleration must be considered when considering the cutting force. With different eccentric profiles different acceleration can be achieved. This paper aims to create an electromechanical model for the drive train of a hedge trimmer and to examine the behavior of the system based on its variables.

Keywords: *hedge trimmer, cam-follower mechanism, Lagrange-equation, differential equation, Scilab*

1. Introduction

Hedge trimmers are used mostly in households, in some cases as professional equipment as well, i.e. gardening companies. A key aspect for a hedge trimmer – based on the fact that it is a handheld power tool – is, that it has to be lightweighted. In technical systems engineers widely use linkage systems for translating a rotating motion into an alternating one. The reason for this is, that the linkage system is a known and applied solution, but its size is a limitation when it comes to handheld applications. For this reason, in handheld power tools (jigsaws, hedge trimmers, shrub shears) a cam-follower mechanism is used.

The biggest advantage of this solution is its size and shape, but it must be mentioned, that these systems are sensitive against lubrication and the lack of it. Another advantage of the cam – follower mechanism that by alternating the curvature of the cam, different displacement, velocity, and acceleration functions can be achieved. These can result in more beneficial cutting force characteristics, which can improve user satisfaction – which is one of the most important things to be considered when the topic is consumer goods. As every technical solution, the altered cam – follower profile has its limitations as well. The altered profile can cause higher noise and vibration results, that have to be avoided. Therefore, the task is given for the engineer to create a profile, that provides an appropriate cutting force profile, however, will not cause unwanted noise and vibration results.

There are different techniques to determine the cam profiles, the often applied is the analytical solution (Iriarte et al., 2024; Hsieh, 2010). To improve the efficiency of the designed profile, optimization techniques are used (Qiu et al., 2015; Borboni et al., 2020; Angeles et al. 2012).

This paper aims to create a mechanical model that can be used when examining a system of a hedge trimmer.

2. The electromechanical model for the hedge trimmer

The first step is to create an electromechanical model for the examined mechanism (Rothbart, 2004; Norton, 2009). *Figure 1* shows the schematic model of a hedge trimmer with its main parameters.

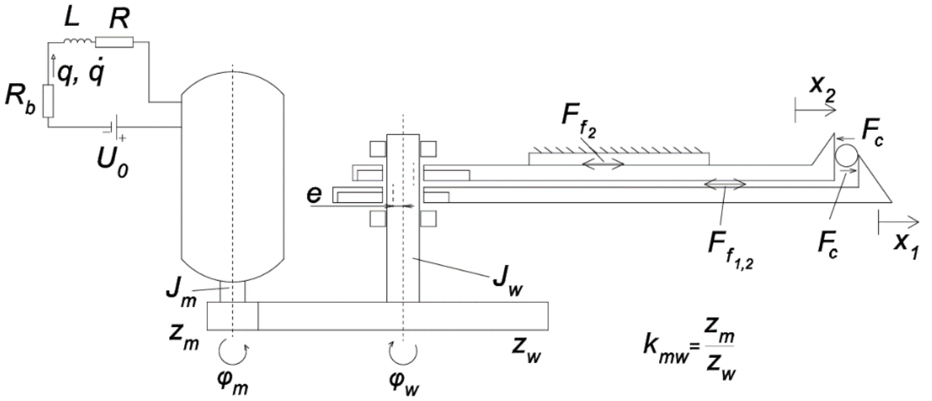

Figure 1. *Electromechanical model of the drive train of a hedge trimmer*

In *Figure 1* q is the electric charge flowing in the electric circuit of the tool *[C]*, $\dot{q}$ is the electric current flowing in the electric circuit of the tool *[A]*, R_b is the inner resistance of the battery *[Ω]*, L is the inductance of the motor *[H]*, R is the resistance of the coil of the motor *[Ω]*, U_0 is the supply voltage of the battery *[V]*, J_m is the moment of inertia of the motor *[kgm²]*, z_m is the number of teeth of the smaller gear, on the rotor [–], φ_m is the angular displacement of the rotor [–], z_w is the number of teeth of the bigger gear, on the driven axis [–], φ_w is the angular displacement of the driven axis [–], J_w is the moment of inertia of the driven axis, including the inertia of the driven gear *[kgm²]*, F_{f2} is the friction force between the fixed support bar and the upper knife *[N]*, $F_{f1,2}$ is the friction force between the upper and lower blade [*N*], x_1 is the longitudinal displacement of the upper knife [*m*], x_2 is the longitudinal displacement of the lower knife *[m]*, F_c is the cutting force *[N]*, $k_{mw} = z_m / z_w$ is the drive ratio between the driver and driven axis, considering that the system has a speed reduction, it will be lower than *1* [–]. The system now can be described with a Lagrangian function, where the variables will be the angular displacements and the longitudinal displacements. The general form of the Lagrangian function may be written as

$$L = W_m^* - W_e + T^* - U_{pot} \tag{1}$$

where the variables are L is the Lagrangian function, W_m^* is the complementary magnetic energy, W_e is the electrical energy, T^* is the complementary kinetic energy, U_{pot} is the potential energy. This equation may be simplified, since a hedge trimmer has no capacitor in it, therefore it will not include any electrical energy, furthermore the parts in the model are rigid bodies, thus the potential energy will be zero. The Lagrangian function can be unfolded with the considerations mentioned above as

$$\mathrm{L} = \frac{1}{2}L\dot{q}^2 + \frac{1}{2}J_m\dot{\varphi}_m^2 + \frac{1}{2}J_w\dot{\varphi}_w^2 + \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 \tag{2}$$

where m_1 is the mass of the upper knife *[kg]*, and m_2 is the mass of the lower knife [*kg*]. The other quantity that has to be considered is the virtual work associated to the non-conservative elements, for this system this work takes the sum of

$$\begin{aligned}\overline{\delta W_{nc}} = &\left(U_0 - \dot{q}R_b - \dot{q}R - U_i\right)\delta q + \left(M_m - b\dot{\varphi}_m\right)\delta\varphi_m - sgn\left(\dot{x}_2\right)F_{f2}\delta x_2 - \\ &-sgn\left(\dot{x}_1 - \dot{x}_2\right)F_{f12}\left(\delta x_1 - \delta x_2\right) - sgn\left(\dot{x}_1\right)F_c\delta x_1 - sgn\left(\dot{x}_2\right)F_c\delta x_2\end{aligned} \tag{3}$$

where U_i is the voltage induced in the motor [*V*], b is the air drag of the motor *[Nms]*, the signum function [–] is

$$sgn\left(x_i\right) = \begin{cases} 1 \; if \;\; x_i > 0 \\ 0 \; if \;\; x_i = 0 \\ -1 \; if \;\; x_i < 0 \end{cases}$$

It is advantageous to express x_1 and x_2 longitudinal displacements, and their derivates to as the dependents of the angular displacements of the driven axis (φ_w), this is shown in *Figure 2*.

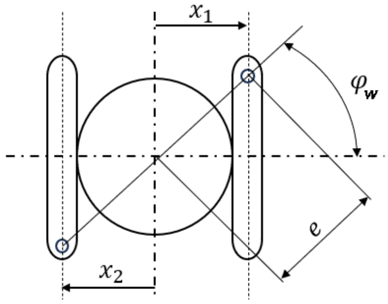

Figure 2. *Expressing variables using* φ_w

The x_1 and x_2 longitudinal displacements and the angular displacement are connected through the e eccentricity *[m]* and trigonometric functions according to

$$x_1 = e \cdot \cos\left(\varphi_w\right), \tag{4}$$

$$x_2 = e \cdot \cos\left(\varphi_w + \pi\right), \tag{5}$$

$$\dot{x}_1 = -e \cdot \dot{\varphi}_w \sin\left(\varphi_w\right), \tag{6}$$

$$\dot{x}_2 = -e \cdot \dot{\varphi}_w \sin\left(\varphi_w + \pi\right), \tag{7}$$

$$\delta x_1 = -e \cdot \sin\left(\varphi_w\right)\delta\varphi_w, \tag{8}$$

$$\delta x_2 = -e \cdot \sin\left(\varphi_w + \pi\right)\delta\varphi_w. \tag{9}$$

To decrease the number of unknown quantities it is beneficial to write *Equations (4)–(9)* not as dependents of the angular displacement of the driven axis, but as dependents of the angular displacement of the rotor. First a connection is needed between the angular displacement of the rotor and angular displacement of the driven axle, this can be written as

$$\varphi_w = \frac{z_m}{z_w}\varphi_m = k_{mw}\varphi_m. \tag{10}$$

If both sides of *Equation (10)* are derivated, the result is the

$$\dot{\varphi}_w = k_{mw}\dot{\varphi}_m \tag{11}$$

correlation between the angular velocity of the driven axle and the rotor. Using *Equation (10)* and *(11)*, *Equation (9)* can be rewritten as

$$\delta x_2 = -e \cdot k_{mw}\sin\left(k_{mw}\varphi_m + \pi\right)\delta\varphi_m. \tag{12}$$

Using *Equations (5)–(8)*, *(10)* and *(12)* the main *Equation (2)* and *(3)* can be readjusted as

$$\mathrm{L} = \frac{1}{2}L\dot{q}^2 + \frac{1}{2}\left(J_m + k_{mw}^2\left[J_w + \left(m_1 + m_2\right)e^2\sin^2\left(k_{mw}\varphi_m\right)\right]\right)\dot{\varphi}_m^2, \tag{13}$$

$$\begin{aligned}\overline{\delta W_{nc}} = {} & U\left(t\right)\delta q - \left(R_b + R\right)\dot{q}\delta q - k_e\dot{\varphi}_m\delta q + k_m\dot{q}\delta\varphi_m + F_{f2}sgn\left(\dot{x}_2\right)ek_{mw}\sin\left(\varphi_m k_{mw} + \pi\right)\delta\varphi_m + \\ & + 2F_{f12}sgn\left(\dot{x}_1 - \dot{x}_2\right)\left[ek_{mw}\sin\left(\varphi_m k_{mw}\right)\right]\delta\varphi_m + 2F_c sgn\left(\dot{x}_1\right)ek_{mw}\sin\left(\varphi_m k_{mw}\right)\delta\varphi_m - b\dot{\varphi}_m\delta\varphi_m,\end{aligned} \tag{14}$$

where the new variables k_e is the electrical machine coefficient [*Vs/rad*], k_m is the torque machine coefficient [*Nm/A*], $U_i = k_e\dot{\varphi}_m$ is the induced voltage [*V*], $M_m = k_m\dot{q}$ is the torque of the motor [*Nm*]. Now the Lagrange – equations can be created from *Equation (13)* and *(14)*, with variables q and φ_m. The Lagrange – equations of the second kind are showing how the derivates of the Lagrangian function and the virtual work associated to the non-conservative elements are related, their general form may be written as

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{q}}\right) - \frac{\partial \mathrm{L}}{\partial q} = Q_i, \tag{15}$$

where q_i is the generalized displacement coordinate (it can be spatial displacement, angular displacement, electrical charge, etc.) and Q_i is the nonconservative load belonging to the generalized coordinate (e.g.: F_i, M_i, U_0). Using *Equation (15)* for q it can be written, that

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{q}}\right) - \frac{\partial \mathrm{L}}{\partial q} = U\left(t\right) - \left(R_b + R\right)\dot{q} - k_e\dot{\varphi}_m. \tag{16}$$

After executing the partial derivation on the left side of the equation, and doing some reorganization, the equation becomes

$$L\ddot{q} + \left(R_b + R\right)\dot{q} + k_e\dot{\varphi}_m = U\left(t\right). \tag{17}$$

Similarly, to q the same can be done to φ_m, so the Lagrange – equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_m}\right)-\frac{\partial L}{\partial \varphi_m}=k_m\dot{q}+F_{f2}sgn\left(\dot{x}_2\right)ek_{mw}\sin\left(\varphi_m k_{mw}+\pi\right)+ \\ +F_{f12}sgn\left(\dot{x}_1-\dot{x}_2\right)2ek_{mw}\sin\left(\varphi_m k_{mw}\right)+2F_c sgn\left(\dot{x}_1\right)ek_{mw}\sin\left(\varphi_m k_{mw}\right)-b\dot{\varphi}_m \tag{18}$$

In order to have less components in the equations it is worth to introduce new quantities, their purpose is to combine relevant components one of them for the friction $-M_f(\varphi_m)$ and the other for the cutting force $-M_c(\varphi_m)$ according to the

$$-M_f\left(\varphi_m\right)=F_{f2}sgn\left(\dot{x}_2\right)ek_{mw}\sin\left(\varphi_m k_{mw}+\pi\right)+F_{f12}sgn\left(\dot{x}_1-\dot{x}_2\right)2ek_{mw}\sin\left(\varphi_m k_{mw}\right), \tag{19}$$

$$-M_c\left(\varphi_m\right)=2F_c sign\left(\dot{x}_1\right)ek_{mw}\sin\left(\varphi_m k_{mw}\right) \tag{20}$$

formulas. After executing the derivation in Eq. (18) and using Equations (19)–(20), the equation takes the

$$\left\{J_m+k_{mw}^2\left[J_w+\left(m_1+m_2\right)e^2\sin^2\left(\varphi_m k_{mw}\right)\right]\right\}\ddot{\varphi}_m+ \\ +\frac{1}{2}\dot{\varphi}_m^2 k_{mw}^3\left(m_1+m_2\right)e^2 2\sin\left(\varphi_m k_{mw}\right)\cos\left(\varphi_m k_{mw}\right)+b\dot{\varphi}_m=k_m\dot{q}-M_f\left(\varphi_m\right)-M_c\left(\varphi_m\right) \tag{21}$$

nonlinear form. Introducing a new quantity for the inertias as well, marked as J_Σ according to the following

$$J_\Sigma\left(\varphi_m\right)=J_m+k_{mw}^2\left[J_w+\left(m_1+m_2\right)e^2\sin^2\left(\varphi_m k_{mw}\right)\right], \tag{22}$$

$$\frac{\partial J_\Sigma\left(\varphi_m\right)}{\partial \varphi_m}=k_{mw}^3\left(m_1+m_2\right)e^2 2\sin\left(\varphi_m k_{mw}\right)\cos\left(\varphi_m k_{mw}\right). \tag{23}$$

Rewrite *Equation (21)* into the

$$J_\Sigma\left(\varphi_m\right)\ddot{\varphi}_m+\frac{1}{2}\dot{\varphi}_m^2\frac{\partial J_\Sigma\left(\varphi_m\right)}{\partial \varphi_m}+b\dot{\varphi}_m=k_m\dot{q}-M_f\left(\varphi_m\right)-M_c\left(\varphi_m\right) \tag{24}$$

simplified form. To solve the nonlinear *Equation (2.17)* and *(2.24)* it is worth to introduce new variables for q, $\dot{q}$, φ_m and $\dot{\varphi}_m$ as of

$$y_1=\dot{q}, \tag{25}$$

$$y_2=\varphi_m, \tag{26}$$

$$y_3=\dot{\varphi}_m, \tag{27}$$

$$y_4=q, \tag{28}$$

$$\dot{y}_4=y_1, \tag{29}$$

$$\dot{y}_2=y_3. \tag{30}$$

It is obvious, that $\dot{y}_1 = \ddot{q}$ and $y_3 = \ddot{\varphi}_m$, this means that after dividing (17) with *L*, and using *Equations (25)–(28)* we receive the

$$\dot{y}_1 = -\frac{(R_b + R)}{L} y_1 - \frac{k_e}{L} y_3 + \frac{U(t)}{L} \tag{31}$$

differential equation. Applying similarly to *Equation (24)*, this resulting the

$$\dot{y}_3 = \left[-\frac{1}{2} J_\Sigma(y_2) y_3^2 + k_m y_1 - M_s(y_2) - M_v(y_2) - b y_3 \right] \frac{1}{J_\Sigma(y_2)} \tag{32}$$

differential equation. *Equations (2.29)–(2.32)* can be solved with numerical software, such as MATLAB or Scilab.

3. Solving the differential equations

The differential equations were solved in Scilab software by Adaptive Runge-Kutta of order 4 (RK4) method. One temporary limitation is that the model considers unloaded operation. This is because of two reasons: the hedge trimmer during its operation in a soft vegetation is almost running in a no-load environment, the other reason is that introducing various cutting forces would make the calculations way more complicated and the aim of this paper is to examine the overall electromechanical model of the hedge trimmer. The parameters used for the calculations were: $J_m = 5.5 \cdot 10^{-6}$ kgm^2, $J_w = 1.45 \cdot 10^{-4}$ kgm^2, $k_{mw} = 4/57$, $e = 7.5$ *mm*, $m_1 = 0.299$ *kg*, $m_2 = 0.3$ *kg*, $F_f = 8$ *N*, $F_c = 3$ *N*, $L = 10^{-3}$ *H*, $R = 0.2$ *Ω*, $R_b = 0.135$ *Ω*, $k_e = 0.0145$ *Vs/rad*, $k_m = 0.0145$ *Nm/A*, $U_0 = 36$ *V* and $b = 1.24 \cdot 10^{-4}$ *Nms*. 0 shows the knives movement of the examined hedge trimmer.

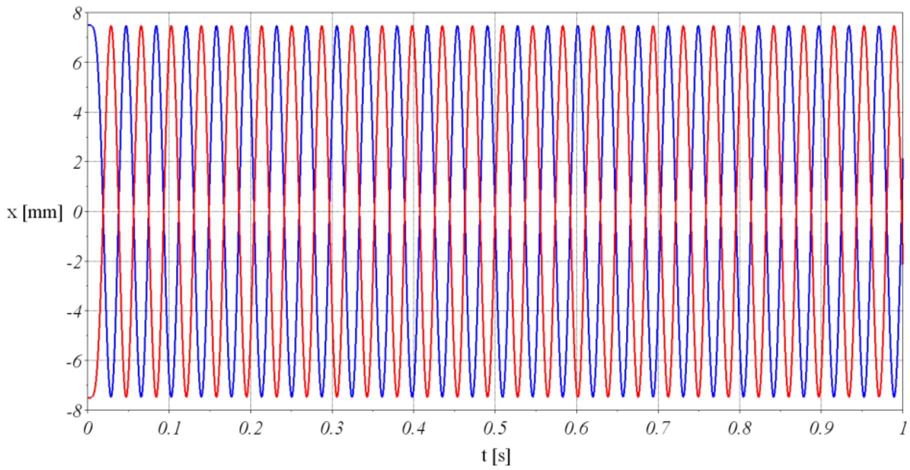

Figure 3. *Movement of the knives in the hedge trimmer*

The two knives move as the mirror images of each other, which reflects the reality. The rotor speed of the driving mechanism can be seen in *Figure 4*.

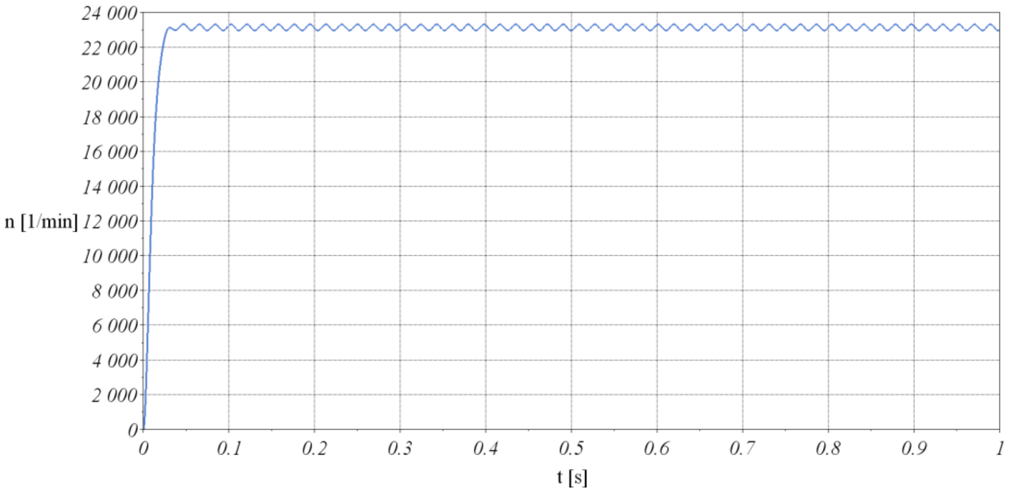

Figure 4. *Change of rotor speed in the hedge trimmer*

It can be seen that the Scilab computation results the actual rotor speed of the examined hedge trimmer. The speed of the driven axle, including the eccentric is shown in *Figure 5*.

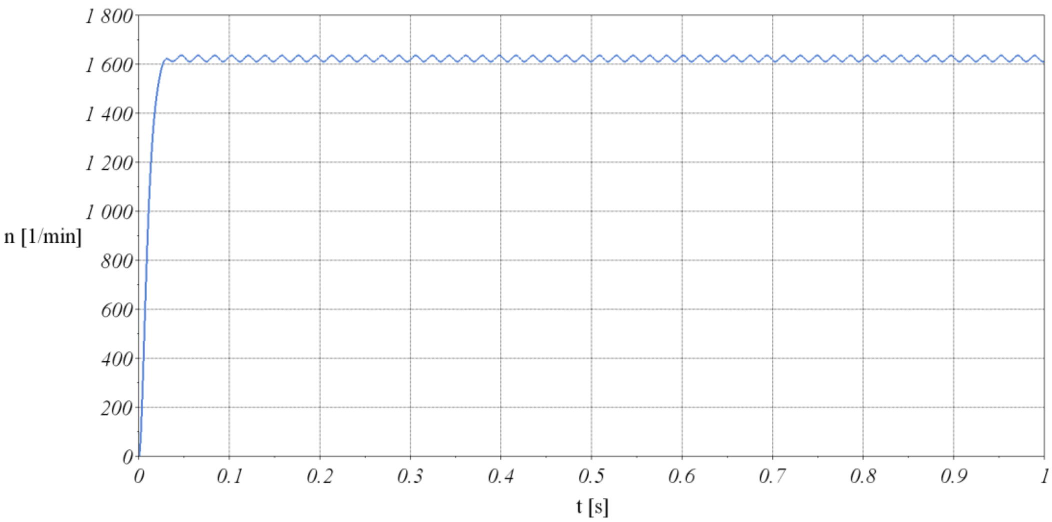

Figure 5. *Change of driven excenter speed in the hedge trimmer*

The actual speed of the driven axle is *1600 rpm*, therefore the model computes this value as well. Finally, only the current consumption remains to be examined, this can be seen in *Figure 6.*

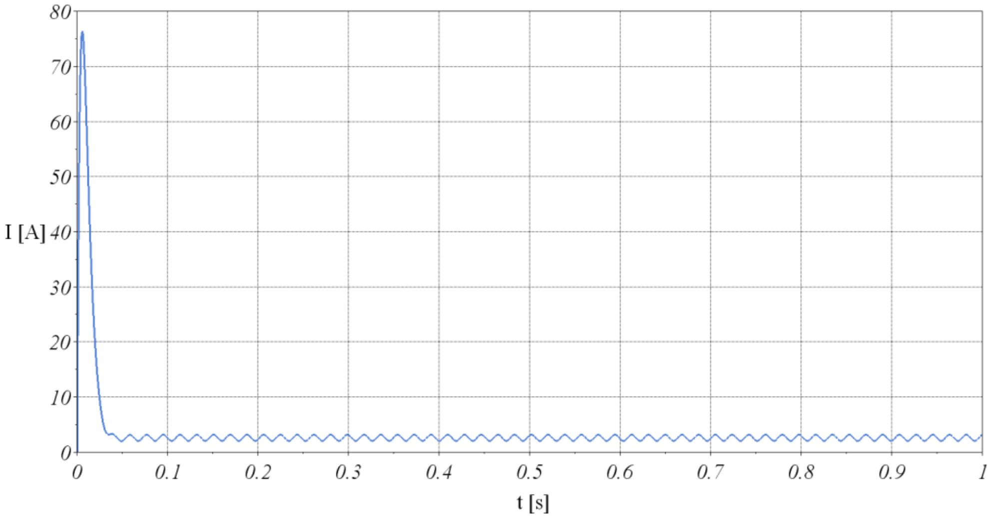

Figure 6. *Change of driven excenter speed in the hedge trimmer*

The steady state current consumption of the machine is approximately *2 A*, this can be seen on the graph as well. The question mark for the future studies is that whether the peak at the beginning of the graph is reliable or not.

4. Summary

In this paper after a short introduction a longer elaboration was done about the electromechanical model of a hedge trimmer based on its drive train. It was shown how the Lagrangian – function can be derived, and how the Lagrange – equation can be written using it. After some work two nonlinear differential equations were created. At the end these two equations were solved using Scilab software. The major aim of this paper was to create a model, and this aim was fulfilled – even though there is still place for minor adjustments.

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