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INTERPOLATED SPLINE METHOD FOR A THERMAL DISTRIBUTION OF A PIPE WITH A TURBULENT HEAT FLOW

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Abstract

This work presents an interpolated spline method to mathematically represent experimental data of a thermal distribution on a tube with heat flux. Linear regression was compared with the double linear interpolation process with an optimization algorithm and cubic spline curve method with the proposed problem. The results show that the interpolated experimental data can highly improve the efficiency of the cubic spline curves and lead to a smooth empirical equation for the experiments. The optimization algorithm chooses the interpolated points in a way that provides more minor errors.

Keywords: *Interpolated spline; Optimization; Curve fitting; Cubic spline; Linear regression; Heat exchanger, Turbulent flow.*

1. Introduction

Optimization algorithms (Ghafil and Jármai, 2020a) are powerful techniques to find the best possible solution among many other feasible or unfeasible solutions. Artificial bee colony (Ghafil and Jármai, 2018) and particle swarm optimization (Alsamia et al., 2021) are famous examples of metaheuristics (Almufti, 2019) which can be inspired by natural or human-made phneomina (Ghafil et al, 2021). One of the critical applications for optimization is curve fitting (Chen et al., 2005) which is a traditional engineering concept. Many methods were employed to find the best equation representing Cartesian space points like mean square error (Sarbishei and Radecka, 2011), linear regression (Yan and Su, 2003), and splines (De Boor, 1978). Cubic spline curves are piecewise polynomials consisting of adjacent segments. The most crucial engineering application for splines is path planning (Mahmood et al., 2019) in robotics (Ghafil and Jármai, 2019). The heat exchanger (Bouchenna et al., 2021) is a device used to heat transfer between two or more fluids for various applications, including power plants, nuclear reactors, refrigeration and air condition system, automotive industries, heat recovery system, chemical processing and food industries. In order to calculate the heat transfer. Acknowledge the prevailing temperature field is the first requirement. Most calculating the temperature distribution methods within a tube wall are given in the literature by constant wall temperature (Askar et al., 2020) or constant heat flux (Karamallah et al., 2016). The heat transfer equipment industry is a vast and expensive part of the world economy due to the cost and size of the heat transfer equipment (Rohsenow et al., 1998). A large part of this cost to the industry is due to rising fuel prices. As a result, methods to enhance the heat transfer process are of great interest. Even small reductions in fuel costs could make the difference between a successful and failing in the power industry (Siddique et al., 2010).

In this paper, interpolated data using a metaheuristic algorithm is used to enhance the efficiency of the cubic spline curve to fit experimental data of a heat exchanger. First of all, the initial control points on the spline are chosen the same as the experimental data; then in-between point is created and chosen by the optimization algorithm. Axiomatically, the position of the control points is affecting the final shape of the spline, so the optimal position of the in-between points is crucial to develop an optimal spline that well-fit data. The interpolated spline method is compared with the pure spline method and linear regression method.

2. Problem definition

The physical problem is chosen to be a pipe of (1.58 cm) outer diameter and (1.4 cm) inner diameter, with (150 cm) length as shown in ***Figure 1.*** The outer surface of the tube is electrically heated by a coil made from Nichrome material (Chemical Composition: 80% Ni, 20% Cr, and has a melting point: 1400°C) connected to an AC power supply to generate heat flux. It is (16.7 mm) long and (1.25 mm) diameter of wire with (1.36 Ω/m) resistance, and (AWG is 16). An electric insulator of Fiberglass is wrapped around the tube. Drilled ceramic bead elements are inserted around the wire heater to insulate the electrical heater, and then the wire heater is wrapped around the pipe. An Aluminum foil and sectional pipe insulation of glass wool type with (1.9 cm) internal diameter and (6.35 cm) outer diameter thermal insulation operating in temperature up to (230 °C) nominal density is (64 kg/m^3) used to insulate the testing tube. The surface temperature distributions along the tube length for different flow rate values are illustrated in ***Table 1.*** and ***Figure 2.*** The symbol *f* in ***Figure 1.*** and ***Figure 2.*** denotes the flow rate in the pipe.

Table. 1. *Experimental data of the tube subjected to heat flux.*

X (m)	*T* (°C)								
0	45.1	45.6	45.5	45.5	45.4	45.5	45.3	45.1	45.2
0.22	62.1	62.3	60.1	59.5	58.9	56.3	56.2	56.1	55.2
0.44	65.3	63.6	61.4	61.2	61.2	57.8	57.1	56.5	55.7
0.66	64.4	62.9	60.2	60.1	60	57.4	56.5	55.7	55.5
0.88	64.7	63.4	60.7	59.9	59.7	57.9	56.8	55.9	55.8
1.1	65.6	65.1	62.1	61	60.2	58.8	57	56.6	56.4
1.32	66.8	66.9	63.4	62.3	61.5	59.7	57.7	57.3	56.7
1.54	55.3	53.5	52.2	51.3	50.6	50.2	49.9	49.6	49.4
flow rate (L/min)	1	1.5	2	2.5	3	3.5	4	4.5	5

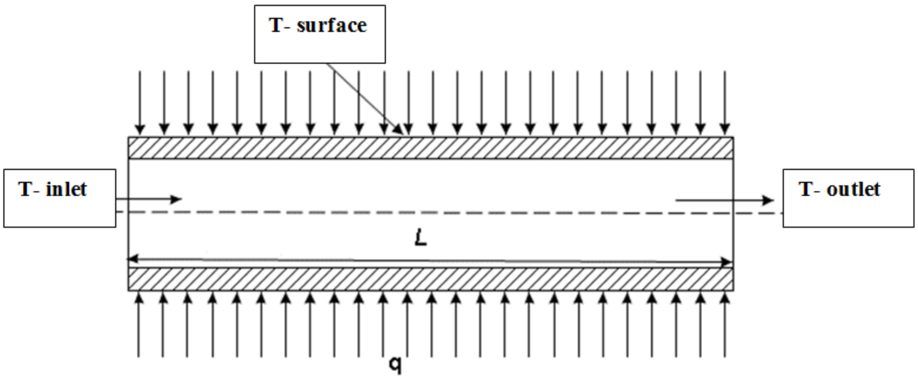

Figure 1. *Tube with different flow rates.*

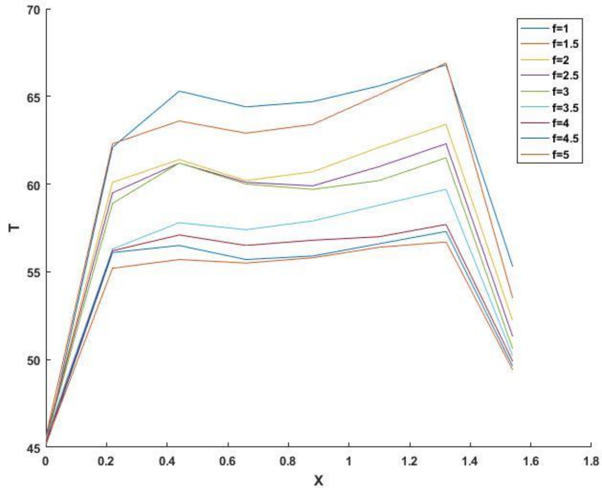

Figure 2. *Thermal distribution on the tube with different flow rates.*

3. Polynomial equation

In this section, a polynomial of the fourth order was proposed to represent the data in section 2, as shown in equation (1). Polynomial regression analysis (Montgomery et al., 2021) was used to find the coefficients of the equation a_o, a_1, a_2, a_3, and a_4.

$$T(x) = a_o x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4, \quad (1)$$

The coefficients in equation (1) are calculated by the polynomial regression method, and they have been found as follows:
$a_o = -82.344$, $a_1 = 264.270$, $a_2 = -291.969$, $a_3 = 130.381$, $a_4 = 45.048$.

Figure 3., illustrates the real data curve vs polynomial curve denoted as fitting data, for the case flow rate equal to 1.

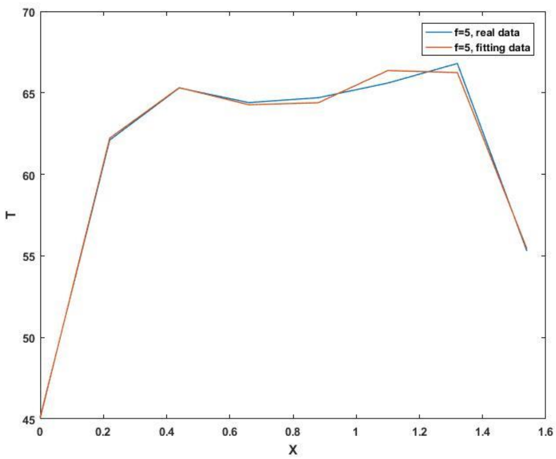

Figure 3. *Real data vs polynomial regression curve.*

4. Cubic Spline curves

Knot spline curve (Likhachev, 2021) is used in this study to represent the experimental data by defining the data set as a control point on the spline. ***Figure 4.*** shows the comparison between the knot spline curve and real data from experiments. In this section, knot spline was used purely by defining its control points in terms of real data.

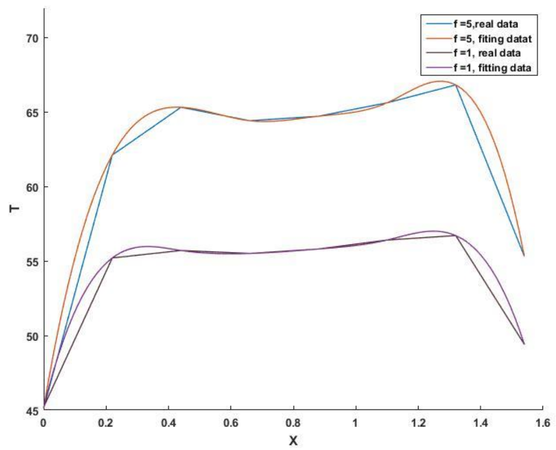

Figure 4. *Knot spline curve compared with real data curve.*

5. Interpolated spline curve

In this section, the Dynamic Differential Annealed Optimization (DDAO) algorithm is presented and used to find the best fitting equation and the results will be compared with other methods.

5.1. Dynamic Differential Annealed Optimization

Dynamic differential annealed optimization is a new metaheuristic that is stimulating the manufacturing process of the dual-phase (DP) steel. ***Figure 5.*** reveals the physical scheme behind the algorithm which is used for continuous domains.

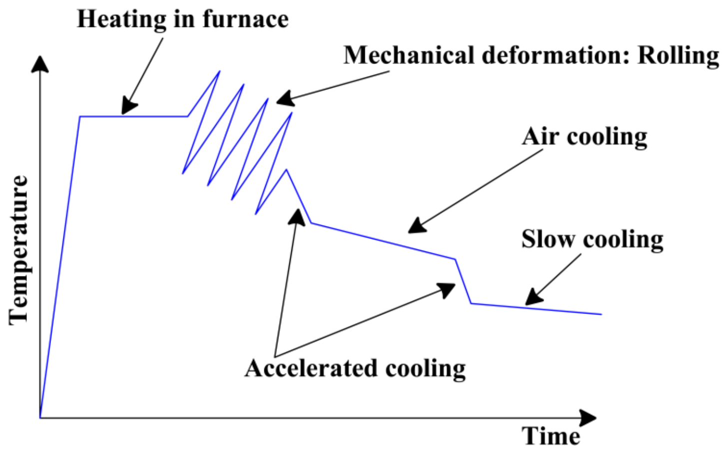

Figure 5. *Description of dual-phase steel production process.*

DDAO simulates the numerous cooling rates during manufacturing of the DP steel by the following equation:

$$S_k = (Sc_i - Sc_j) + S_r, \tag{2}$$

where S_k is a new solution proposed for the iteration number (k), $k = 1\ldots n$, where n is the number of iterations, and Sc_i and Sc_j, are randomly chosen solutions from the population with random (i) and (j) indices. S_r is a randomly generated solution within the search space of the problem out of the population. Equation (2) represents the simulation of the mechanical operations during the production

$$F = \begin{cases} 1 & if\ \ rem(iteration,2) = 1 \\ random\,[0,1] & if\ \ rem(iteration,2) = 0 \end{cases}, \tag{3}$$

where F is called forging parameter and *rem* is the remainder after division by 2, and this parameter has two values; 1 at odd iteration numbers, and a random number [0,1] if the iteration number is even. Since forging is done in parallel with cooling, Equation (2) can be modified as follows:

$$S_k = (Sc_i - Sc_j) + S_r * F. \tag{4}$$

In real production of DP steel, formation of new phases can be occurred at high temperatures more than at low temperatures. To mimic that in mathematics, DDAO has proposed equations (5) and (6):

$$P = e^{\frac{-\Delta E}{T}}, \tag{5}$$

$$\Delta E = \frac{Cost(S^k) - Cost(S_L)}{Cost(S_L)}, \tag{6}$$

where P is the probability of having a new solution, ΔE is the difference between the objective value of the proposed solution from equation (4) and the objective value of the solution S_L, which is a solution of index L in the population, L =1,…, population size. T is the temperature variable, which should be damped from a high value to a lower value. ***Figure 6.*** shows the pseudocode of the algorithm which is also can be freely downloaded at:

https://www.mathworks.com/matlabcentral/fileexchange/75526-dynamic-differential-annealed-optimization-ddao

```
Initialize population Xi (i=1,2,…,n)
Initialise parameter T, cooling rate
Calculate the cost of each solution
Xb= The best solution
While (t<Max iteration)
Initialise sub-population S
Calculate the cost of the sub-population
Sort sub-population
Sr = Best solution in sub-population
Choose two random solutions Xm and Xn from population
Calculate Sk from equation (3)
Sort population X
  foreach solution in population X
    if there is an improvement
      Xi = Sk
      otherwise, replace the worst solution in population X using equations (4) and (5)
    endif
  endfor
Update Xb
T=T*cooling rate
t=t+1
endwhile
return Xb
```

Figure 6. *Pseudocode of proposed DDAO algorithm.*

5.2. The objective

The objective function for the fitting problem can be summrized as the summation of differences among y-axis of the experimental data and the correponding y-axis on the fitting curve on the same x-axis value. The perfect cost function value that can be reached for the fitting problem is zero.
The objective function is described by the following

$$O = \sum_{i=1}^{n} \left\| y_{p\,i} - y_{a\,i} \right\|, \tag{7}$$

where O is the objective function that should be minimized, y_p is the y-coordinates on experimental data, y_a is the y-coordinates on the fitting curve, and i is the index of the experimental data in the set of n experiments. ***Figure 7.*** shows the representation of the objective function which is described by equation (7).

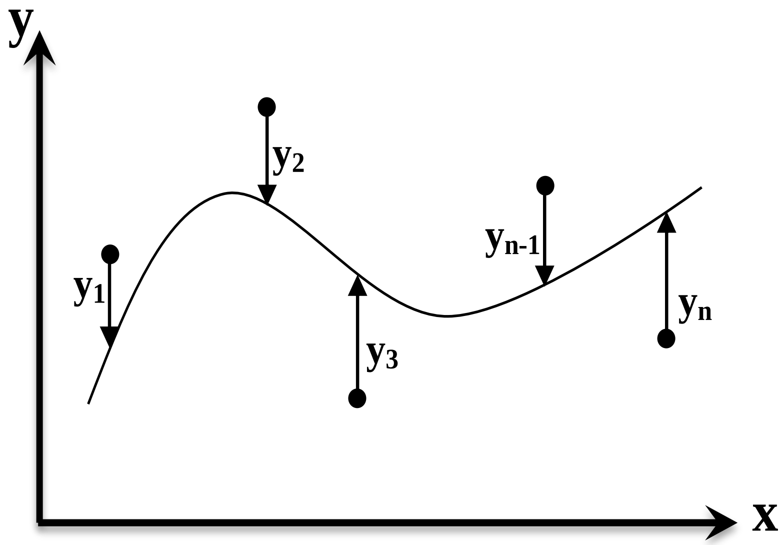

Figure 7. *Representation of the objective function.*

5.3. Experiment

The efficiency of the knot spline curve is improved by considering interpolated points represented by cross symbol in ***Figure 8.*** The position of the interpolated points is highly affecting the final shape of the spline, consequently, the efficiency of the fitting process. The domain between adjacent segments is continuous. Therefore, an optimization algorithm is proposed to create and optimize the interpolated points. Dynamic differential annealed optimization (Ghafil and Jármai, 2020b) is used for the optimization problem where it is responsible for the optimal positions of the interpolated points on each successive control point. ***Figure 9.*** reveals the interpolated spline curve with experimental data where it is clear that the developed curve greatly matches the experiments.

The cubic spline in ***Figure 9.*** representing the experimental data on the continuous domain [0 , 1.6] smoothly and perfectly except for a bit of subdomain [0.23 , 0.28]. This is an acceptable error on the method that can be overcome by increasing the number of interpolated points developed by the optimization algorithm. The eight control points which are used in this experiment are the same as the experimental data in ***Table 1.*** for a flow rate equal to five. Thus the number of interpolated points are seven considering using one interpolated point between each two successive control points. Each interpolated point should be in the best possible position in order to have the best curve that fits the data. This best position or coordination is estimated by the objective function described in equation (1). The objective function is used by DDAO to return coefficients for equation (1) that are corresponding to the minimum cost function. The output of the DDAO is the red curve in ***Figure 8.*** which is consists of 100 points. In other words, We have used the eight control points from experimental data to generate a curve with 100 points using the DDAO with the help of seven interpolated points. The comparison between the performance of the interpolated spline method in ***Figure 9.*** and spline method in ***Figure10.*** reveals the efficiency of the proposed method in this article. ***Table 2.*** shows the statistical results of comparison pure spline and interpolated spline methods on fitting experimental data in ***Table 1.***

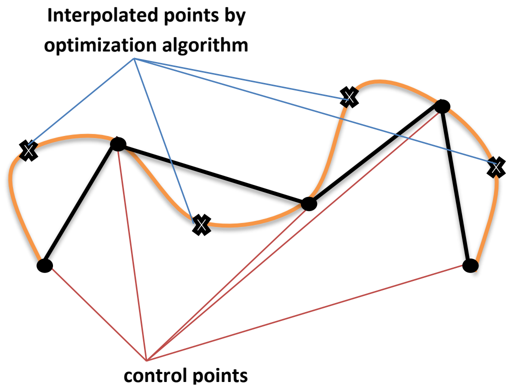

Figure 8. *Interpolated spline method.*

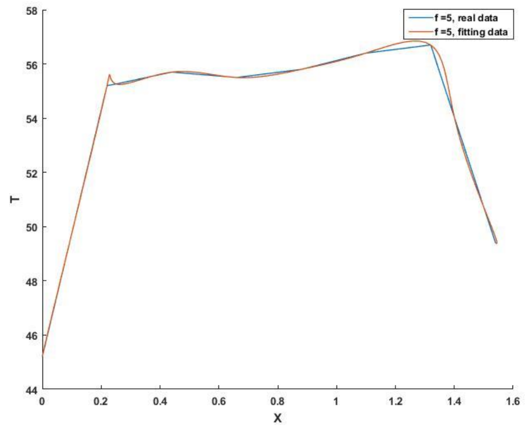

Figure 9. *Interpolated spline method compared with experimental data.*

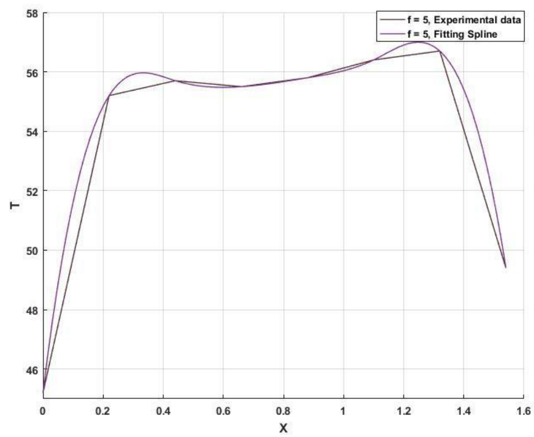

Figure 10. *spline method compared with experimental data.*

The error percentage was calculated using the following equation

$$Error\% = \frac{|y_f - y|}{y} * 100 \quad (7)$$

where y is the y-coordinate of the experimental data and y_f is the y-coordinate of the corresponding y-coordinate on the fitting curve. The statistical results reveal that the efficiency of the interpolated spline method is better than spline in fitting experimental data. In many cases, interpolated spline curve is matching the experimental data, and in very narrow ranges there are small overshots that can be an acceptable limitation on the method. The best results were written in bold line in ***Table 2.***

Table. 2. *Results of the comparison beween spline and interpolated spline methods.*

Experimental data		Method	y_f -axis	error%
x-axis	y-axis			
0.1	49.74	Interpolated spline	49.74	**0%**
		spline	55.89	12.3%
0.3	55.365	Interpolated spline	55.3	**0.11%**
		spline	55.9	0.96%
0.5	55.645	Interpolated spline	55.715	**0.12%**
		spline	55.558	0.15%
1.2	56.53	Interpolated spline	56.73	**0.35%**
		spline	56.9	0.65%
1.4	54.06	Interpolated spline	54.06	**0%**
		spline	55.401	2.4%

6. Conclusion

In this paper, interpolated cubic spline curve was developed to fit experimental data and represent them in the best possible equation. The interpolated spline was compared with the linear regression method and pure knot spline on the same experiments. The proposed interpolated spline is overcome linear regression and pure knot spline in fitting the data set. However, the efficiency of the proposed method is depending on the degree of the polynomial in the regression and the number of control points in the spline interpolation.

7. Acknowledgments

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