Production Systems and Information Engineering
Volume 7 (2015), pp. 15–31

THE EFFECT OF THE SUPPLY ACCURACY ON THE INVENTORIES

János Korponai
József Hatvany Doctoral School in Information Sciences
janos.korponai@gmail.com

Ágota Bányainé Tóth
University of Miskolc, Hungary
Institute of Logistic
altagota@uni-miskolc.hu

Béla Illés
University of Miskolc, Hungary
Institute of Logistic
altilles@uni-miskolc.hu

[Received September 2015 and accepted November 2015]

Abstract. Our theories applied in the case of the economic order quantity are influenced by the fact that disturbances can occur in our internal and external processes that together create an uncertainty regarding the arrival date of purchased parts and the development of the stock level. By walking around the continuous review model during our analysis, we will present the relations between the fluctuation of supply accuracy and the safety stocks, and we will define the formula for the quantification of the safety stock. The definition of the service level provides an opportunity to handle the fluctuation of replenishment period with a predetermined safety, and to quantify the stock level required for the operation.

Keywords: supply accuracy, replenishment period, safety stock, standard normal distribution, random variable

1. Introduction

All production companies require for operation the acquisition of items from external resources, and their arrival by the specified deadline. When the stock replenishment can be exactly on time realized, the level of the closing stock, the quantity of the placed order and the actual date of the order can be clearly defined with the knowledge of the initial stocks, the production demand and the restocking of supplies. However, this initial condition is very rare in practice. There are several unpredictable factors influencing the supply accuracy that will affect the operation of the production company. The objective of the logistics management is to guarantee the stock level required for the adequate handling of production at the lowest possible level of costs [1].

2. The relationship between the deterministic supply accuracy and the stock level

The importance of supply accuracy on the inventories and supply chain efficiency is growing. The operation research and the management sciences literature includes a huge number of methods and models to support the design, optimisation and control of purchasing processes from the point of view supply accuracy [2].

The integration is a core problem of supply in the field of logistics, because the processes of purchasing and production have bidirectional effects. The integration of the first two functional parts of logistics (purchasing, production) is especially important in the case of multistage supply [3].

The supply chain problem includes a wide area of decisions: supplier selection [4], payment management of supply processes [5], coordination of multilevel, vendor managed inventory systems [6], consignment stock management [7], risk management of supply chain and stock management [8]. The modelling of supply processes can be described from the point of view analysis and optimization of warehouses, and there are important sources proposing approach to reduce inefficiencies using value stream mapping [9]. The supply chain accuracy is influenced by the size, operation and scheduling of fleet [10]. It means, the control of the supply processes includes external and internal factors to be optimised. Within the frame of this paper authors are focusing on the supply accuracy on the inventories.

As a simplified model, Figure 1 shows the inventory mechanism and the modification of stocks in case of a deterministic demand and stock replenishment. The initial conditions of the model include the continuous use at a steady pace, a steady t periodic supply, the minimum stock level $Q_{\min}$ equal to zero starting from the inadmissible stock shortage and the zero safety stock, and the Q reorder quantity until the admissible $Q_{\max}$ maximum stock level. The calculable τ replenishment time and the t_i date of receipt determine the latest t_r date, when the order must be placed [11]. The Q_r stock level assigned to time t_r represents the minimum stock level that covers the usage within the time period necessary from the placement of the order until the actual receipt of the ordered quantity [12].

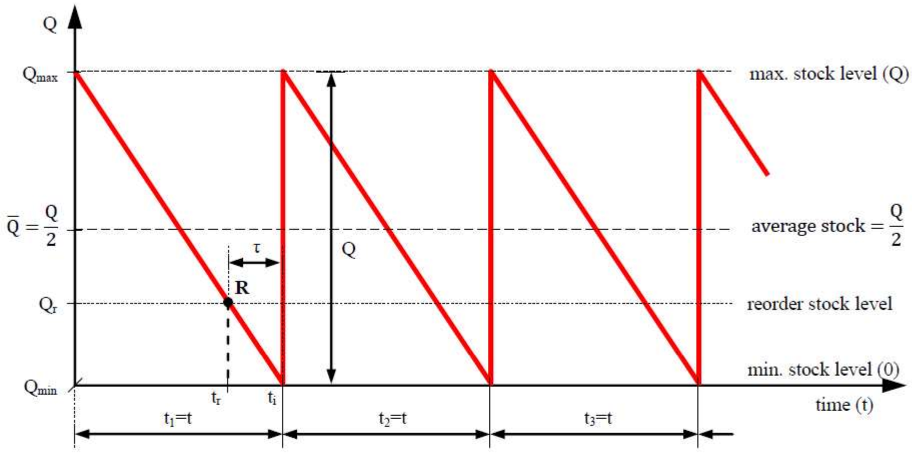

Figure 1. Development of stock level in case of deterministic demand and replenishment time

Furthermore, the initial conditions of the model include the fact that the time demand of stock replenishment is shorter than the periodic time ($\tau < t$), meaning that maximum one supply can be in progress at a given moment. This simplified model shows that the maximum stock level is equal to the quantity per order ($Q_{\max} = Q$), the stock drops to zero at the end of the period ($Q_{\min} = 0$), thus the average stock can be defined with the following formula [13]:

$$\overline{Q} = \frac{Q_{\max} - Q_{\min}}{2} = \frac{Q}{2} \tag{2.1}$$

where

$\overline{Q}$ – average stock,
$Q_{\max}$ – maximum stock level,
$Q_{\min}$ – minimum stock level,
Q – ordered quantity.

3. The relationship between the stochastic supply accuracy and the stock level

By using the principle of ceteris paribus, we carry out the analysis by emphasizing one initial condition of the model, the development of the arrival time of purchased parts. In this case, the model changes to the extent that we allow a deviation in any direction and of any extent between the lengths of the different periods, by specifying that the demand of the respective period is continuous and constant within the period (almost stationary) [11]. The practical applicability of the model justifies this kind of modification of the conditions, since it is hard to imagine for the operation of producing organizations to respect the delivery deadline with complete accuracy at every moment. This change requires the actual supply of the planned demand, thus the model reflects only the replenishment period and not the quantitative deviations from the planned demand.

In case a disturbance occurs in the process of at one of the supply chain's actors, it can happen that the actual arrival date of goods will deviate from the specified deadline. By continuous use, the delay of the arrival of goods would lead to the shortage of stocks. Figure 2 shows that, compared to Figure 1, in case of a delayed goods arrival the stock level can drop below the previous level $Q_{\min} = 0$.

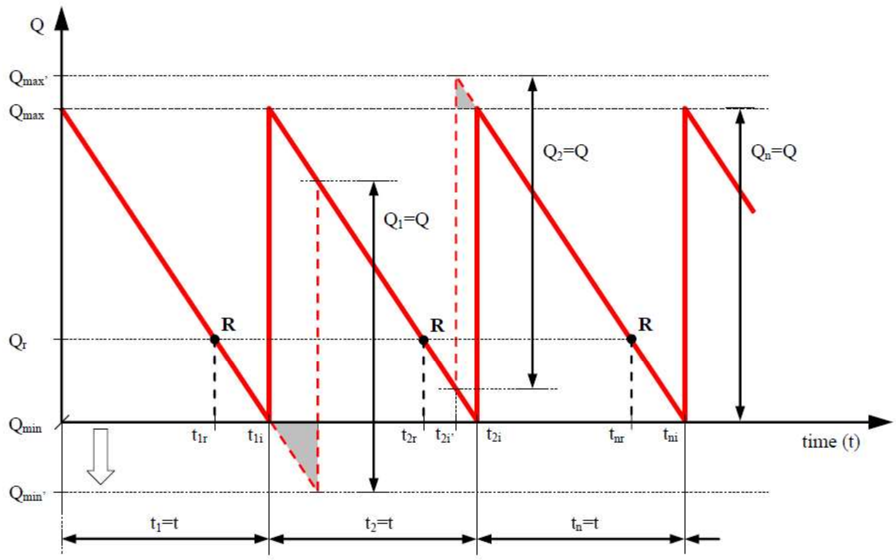

Figure 2. The relationship between supply accuracy of different extents and stock levels

In case of a supply management strategy that does not allow a stock shortage, this means in practice a shift of the Qmin level towards the value $Q_{\min'} = 0$. In order to simplify the model and by applying the principle of ceteris paribus once again, the stock replenishment quality Q remains constant for every order, so it can lead to a maximum stock level $Q_{\max'}$ of different periodical values in relation to the actual date of arrival. The fields marked with grey on Figure 2 show the $Q_{\min}$–$Q_{\min'}$ stock shortage generated by the delayed arrival of goods, and the $Q_{\max'}$–$Q_{\max}$ surplus stock due to early arrival compared to the planned maximum stock level $Q_{\max}$ [14].

The average stock can be defined using two different approaches. With the first method, we divide the whole analyzed period into shorter periods by taking the planned and the actual arrival dates of goods into account. The average stock of the various shorter periods can be quantified separately, and the average stock of the whole analyzed period can be determined by using the average stocks of the individual periods and the length of the different intervals (Figure 3). According to the other method, we take the opening and closing stocks into account conform to the periods with the original length values, and the utilization assumed as uniform during the whole period.

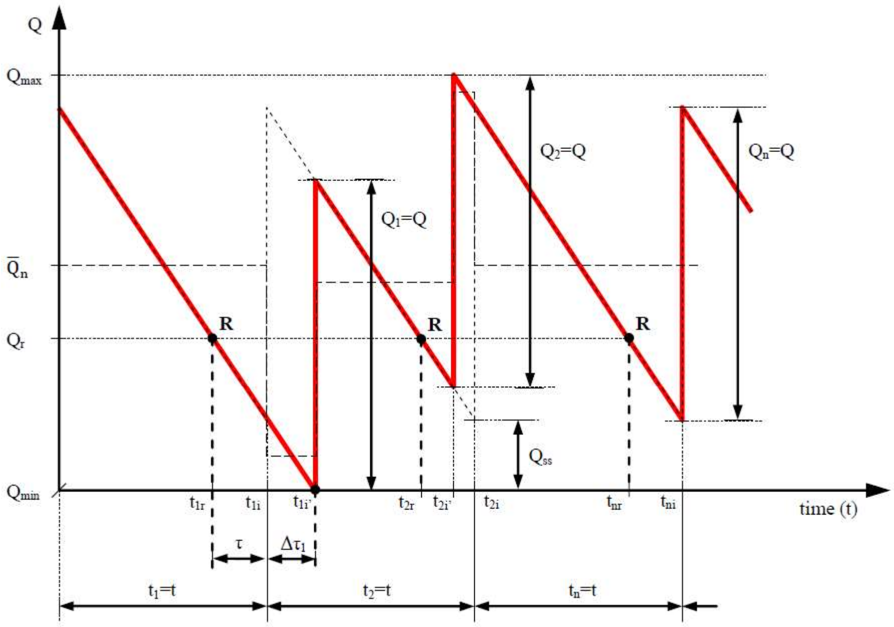

Figure 3. The average stock of the various shorter periods

In practice, the deviation from the planned arrival date can be attributed to various root causes, e.g. unexpected traffic obstacles, disturbances in the supplier's processes, cancellation of supply due to a lack of capacity, delayed or early supply due to an oversight etc.

When analyzing the direction and extent of the deviation of supply accuracy, we can start from the long-term analysis of one product, but we can analyze product groups, product scopes assigned to suppliers, and also complete product portfolios. In case the analysis is limited to the monitoring of one product, the deviations from the target must be analyzed in relation to each other for comparability and cumulability.

In case of an analysis of an adequate number of samples, the deviations will be typically situated along the probability density function of a standard normal unimodal distribution (Figure 4).

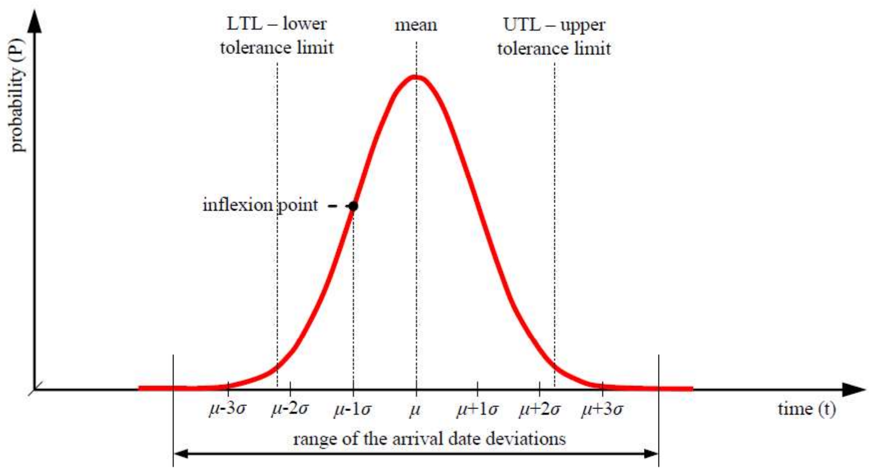

Figure 4. The probability density function of the standard normal distribution and its features

In case the partners have agreed on a maximum number of allowed deviations, the respective range of accepted deviations can be defined in the form of lower and upper tolerance values. The well-known distribution and density functions popular in production management and quality management circles for the presentation of quantity features of certain products can be used in logistics as well. With them, we can show the standard deviation of the actual arrival time from the plan, the situation of deviations compared to the target, the distribution of deviations, the intervention points or tolerances, and the range of the changing replenishment period.

Since the processes between the order placement and the actual arrival of goods are influenced by accidental factors too, the actual date of arrival can be defined only with random variables.

4. The characteristic of the deterministic stock replenishment time

The simplified stock management model showed on Figure 1 and characterized with a deterministic stock replenishment time and a utilization with constant intensity can be completed with the maximum extent of the deviation of supply accuracy in two directions, so that Figure 5 is modelling the deviations occurring compared to the planned utilization. Since the direction and extent of the deviation compared to the plan is stochastic, i.e. it cannot be exactly defined in advance, thus the development of the actual arrival date and of stocks can be determined only with random variables that are represented on the Figure 5 by the probability density functions of the standard normal distribution.

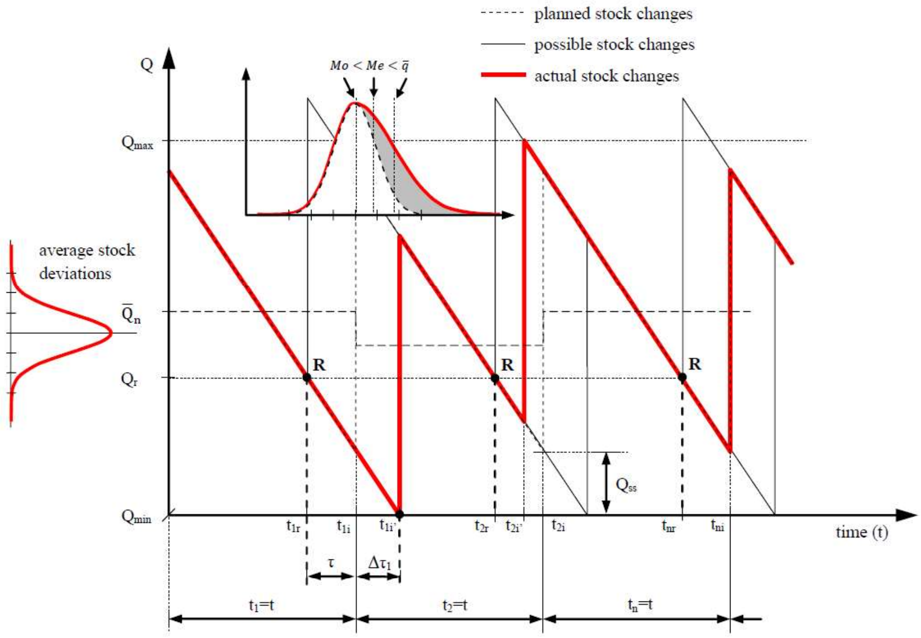

Figure 5. Continuous review in case of stochastic replenishment period

The arrival date of the ordered product is theoretically limited from below, meaning that the product ordered at the R date cannot arrive sooner than the R date of the order placement, and in general, a delay of any extent can occur in the actual arrival date by varying degrees of probability. The form of frequency curves limited from below and characterized with a left-side asymmetry resembles the density function of the F-distribution and the χ^2

distribution. However, in practice – even if by a very low probability – it can occur that the item ordered at the R date arrives before the order placement due to some kind of error, e.g. in case of a supply of identical products in every period, the supplier sends a certain amount even if an order was not placed in the respective period. This means that the date of arrivals is not limited from below in practice, a fact that justifies the application of the probability density function of a standard normal distribution.

The situation and form of the density function shows these two approaches in the Figure 5. The density function reaches a little bit over order rate R from the left side, showing the low probability of the item arriving before the order placement. The left-side asymmetry of the density function marked with grey shows the distortion of the probability of the delay and the early arrival towards the delays.

The longer the stock replenishment time τ, and the higher the performance accuracy towards the supplier (service level), the smaller the shift or asymmetry of the probability density function of standard normal distribution in any direction. Accordingly, in order to simplify the model, we will use a symmetric unimodal function without the shift of the mean value.

In case we only keep stocks to cover the demands of the planned length of the period, according to our stock management strategy, the delayed receiving of the purchased parts compared to the planned $t_{i'}$ arrival date would result in a drop to zero of the stocks before the ti date of receipt. This is inacceptable in case we start from a strategy that does not allow a stock shortage, a certain amount of safety stock (Q_{ss}) should be kept in order to avoid the risks and costs due to delayed deliveries.

Since the fluctuation of supply accuracy has a stochastic character, the possible results are random variables equivalent to the standard normal distribution. Figure 5 shows that if the date of arrival is a random variable, the occurrence of extreme deviations has a very low probability, thus the stock shortage can be most likely avoided only with a safety stock with an infinite high level. In practice however, this is not feasible due to different economic efficiency reasons, thus a compromise must be reached between the consequences caused by the stock shortage and the sacrifices brought to avoid the shortage [15].

In order to be able to define the relation between the supply accuracy and the necessary safety stock, we need to convert the delivery time to used quantity. Earlier, we measured the supply accuracy in time and not in quantity, so we have to quantify the quantity used within the period between the planned stock replenishment time and the deviation compared to the deadline. By assuming a use of uniform D intensity, the used amount due within the planned

τ stock replenishment:

$$Q_t = D \cdot \tau \tag{4.1}$$

where

Q_t – utilization during the stock replenishment time,
D – intensity of the demand,
τ – lead time of stock replenishment.

By assuming a use of uniform D intensity, the used amount due within the fluctuation by $\Delta\tau$ of the stock replenishment time:

$$Q_{\Delta\tau} = D \cdot \Delta\tau \tag{4.2}$$

where

$Q_{\Delta\tau}$ – fluctuation of the utilization caused by the fluctuating stock replenishment time,
$\Delta\tau$ – fluctuation of the stock replenishment time.

5. The effect of the safety stock level on the probability of the stock coverage and shortage

The density function detailed on Figure 6 shows the probability of the used amount due within the actual stock replenishment period, which reflects that the frequency of occurrences is more typical around the expected value, while their probability decreases towards the two ends of the function [16, 17].

The Figure 6 shows clearly that the extent of the given stock level covering the demand during the replenishment period can be defined in relation to any arbitrary q_n stock level and the actual demand during the replenishment period. The stock level above the actual demand during the replenishment period is the safety that covers the fluctuation of the arrival date of purchased parts. The safety stock level can be calculated with the following formula:

$$Q_{ss} = q_n - \mu \tag{5.1}$$

where

Q_{ss} – safety stock level,
q_n – arbitrary stock level,
μ – the actual demand or the expected demand value during the replenishment period.

The formula above demonstrates that the safety stock can have a negative level, if we allow the probability of the occurrence of a stock shortage during our stock management strategy to reach an extent where the stock we

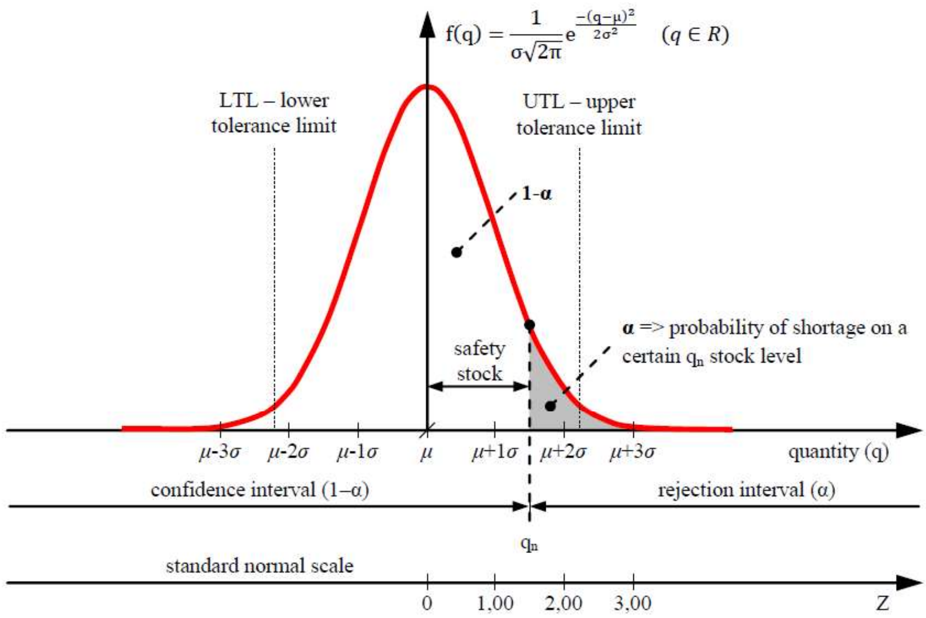

Figure 6. The effect of the safety stock on the occurrence probability of the stock shortage

keep in certain periods is not enough to cover the planned demands of the replenishment period:

$$q_n < \mu \text{ in case of } Q_{ss} < 0 \tag{5.2}$$

In the knowledge of the standard deviation and the supply accuracy, the probability of the occurrence of a shortage can be defined in relation to any replenishment period and stock level. In case we adjust the stocks only to the planned demand level, i.e. we do not keep a safety stock ($q_n = \mu$), the probability of the occurrence of a shortage in case of a supply accuracy with standard normal distribution will correspond to the probability of the non-occurrence of the shortage (Figure 7). Since the combined probability of the two possible results is 100%, the following relation is valid:

$$P_{(\text{shortage})} = P_{(\text{coverage})} = 0.5 \tag{5.3}$$

where

$P_{(\text{shortage})}$ – probability of stock shortage,
$P_{(\text{coverage})}$ – probability of stock coverage.

The distribution function on Figure 7 shows the probability of the coverage of replenishment period fluctuations in relation to the changes of the stock level. The probability of the occurrence of the shortage decreases proportionally with the increase of the safety stocks; however, the complete safety can be guaranteed only by a stock with an infinite level. The introduction of the distribution function is justified by the fact that the function reflects well the probability of the occurrence of the stock shortage in connection with the stock level. The probability can be expressed as a risk as well, to which actual costs can be assigned.

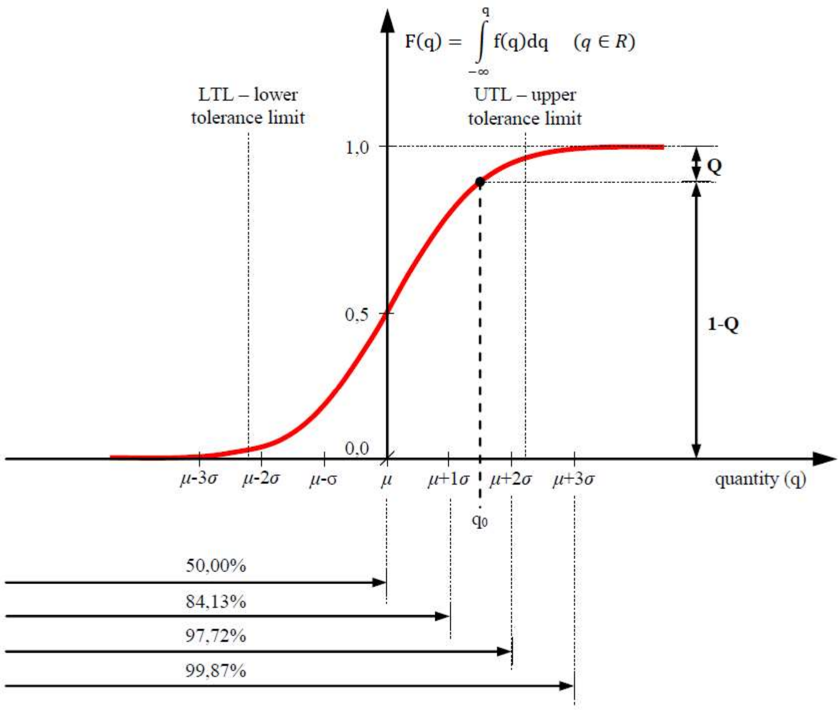

Figure 7. The probability of the coverage of supply accuracy fluctuations in relation to the change of the stock level

6. The method of the safety stock calculation

When determining the order quantity due for the next period and the stock level guaranteeing a continuous operation, five tasks must be completed.

1. The service level of our stock management strategy has to be defined, i.e. the frequency of shortage acceptance.

2. Define the planned duration of the stock replenishment.
3. Define the expected duration of the acceptable stock replenishment by the respective service level by considering the expected fluctuation of the stock replenishment time.
4. Define the stock level that covers the utilization demand due within the expected duration of the acceptable stock replenishment at the respective service level by avoiding the occurrence of a shortage.
5. Verify whether the stock available at the time of the order does cover the utilization demand defined by considering the service level and the expected fluctuation of the stock replenishment time.

Under service level we mean the extent of acceptance regarding the shortage, which can be defined in two ways [1]. It can be determined as a relation of the number of periods allowing shortages and the total number of periods analyzed, or we can define the allowed probability of the occurrence of the shortage. In case we allow 2 periods uncovered by stocks for a complete period, e.g. one year, the service level can be defined by a weekly period as follows:

$$\frac{SL \cdot t}{T} = \frac{2 \cdot 7}{360} = 0.0389 \tag{6.1}$$

where

SL – service level,
t – the length of a given period,
T – the length of the complete period.

From the standard normal distribution table we get the value $Z = 1.765$ matching the probability value 0.0389. Another way to define the service level is to define the maximum allowed level of the probability of the shortage's occurrence (e.g. 0.02) and to find the matching $Z = 2.33$ value. Irrespective of the definition of the service level, we apply the resulting value in a consistent way.

Subsequently, we must define the planned duration of the stock replenishment of the n-th period:

$$\tau_n = t_{ni} - t_{nr} \tag{6.2}$$

where

τ_n – the planned duration of the stock replenishment of the n-th period,
t_{ni} – the planned arrival date of the order belonging to the respective n-th period,
t_{nr} – the date of the order placement belonging to the respective n-th period.

The fluctuation expected for the duration of the next period can be defined based on the service level and the expected demand value during the replenishment period. By using the formula the distribution function (6.3), we can determine the $F(q)$ value of the distribution function for any arbitrary q value [18, 19, 20]:

$$F(q) = \int_{-\infty}^{q} f(q)\,\mathrm{d}q = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{q} e^{\frac{-(q-\mu)^2}{2\sigma^2}}\,\mathrm{d}q = \Phi\left(\frac{q-\mu}{\sigma}\right) = Z \tag{6.3}$$

where

$F(q)$ – the formula of the distribution function,
Z – the standardized value of a normally distributed random variable.

In case the value of the standard normal distribution function is equal to the service level, the stock level q barely covers the expected utilization demand μ derived from the replenishment period and constant utilization that can be characterized with an σ standard deviation by an SL reliability level:

$$\Phi\left(\frac{q-\mu}{\sigma}\right) = SL = Z \tag{6.4}$$

where

q – the demand during the stock replenishment time expired until the actual arrival of goods,
σ – standard deviation of the demand, caused by the fluctuation of the length of the stock replenishment period.

We have to define the expected duration of the acceptable stock replenishment by the respective service level by considering the expected fluctuation of the stock replenishment time. We can transform the above-mentioned formula in a way that q equals the demand during the $t_{ni'}$ stock replenishment time expired until the actual arrival of goods accepted by the given SL level, and μ equals the demand during the $\tau_n = t_{ni} - t_{nr}$ planned stock replenishment period, it means:

$$q = D \cdot (t_{ni'} - t_{nr}) \text{ and } \mu = D \cdot (t_{ni} - t_{nr}) = D \cdot \tau \tag{6.5}$$

where

$t_{ni'}$ – stock replenishment time expired until the actual arrival of goods.

From the formula above it follows:

$$F(q) = \Phi\left(\frac{q-\mu}{\sigma}\right) \rightarrow \Phi\left(\frac{D \cdot (t_{ni'} - t_{nr})}{\sigma}\right) = \Phi\left(\frac{D \cdot \Delta\tau_n}{\sigma}\right) = SL \tag{6.6}$$

Formula (6.5) and (6.6) show that the Q_{ss} safety stock level belonging to the SL service level is the difference of the stock level and during the planned replenishment period expected demand level $q - \mu$, which can be defined with the help of the standard normal distribution $\Phi(q)$ table, it means:

$$Q_{ss} = q - \mu \tag{6.7}$$

The extent of the expected demand due by the end of the period between the ordering point and the receipt of products (replenishment period) can be quantified similarly to formula (6.5):

$$\mu_\tau = \tau \cdot \overline{D}_\tau \tag{6.8}$$

where

μ_τ – expected value of demand due within the stock replenishment period,
$\overline{D}_\tau$ – average demand of the period between the ordering point and the period's end.

During the quantification of the safety stocks, it is not enough to cover the utilization demands expected for the remaining τ period of the ordering period; the temporal deviations during stock replenishment must be taken into account as well. Figure 8 demonstrates the above-mentioned relations [1].

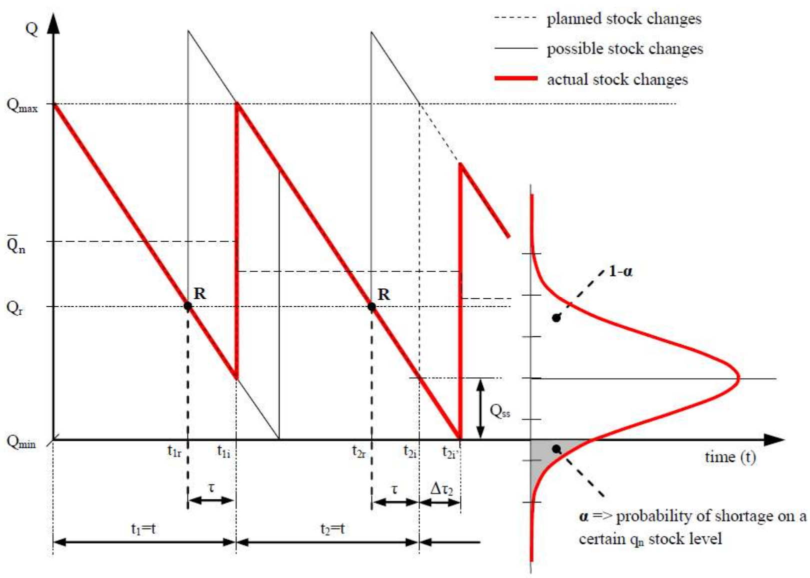

Figure 8. The relations of the continuous review

7. Summary

Several unpredictable factors can occur in the supply chain, that influencing the operation of the production company. The objective of the logistics management is to guarantee the stock level required for the adequate handling of production at the lowest possible level of costs. The more stringent expectation we define towards the frequency of stock shortages in our stock management strategy, the higher the expenses will be for the avoidance of the stock shortage. Fundamentally, we can guarantee the achievement of the target level in two directions:

- with a stricter supplier qualification, we can reduce the uncertainty of supplies,
- we can increase the safety stocks to cover the fluctuating arrival date of purchased parts.

The management of unpredictable supply accuracy urges the management of the production company to reach a compromise. The shortage of stocks can cause serious disturbances in the production supply and customer service, the costs of which are often unquantifiable. The avoidance of stock shortages is an important objective for every organization; however, it would be possible only with the management of an infinite stock level due to the stochastic nature of demands.

Acknowledgements

This research was partially carried out in the framework of the Centre of Excellence of Mechatronics and Logistics at the University of Miskolc.

The presented research work was partially supported by the grant TÁMOP-4.2.2.B-15/1/KONV-2015-0003.

REFERENCES

[1] KOLTAI T.: *Production Management* (in Hungarian: *Termelésmenedzsment*), Published by Typotex Kiadó, Budapest, 2009, ISBN 978 963 279 035 0, ISSN 1787-9655

[2] WANG, Y, WALLACE, S. W., SHEN, B, CHOI, T-M.: *Service supply chain management: A review of operational models.* European Journal of Operational Research, Volume 247, Issue 3, 16 December 2015, pp. 685-698

[3] MASOUD, S. A., MASON, S. J.: *Integrated cost optimization in a two-stage, automotive supply chain.* Computers & Operations Research, Volume 67, March 2016, pp. 1-11.

[4] Cardenas-Barron, L. E., Gonzalez-Velarde, J. L., Trevino-Garza, G.: *A new approach to solve the multi-product multi-period inventory lot sizing with supplier selection problem.* Computers & Operations Research, Volume 64, December 2015, pp. 225-232.

[5] Zhou, Y.-W., Wen, Z.-L., Wu, X.: *A single-period inventory and payment model with partial trade credit.* Computers & Industrial Engineering, Volume 90, December 2015, pp. 132-145.

[6] Lee, J.-Y., Cho, R. K., Paik, S. K.: *Supply chain coordination in vendor-managed inventory systems with stockout-cost sharing under limited storage capacity.* European Journal of Operational Research, Volume 248, Issue 1, 1 January 2016, pp. 95-106.

[7] Zahran, S. K., Jaber, M. Y., Zanoni, S., Zavanella, L. E.: *Payment schemes for a two-level consignment stock supply chain system.* Computers & Industrial Engineering, Volume 87, September 2015, pp. 491-505.

[8] Cedillo-Campos, M. G., Cedillo-Campos, H. O.: *Security risk level classification of stock keeping units in a warehouse.* Safety Science, Volume 79, November 2015, pp. 358-368.

[9] Dotoli, M., Epicoco, N., Falagario, M., Costantino, N., Turchiano, B.: *An integrated approach for warehouse analysis and optimization: A case study.* Computers in Industry, Volume 70, June 2015, pp. 56-69.

[10] Sha, M., Srinivasan, R.: *Fleet sizing in chemical supply chains using agent-based simulation.* Computers & Chemical Engineering, Volume 84, 4 January 2016, pp. 180-198.

[11] Cselényi J., Illés B.: *Planning and controlling of material flow systems* (in Hungarian: *Anyagáramlási rendszerek tervezése és irányítása*), Published by Miskolci Egyetemi Kiadó, Miskolc, 2006, ISBN 963 661 672 8

[12] Prezenszki J.: *Warehousing-logistics* (in Hungarian: *Raktározás-logisztika*), Published by Ameropa Kiadó, Budapest, 2010, ISBN 978-963-06-8670-9

[13] Szegedi Z., Prezenszki J.: *Logistics Management* (in Hungarian: *Logisztika-menedzsment*), Published by Kossuth Kiadó, Budapest, 2003, ISBN 963 09 4434 0

[14] Prezenszki J. (ed.): *Logistics I.* (in Hungarian: *Logisztika I.*), Published by Kossuth Kiadó, Budapest, 2006, ISBN 963 431 796 0, ISSN 0865-3313

[15] Vörös J.: *Production and service management* (in Hungarian: *Termelés- és szolgáltatásmenedzsment*), Published by Akadémia Kiadó, Budapest, 2010, ISBN 978 963 05 8835 5, ISSN 2061-6430

[16] Kulcsár B.: *Industrial logistics* (in Hungarian: *Ipari logisztika*), LSI Oktatóközpont, Budapest, 1998, ISBN 963 577 242 4

[17] Kummer, S., Grün, O., Jammernegg, W.: *Basics of purchasing, production, logistics (in German: Grundzüge der Beschaffung, Produktion, Logistik)*, Published by Pearson Studium, München, 2009, ISBN 978-3-8273-7351-9

[18] CSERNYÁK L. (ED.): *Probability theory – mathematics for business administration* (in Hungarian: *Valószínűségszámítás – matematika üzemgazdászoknak*), Published by Nemzeti Tankönyvkiadó, Budapest, 1998, ISBN 963 18 8413 9

[19] CSERNYÁK L. (ED.): *Analysis – mathematics for business administration* (in Hungarian: *Analízis – matematika üzemgazdászoknak*), Published by Nemzeti Tankönyvkiadó, Budapest, 1998, ISBN 963 19 0307 9

[20] KORPÁS A.-NÉ (ED.): *Elementary statistics II.* (in Hungarian: *Általános statisztika II.*), Published by Nemzeti Tankönyvkiadó, Budapest, 1997, ISBN 963 19 5508 7