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# PARAMETER APPROXIMATION FOR TOOL LIFE EQUATIONS

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**Abstract.** The aim of this research is to investigate a model for estimating tool life (Kundrák, 1996) during high speed hard turning based on the relationship between the wear progress (speed) and time. The tool life equation is valid in the whole cutting speed range for a given feed rate and depth of cut, and has two extreme values. Here the coefficients $A$, $B$ and $K$ of the equation are calculated under different cutting conditions and the results are in good agreement with experimental values. It is also shown that different feed rate and depth of cut values change the location of the two extreme values of the tool life curve. A Matlab model was developed to determine the parameters of the tool life equation. Furthermore the properties of estimated parameters were investigated by Statistica 9 program package.

*Keywords*: tool life equation, least squares, Levenberg-Marquardt

## 1. Introduction

The time spent cutting up to the allowed wear of the tool is considered tool life. Among the cutting data it is the cutting speed that has the most significant effect on the wear intensity.

The famous formula for estimating tool life was established by Taylor [11] about one hundred years ago. It gave the tool lives belonging to different cutting speeds by $v_c$-T tool life curve ('Taylor relation'), which is still in use.

The prediction of tool deterioration is done even today by calculating the tool life based on experimental work, using the empirical tool life equations.

Although Taylor's equation gives a simple relationship between tool life and cutting speed, and it is easy to use, at the same time the information is limited only to one cutting datum. In addition, Taylor equations consume a lot of money and time since they give reliable results only in a relatively narrow range of cutting speed [4, 7, 11].

The results of a wide range of tool life experiments proved that the real tool life change depending on the cutting speed has a relative maximum and a relative minimum value.

This is caused first of all by the layers of different thicknesses and origins (for example metallic and non-metallic build-ups) which are created by the resultant of highly complicated mechanical, physical and chemical procedures.

In the last one hundred years a great number of suggestions have been made to describe the connection between the cutting parameters and tool life using simplified approximation curves (Safonov, Temchin, Wu, Kronenberg, Metchisen, Granovski, König-Depiéreaux etc) [2, 8].

Among the relationships found in technical literature only some are able to describe tool life in a wide range of cutting parameters.

A new tool life equation valid for the whole cutting speed range created by Kundrák [6] is suggested to be used for the description of tool life [5, 7, 8,9].

$$T = \frac{K}{V^3 + AV^2 + BV}, \tag{1.1}$$

(time $T$, speed $V$, parameters $A, B$ and $K$).

This tool life equation is valid in the whole cutting speed range for a given feed rate and depth of cut, and has two extreme values. Here the coefficients $A$, $B$ and $K$ of the equation are calculated under different cutting conditions and the results are in good agreement with experimental values. It is also shown that different feed rate and depth of cut values change the location of the two extreme values of the tool life curve.

A Matlab simulink model was developed to simulate the tool life based on the flank wear rate [1].

In this paper a good iterative, numerical algorithm is given with starting values. Furthermore the properties of estimated parameters are investigated by Statistica 9 program package.

## 2. Mathematical description of tool life

Denote speed by $x$ and tool life by $f(x)$. From the former investigations we have the following conditions for the approximation of the tool life curve:

F1 $f:(0,\infty)\to(0,\infty)$.

F2 $\lim_{x\to 0+0} f(x)=+\infty$.

F3 $\lim_{x\to +\infty} f(x)=0$.

F4 There exist $0<a<b$ such that $f'(a)=f'(b)=0$.

F5 $\lim_{x\to 0+0} f'(x)=-\infty$.

F6 $\lim_{x\to +\infty} f'(x)=0$.

The relationship suitable to describe the complete tool life curve can be written in the following form using the symbols applied in cutting:

$$f(x)=\frac{K}{x^3+Ax^2+Bx}\,. \tag{2.1}$$

**Remark.** The function $f$ was given by investigating reciprocal polynomials. Conditon F4 implies that the degree of polynomials is at least 3 and the constant of polynomials is equal to 0 by condition F2.

$$f(x)=\frac{K}{x^3+Ax^2+Bx+C} \tag{2.2}$$

$$f'(x)=\frac{-K(3x^2+2Ax+B)}{(x^3+Ax^2+Bx+C)^2} \tag{2.3}$$

The expansion of further constraints

1. $K > 0.$
2. $C = 0.$
3. $A^2 < 4B$
4. $A^2 > 3B$
5. $A < 0$ and $B > 0.$

Thus $C = 0.$ The possible place of extrema are

$$x_1 = -\frac{1}{3}A - \frac{1}{3}\sqrt{A^2 - 3B}, \quad x_2 = -\frac{1}{3}A + \frac{1}{3}\sqrt{A^2 - 3B}. \tag{2.4}$$

The second derivative

$$f''(x) == \frac{2K(3x^2 + 2Ax + B)^2}{(x^3 + Ax^2 + Bx)^3} - \frac{K(6x + 2A)}{(x^3 + Ax^2 + Bx)^2} \tag{2.5}$$

implies that there is a local minimum at $x_1$ and a local maximum at $x_2$.

By the classical relation of roots and cofficients we have

$$x_1 + x_2 = -\frac{2A}{3} \text{ és } x_1 x_2 = \frac{B}{3}. \tag{2.6}$$

**The optimization problem can be defined as follows:** Given the measurements

$$(x_1 y_1), (x_2 y_2), \ldots, (x_n y_n), \tag{2.7}$$

determine the parameters $K$, $A$ and $B$ such that

$$\sum_{i=1}^{n} \left( y_i - \frac{K}{x_i^3 + Ax_i^2 + Bx_i} \right)^2 \to \min. \tag{2.8}$$

Let

$$Q := \sum_{i=1}^{n} \left( y_i - \frac{K}{x_i^3 + Ax_i^2 + Bx_i} \right)^2. \tag{2.9}$$

Then

$$\frac{\partial Q}{\partial K} = \sum_{i=1}^{n} -2\left( y_i - \frac{K}{x_i^3 + Ax_i^2 + Bx_i} \right)\left( x_i^3 + Ax_i^2 + Bx_i \right)^{-1} = 0, \tag{2.10}$$

$$\frac{\partial Q}{\partial A} = \sum_{i=1}^{n} 2\left( y_i - \frac{K}{x_i^3 + Ax_i^2 + Bx_i} \right) Kx_i^2\left( x_i^3 + Ax_i^2 + Bx_i \right)^{-2} = 0, \tag{2.11}$$

$$\frac{\partial Q}{\partial B} = \sum_{i=1}^{n} 2\left( y_i - \frac{K}{x_i^3 + Ax_i^2 + Bx_i} \right) Kx_i\left( x_i^3 + Ax_i^2 + Bx_i \right)^{-2} = 0. \tag{2.12}$$

This is a system of nonlinear equations where the order of magnitude of parameters is determined strictly by data. We can solve this system by an iterative method (Levenberg-Marquardt, trust region) [3, 4]. The starting values are obtained by the modified optimization problem

$$\sum_{i=1}^{n} \left( \frac{1}{y_i} - \frac{x_i^3 + Ax_i^2 + Bx_i}{K} \right)^2 . \tag{2.13}$$

If we solve this modified least squares problem, then the system of equations is linear and its solution is a good starting value for the original least squares problem. By further investigations it was obtained that the solution of the problem

$$\sum_{i=1}^{n} \left( \frac{1}{y_i x_i} - \frac{x_i^2 + Ax_i + B}{K} \right)^2 \tag{2.14}$$

would be better for starting value.

## 3. Experimental results

Table 1 contains the data of six experiments and the results of our calculations. Having fixed the values of feedrate and depth of cut, tool life was measured depending on different cutting speeds. The bottom part of the table shows the calculated parameters of the six tool life curves and the coordinates of the extrema on them where $E$ is the error of the approximation.

**Table 1.** Data and results of calculations

| **$v_c$ [m/min]** | **T [min]** | | | | | |
|---|---|---|---|---|---|---|
| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| 11 | 450 | 300 | 260 | 400 | 310 | 290 |
| 20 | 310 | 220 | 210 | 270 | 240 | 210 |
| 29 | 260 | 210 | 190 | 260 | 220 | 210 |
| 35 | 260 | 220 | 200 | 250 | 220 | 210 |
| 40 | 260 | 230 | 210 | 260 | 230 | 220 |
| 50 | 270 | 210 | 160 | 270 | 230 | 220 |
| 59 | 270 | 170 | 110 | 260 | 200 | 180 |
| 68 | 270 | 110 | 60 | 220 | 150 | 130 |
| 80 | 230 | 60 | 30 | 150 | 90 | 70 |
| 92 | 160 | 40 | 30 | 90 | 50 | 40 |
| 105 | 100 | 20 | 10 | 60 | 30 | 20 |
| 120 | 60 | 10 | 6 | 30 | 20 | 10 |
| 150 | 20 | 4 | 2 | 10 | 7 | 5 |
| **f[mm/rev]** | 0.025 | 0.075 | 0.125 | 0.05 | 0.05 | 0.05 |
| **$a_p$[mm]** | 0.1 | 0.1 | 0.1 | 0.05 | 0.15 | 0.25 |
| K | $26.03 \times 10^6$ | $7.67 \times 10^6$ | $5.05 \times 10^6$ | $16.12 \times 10^6$ | $9.86 \times 10^6$ | $8.06 \times 10^6$ |
| A | -146.61 | -102.97 | -90.44 | -125.44 | -112.03 | -107.66 |
| B | 6772.17 | 3373.07 | 2647.84 | 4975.66 | 3959.55 | 3623.01 |
| E | 145.30 | 152.51 | 450.12 | 255.94 | 104.11 | 95.93 |
| minx | 37.43 | 26.99 | 25.02 | 32.33 | 28.70 | 26.93 |
| miny | 258.94 | 215.06 | 199.69 | 253.72 | 219.15 | 206.57 |
| maxx | 60.31 | 41.66 | 35.27 | 51.30 | 45.98 | 44.85 |
| maxy | 275.35 | 225.00 | 204.03 | 268.12 | 232.46 | 223.01 |

In order to make the discussion of different tool life curves convenient, a user interface is developed with the help of Matlab GUI (Fig. 1). After loading the list of cutting speeds and the corresponding tool life values, the program calculates the parameters of the tool life equation, presents the graphs of v-T and v-L functions and displays the extrema on these functions.

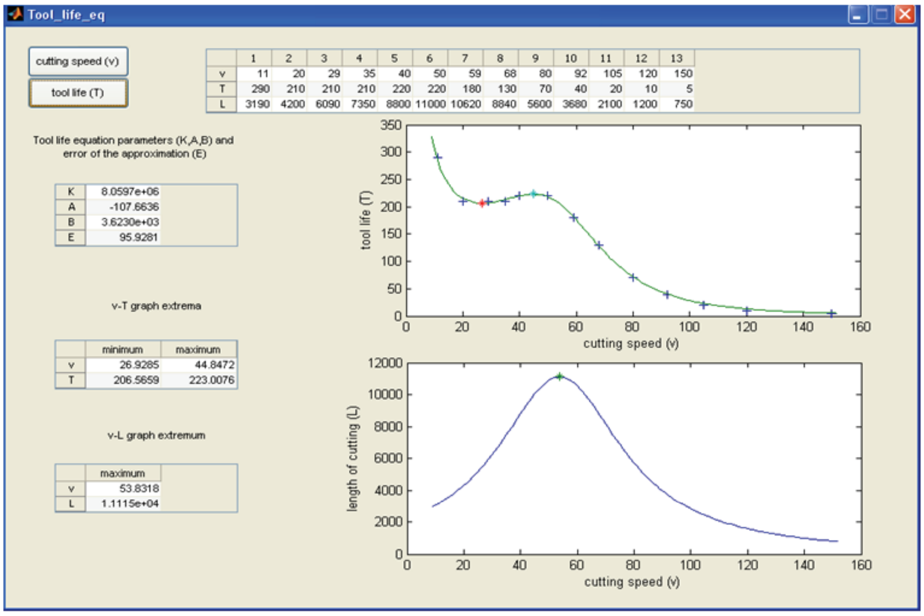


**Figure 1.** Matlab user interface for the problem

## 4. Statistical results

Main results are given by *Statistica 9* software where (x11_Y1Y6) is the name of data file.

Model is: Y2=K/(X^3+B*X^2+C*X); Dependent variable: Y2;

Independent variables: 1; Loss function: least squares;

Final value: 152,51007128;

Proportion of variance accounted for: 0,99877281 R =0,99938622.

A common alternative to the least squares *loss function* is to maximize the likelihood or log-likelihood function (or to minimize the negative log-likelihood function; the term maximum likelihood was first used by Fisher, 1929).

**Table 2.** Iteration history (Statistica 9 software)

| | Model is: Y2=K/(X^3+B*X^2+C*X) (x11_Y1Y6) Dep. Var. : Y2 | | | |
|---|---|---|---|---|
| | Loss Function | K | B | C |
| 1 | 612,0588 | 0 | 0,100 | 0,100 |
| 2 | 603,7063 | 19131 | 0,100 | 0,100 |
| 3 | 600,1535 | 22883 | -1,904 | -0,187 |
| 4 | 592,9964 | 28799 | -4,053 | -1,180 |
| 5 | 579,2200 | 36857 | -5,794 | -5,159 |
| 6 | 546,9349 | 46742 | -6,928 | -17,346 |
| 7 | 525,4859 | 58241 | -7,394 | -22,311 |
| 8 | 523,4034 | 75313 | -7,184 | -19,546 |
| 9 | 519,6936 | 108030 | -6,796 | -14,177 |
| 10 | 513,7193 | 167372 | -6,138 | -4,121 |
| 11 | 503,8548 | 285899 | -4,974 | 17,179 |
| 12 | 489,0267 | 520740 | -3,234 | 64,050 |
| 13 | 468,1469 | 971452 | -1,915 | 171,837 |
| 14 | 436,5744 | 1745041 | -6,741 | 424,369 |
| 15 | 355,1769 | 2638343 | -36,759 | 972,743 |
| 16 | 138,1249 | 4731877 | -80,710 | 2119,442 |
| 17 | 35,0382 | 7526846 | -99,462 | 3173,117 |
| 18 | 12,5452 | 7703253 | -102,898 | 3370,423 |
| 19 | 12,3495 | 7676002 | -102,973 | 3373,188 |
| 20 | 12,3495 | 7675579 | -102,971 | 3373,075 |

**Table 3.** Summary: parameter estimates (Statistica 9 software)

| | Model is: Y2=K/(X^3+B*X^2+C*X) (x11_Y1Y6) Dep. Var. : Y2 Level of confidence: 95.0% ( alpha=0.050) | | | | | |
|---|---|---|---|---|---|---|
| | Estimate | Standard error | t-value df = 10 | p-value | Lo. Conf Limit | Up. Conf Limit |
| K | 7675579 | 196731,1 | 39,016 | 0,000000 | 7237235 | 8113923 |
| B | -103 | 0,7 | -140,242 | 0,000000 | -105 | -101 |
| C | 3373 | 53,2 | 63,393 | 0,000000 | 3255 | 3492 |

The estimated parameter values are rejected by t-test at arbitrary confidence level.

**Table 4.** Correlation of parameters (Statistica 9 software)

| | Model is: Y2=K/(X^3+B*X^2+C*X) (x11_Y1Y6 Dep. Var. : Y2 | | |
|---|---|---|---|
| | K | B | C |
| K | 1,000000 | -0,736468 | 0,909304 |
| B | -0,736468 | 1,000000 | -0,937510 |
| C | 0,909304 | -0,937510 | 1,000000 |

The correlations are high in every case.

**Table 5.** Observed and predicted values (Statistica 9 software)

| | Model is: Y2=K/(X^3+B*X^2+C*X) (x11_Y1Y6 Dep. Var. : Y2 | | |
|---|---|---|---|
| | Observed | Predicted | Residuals |
| 1 | 300,0000 | 295,4953 | 4,50468 |
| 2 | 220,0000 | 223,9542 | -3,95422 |
| 3 | 210,0000 | 215,5498 | -5,54976 |
| 4 | 220,0000 | 220,6083 | -0,60835 |
| 5 | 230,0000 | 224,6363 | 5,36375 |
| 6 | 210,0000 | 211,8833 | -1,88328 |
| 7 | 170,0000 | 167,0517 | 2,94834 |
| 8 | 110,0000 | 113,4403 | -3,44033 |
| 9 | 60,0000 | 62,4896 | -2,48961 |
| 10 | 40,0000 | 35,2962 | 4,70379 |
| 11 | 20,0000 | 20,3845 | -0,38454 |
| 12 | 10,0000 | 11,8089 | -1,80890 |
| 13 | 4,0000 | 4,9073 | -0,90732 |

The approximation function $f$ of observed values is uniform in the whole interval.

## 5. Conclusions

With the appearance of superhard tools, the possibility of precision machining applications has significantly widened. The results reported in this paper should help us to promote the economical application of CBN tools. Application of a new tool life equation is suggested and its advantages have been shown, i.e. it is valid for the whole range of the cutting speed, it considers the joint influence of technological data elements, and it can be applied in practice with our Matlab user interface. This interface is simple, quick, accurate and user friendly.

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