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ALGORITHMS FOR E-MARKETPLACES INTEGRATED WITH LOGISTICS

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Abstract. The E-Marketplace Model Integrated with Logistics (EMMIL) is a family of models that bring together three entity types – seller, buyer and third party logistics service provider - in the same transaction optimising the total cost and/or maximising the revenue. This article is focusing on the allocation algorithms of the buyer oriented EMMIL. The line-haul model is analysed in detail, the combinatorial model is solved for a major class of possible scenarios.

Keywords: B2B e-marketplace, logistics

1. Introduction

Ongoing researches in the fields of supply chain analysis and e-business have an emerging interrelated area of investigation, seeking for new methods in supply chain optimisation [1]. This paper is aimed at contributing to this interdisciplinary research field.

Business-to-Business (B2B) e-marketplaces facilitating trade between businesses are supply chain management (SCM) tools of high importance. Today a substantial part of supply chains are managed across the Internet still they contain a surprisingly high amount of inefficiencies [2].

Logistics services may be handled internally or outsourced to third party logistic (3PL) providers. The standard B2B marketplace models today do not facilitate the integration of logistical solutions into the negotiation between buyers and sellers thus total cost optimisation is practically impossible. E-marketplaces selling goods either do not offer logistical solutions at all or offer a single solution or 2-3 possibilities in different time-cost ranges. The so called integrated marketplaces offer both goods and logistical services, but the goods must be selected first and then logistics providers' market can be reached with a click.

To address part of the SCM inefficiency problems a new model of B2B marketplaces is introduced in [3]. This model integrates three sides of the business into the same transaction - sellers, buyers and third party logistics service providers (3PL) - creating the possibility of a higher level optimisation compared to the traditional e-marketplaces. The new model is called EMMIL, meaning e-market place model integrated with logistics. This paper gives insight to the theory behind the EMMIL model.

2. Related Research in Auction Theory

Auctions are widely used market mechanisms both in traditional and electronic commerce. In the last two decades researchers of auction theory and practitioners have made huge efforts in order to facilitate, support, optimise and standardise electronic negotiations. The aim is to find incentive and computationally manageable auction mechanisms for different business scenarios.

An auction is a resource allocation process, its main components according to [4] are the following: resources, market structure, preference structure, bid structure, matching supply with demand and information feedback.

Resources might be classified as single or multiple items with single or multiple units each having single or multi-attribute specifications. In combinatorial auctions resources are traded in bundles in which case the value of the goods is not equal to the sum of the individual values.

The market structure determines both the number of buyers and sellers participating in a transaction and the mechanism of negotiation. We can distinguish between seller-oriented, buyer-oriented and intermediary marketplaces. Buyer-oriented marketplaces run reverse auctions for an entity with strong buying potential in order to minimise procurement cost.

The bid structure allows the bidders to show their preference structures. We have designed a new bid structure that is much closer to the real world situations than the bid structures discussed in the literature of auction theory.

Matching supply with demand – winner determination – is the crucial point in all auctions. In simple cases is rather straightforward, but in case of general combinatorial auctions is NP-complete so authors in the literature tend to address particular circumstances and find solutions for restricted areas. Linear and integer programming are natural tools for optimisation problems but as the number of participants increases complexity causes difficulties in producing the results within a business feasible duration. Applying metaheuristics, particularly branch and bound algorithms seem to be a very promising direction for tackling these problems [5],[6] that is why we are using a combination of integer programming and branch and bound techniques.

3. General Scheme of EMMILs

In this we give an overview of the general architecture of the EMMIL family of three-sided e-marketplace models that integrate logistics service providers to goods' markets.

Third party logistics providers (3PLs) usually offer one or more of the following services: transportation, warehousing, packaging, unit-load grouping and bulk-breaking. By integrating logistics to goods' marketplaces we mean that the logistics providers are placing their offers step by step during the negotiation of buyers and sellers then each time a combined proposal showing the total cost is created by the marketplace. With this approach the business decisions can be based on the total cost which opportunity has never been offered by any marketplaces.

The general structure of EMMIL can be seen on Figure 1. This framework serves as a base for designing different type of models. In the centre there is the

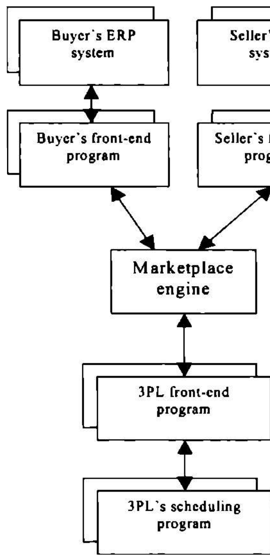

Figure 1. General structure of EMMIL

marketplace engine to which the entities from all the three sides – buyer, seller and 3PL – are joining with the help of front-end processors. It is assumed that all participants have advanced resource planning and scheduling programs that can provide the front-end processors with the relevant data within a short time.

The ratio of the number of buyers and sellers determines if the model is buyer-oriented, seller-oriented or intermediary (exchange) type. All three kinds of marketplaces need a different composite auction mechanism. Here we deal with procurement auctions only since we focus on the buyer-oriented EMMIL models (EMMIL/BM). We use a composite reverse auction with discrete rounds of open bidding that alternate between sellers and 3PLs. The general bidding process is independent from the bidding structure and can be outlined by the flowchart in Figure 2.

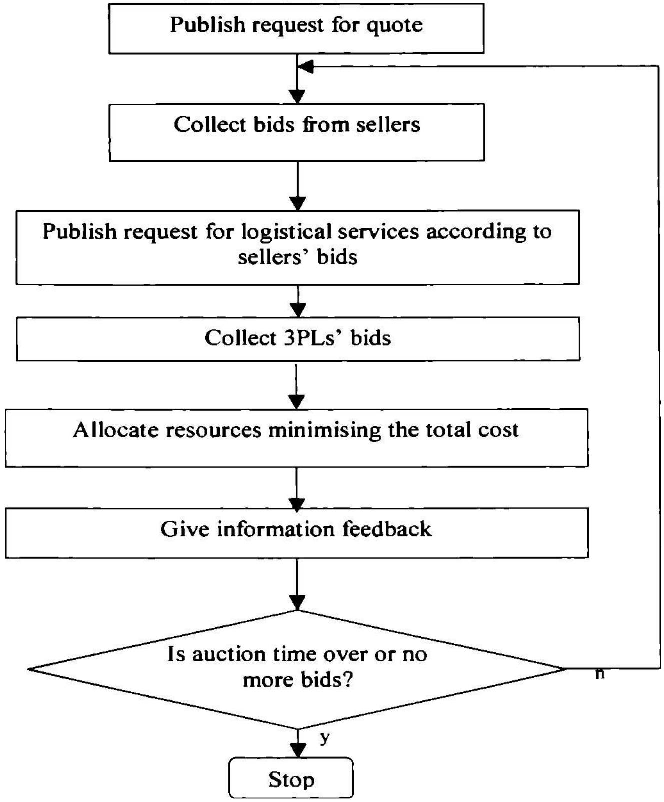

Figure 2. Allocation algorithm for buyer-oriented EMMILs

It can be seen that the auction terminates if no new bids are placed from either sides or a pre-set auction time is over. The auction fails if the set of optimal combination of bids remains empty. In the following chapters we go into the details of buyer oriented EMMIL models and the composite procurement auctions.

4. Formulation of EMMIL/BM

Participants of the marketplace are: a single buyer looking for a set of non-digital goods in specified quantities, many sellers (suppliers) who wish to sell certain quantities of some of the required goods and many third party logistics providers that undertake transportation jobs and warehousing if needed. A seller may act as 3PL as well in case of having logistical capabilities, e.g. vehicles for the transportation.

The products are assumed to be homogenous from the transportation point of view which means they belong to the same transportation class and are packed uniformly. We assume that bulks will not be broken thus required quantities will be integer numbers. We shell focus on road transport.

4.1. Cost Considerations

In order to create an incentive bid structure we are investigating the cost structure of sellers and 3PLs since they have independent private values that is basically the sum of their costs and their target profit margin. Cost structure of EMMIL models are discussed in detail in [7], here only a short summary is given.

Cost accounting [8] classifies costs as variable or fixed according to how they react to the changes in activity. Variable costs - like cost of material built into a product - change linearly with the volume, fixed costs like building depreciation or insurance - do not change for a longer period of time. There are also semi-fixed or step-fixed costs that can be described by a step function. In practice total production and service costs are always semi-variable, they contain both fixed and variable components. We can say that economies of scale can be achieved if the volume of an activity increases up to the limit that can be handled by the same fixed cost value. What we are looking for is an appropriate price structure for the seller side and the 3PL side that reflects the nature of their costs and can be handled by the marketplace engine.

In case of goods the different cost ranges are usually realised by quantity discounts according to pre-set volume ranges. There are several papers in auction literature dealing with quantity discount models, e.g. [9], [10], [11]. In traditional commerce we can also find discount rates based on overall spending on purchases. According to these principles we shall introduce a complex bid structure for sellers in formula (3) allowing any of these two discount types or the combination of them.

Costs of basic logistical activities include handling, transportation and warehousing. Cost calculations should be based on processes but here we are going to simplify the model and use an overall fix and variable cost structure related to a particular tour of a truck regardless of the actual process that takes place between loading the goods at the sellers' plant and unloading them at the buyer.

The fix cost of a tour should cover the organisational overhead and the direct fixed costs. Direct fixed costs of transportation are related partly to the operation of terminals, communication and information systems applied for vehicle tracking, partly to the type of the vehicles, their purchase costs, insurance and general maintenance. Rights-of-way can also form fixed costs if they are pre-paid for a longer period Variable costs relate directly to the movement of a particular load from point A to point B. They include labour (driver's wages), fuel and depreciation of the vehicle as well as maintenance associated with the usage. They depend on the type of the vehicle, the distance, the road conditions, and in case of heavy cargo some of them like cost of fuel can be influenced by the weight of the load as well. The measurement can be [EUR / km] or [EUR/ ton km]. We have to mention that this latter measurement can be misleading if the truck is not full since the major part of fuel and similar costs do occur even if the truck is empty. We may say that these are semi-fixed costs for a line-haul operation where the fix part refers to the cost of moving the empty truck from A point to B and the variable part depends on the weight of the freight - very often non-linearly.

In addition to these variable costs there can be several others that are not proportional to the distance or weight such as casual highway tolls, detention (delay) costs or accommodation costs of the driver in case of long distances. Carriers very often have to cover the back-haul costs as well, i.e. moving back the vehicle from B point to A, adding joint costs to the front-haul transportation. These can only be avoided with back-haul freight or optimised vehicle routing. In case of a consolidation or split-delivery every stop-off means extra costs because of the detention.

Carriers usually apply rates and tariffs for pricing reflecting more or less the fixed and variable costs. They usually set an initial sum that depends on the geographical area and overall requirements of the transportation. Second part of the rate is a unit price related to the distance and the weight or volume of the load, measured e.g. in [EUR / pallet km]. In EMMIL marketplaces carriers should be able to bid for certain routes (line-haul or consolidation tour) with uncertain quantities. A useful cost structure can be given if we create a sum of the indirect and direct fixed cost and the semi-fixed costs of moving the vehicle from starting point (a seller) to the end point (the buyer). This will be the fixed cost of transportation in a bid. The variable cost will be proportional to the volume transported where extra costs of moving the load and handling can be accounted for.

Warehousing cost is composed of handling and storing costs. Handling cost is proportional to the volume so it be aggregated with the previously discussed handling costs. The problem is that storing is proportional with the volume and the time. Still it can be incorporated into the variable cost structure since the time of necessary storing can be calculated before placing thus it can be taken as a fix factor. These considerations lead to the bid structure given in formula (4).

4.2. Bid Structure

First we introduce some basic notations then formulate the buyer's request for quotation (1).

N Number of items (products)

i Item identifier

M Number of suppliers

k Supplier identifier

L Number of 3PLs

l 3PL identifier

Φ type of unit-load

φ class of product

$$R = \{W, \varphi, \Phi, [Q_i, P^h_i], E, U]\} \; i = 1,2,\ldots,N \qquad (1)$$

W Target location (warehouse of the buyer)

Q_i, required quantity of product *i* [unit]

P^h_i upper limit of price of product *i* [EUR/unit]

E earliest arrival time

U latest arrival time

According to the principles set in section 4.1. we shall introduce a complex bid structure for sellers (2) allowing any of these two discount types or the combination of them:

$$B_s^k = \{ W^k, [Q^{kv}_i, P^{kv}_i], [Q^{kh}_i], [\Xi^k_g, \Delta^k_g], E^k, U^k\}, \qquad (2)$$

where

$i \in \{1,\ldots,N\}$, $v = 1,\ldots,V^k_i$ $g = 1,\ldots,G^k$

B_s^k bid of seller *k*

W^k warehouse location of seller *k*

V^k_i Number of quantity discount categories used by seller *k* for item *i*

G^k Number of categories for overall discount at seller *k*

Q^{kv}_i, minimum quantity of item *i* in discount category *v* at seller *k* [unit]

Q^{kh}_i, maximum quantity of item *i* at seller *k* [unit]

P^{kv}_i unit price of item *i* in volume discount category *v* at seller *k* [EUR/unit]

E^k earliest shipping time at seller *k*

U^k latest shipping time at seller *k* ($>=E^k$)

Ξ^k_g lower limit of overall discount category *g* at seller *k* [EUR]

Δ^k_g discount factor of overall discount category *g* at seller *k* (>0, <1)

If $V^k_i = 1$ then no volume discount is given for product *i*. If no overall discount is given then $G^k = 1$ and $\Xi^k_1 = \Delta^k_1 = 0$ is set. If a seller does not have product *i* to sell then sets $V^k_i = 1$ and $Q^{kh}_i = Q^{kv}_i = P^{kv}_i = 0$.

To make further considerations easier we introduce some preliminary concepts. Let A be the set of identifiers of sellers $A = \{1,2,\ldots,M\}$. Denote ***P(A)*** the power set of A.

Now we are going to formulate the bid structure of 3PLs as (3) shows.

$$B_L^{1}=\{[\Gamma^l_j, F^l_j, V^l_j, {}^mT^l_j, {}^hT^l_j]\}, \quad j=1,2,\ldots,G^1, \tag{3}$$

where

$G^l = |\ B_L^l\ |$ number of routes specified in bid of 3PL l

$\Gamma^l_j \in P(A)$ route as a combination of sellers' identifiers

F^l_j Fix cost of delivering a standard size truck of goods on route Γ^l_j

V^l_j Variable cost of delivering goods on route Γ^l_j (cost/unit load) [EUR/unit]

${}^mT^l_j$ Minimum time period needed for delivering all goods on route Γ^l_j [day]

${}^hT^l_j$ Maximum time period needed for delivering all goods on route Γ^l_j [day]

This is a combinatorial bid, we allow 3PLs to bid for consolidation delivery referring to certain combinations of sellers. This means that they are ready to collect the goods from these sellers with as many trucks as it will be needed and it does not mean that they are going to traverse all the sellers with one truck.

5. The Optimisation Problem

In this section we formulate the objective function (4) of the transactions for the EMMIL/BM marketplaces in the most general way:

$$\min\left(\sum_{k=1}^{M}\left(\sum_{i=1}^{N}P_i^kQ_i^k\right)\left(1-\Delta^k\right)+\sum_{l=1}^{L}\sum_{j\in S^l}x_j^l\left(F_j^l\left\lceil\left(\sum_{k\in\Gamma_j^l}\sum_{i=1}^{N}Q_i^k\right)/Z\right\rceil+V_j^l\sum_{k\in\Gamma_j^l}\sum_{i=1}^{N}Q_i^k\right)\right), \tag{4}$$

where:

Q_i^k Purchased quantity of product i from seller k [unit]

P_i^k Unit-price of product i at seller k as a step function of quantity [EUR/unit]

Δ^k Discount given as a step function of the total purchase cost at seller k

Z Standard truckload size [unit]

$x_j^l \in \{0,1\}$ decision variable, $x_j^l = 1 \Leftrightarrow$ offer j of 3PL l is selected as winner.

There are several constraints that have to be formulated. First of all, the buyer wants to buy the required quantity of each product as (5) expresses:

$$\sum_{k=1}^{M} Q_i^k = Q_i, \qquad i = 1,2,\ldots, N. \tag{5}$$

We cannot buy any product from a seller in less quantity than the specified minimum level or more than the maximum (6):

$$Q_i^{k1} \le Q_i^k \le Q_i^{kh} \quad \forall i = 1,2,..N, \forall k = 1,2,... \ M \tag{6}$$

Time constraints in (7) state that the products must be available at the sellers early enough for arriving at the buyer before the latest receiving time but must not arrive too early:

$$(E^k + {}^hT_j^l \le U) \wedge (U^k + {}^mT_j^l \ge E) \forall l \in \{1,2...,L\}, \forall j \in \{1,2,...,G^l\}, \forall k \in \Gamma_j^l. \tag{7}$$

Constraint (8) says that we select a bid $\Gamma_j^l \in D^l$ of a 3PL for delivery only if we purchase some goods from all the suppliers in that combination:

$$x_j^l \le Sign(\sum_{i=1}^{N} Q_i^k), \quad \forall k \in \Gamma_j^l. \tag{8}$$

All sellers selected for the transaction – and only those- must be reached by one and only route as (9) formulates:

$$\sum_{l=1}^{L} x_k^l = Sign(\sum_{i=1}^{N} Q_i^k), \quad \forall k \in \{1,2...,M\}. \tag{9}$$

We apply certain business policies as well. Constraint (10) expresses that the number of suppliers of goods should be kept within specified limits in order to reduce the additional costs of maintaining relationships with suppliers and avoid exposure [9]:

$$S_{\min} \le \sum_{l=1}^{L} Sign(\sum_{j=1}^{G^l} x_j^l) \le S_{\max}, \tag{10}$$

where

S_{min} minimum number of suppliers allowed,

S_{max} maximum number of suppliers allowed.

We have to determine the x_j^l binary and Q_i^k integer variables to minimise the costs and satisfy the constraints. In the following chapter we discuss the possible solution methods.

6. Winner Determination Algorithms

The objective function formulated in (4) is nonlinear and it is not smooth, not even continuous. In order to solve it easily in the following chapters we show a linearisation algorithm. We start with models without discounts then we show how to deal with the discounts as well. We also disregard from the time constraints since the inappropriate offers can be easily filtered first.

6.1. Line-haul Models without Discounts

In this scenario we exclude all the discounts so the objective function will be (11), and constraint (8) will be replaced by (12)

$$\min\left(\sum_{k=1}^{M}\left(\sum_{i=1}^{N}P_i^kQ_i^k+\sum_{l=1}^{L}x_k^l\left(F_k^l\left\lceil\left(\sum_{i=1}^{N}Q_i^k\right)/Z\right\rceil+V_k^l\sum_{i=1}^{N}Q_i^k\right)\right)\right), \tag{11}$$

$$x_k^l \le Sign\left(\sum_{i=1}^{N}Q_i^k\right), \qquad \forall\ k \in A. \tag{12}$$

We would like to derive an equivalent problem with a piece-wise linear objective function from (11) by determining the aggregated minimal logistical cost functions for line-haul transportation from each seller. In order to do this we introduce $g_k^l(Q)$ in (13) which is the logistical cost function of purchasing from seller k and using logistical services of 3PL l. This function has discontinuity at each multiple of the truck capacity Z and it is linear between the jumps. Taking the minimum in l we get $g_k(Q)$, the best logistical cost function for seller k. (14) This function consists of concave, continuous polygons between nZ and $(n+1)Z$ as is illustrated in Figure 3.

$$g_k^l(Q) = F_k^l\lceil Q/Z\rceil + V_k^lQ, \tag{13}$$

$$g_k(Q) = Min_l\, g_k^l(Q). \tag{14}$$

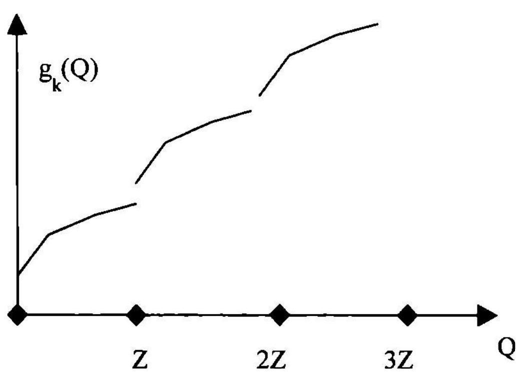

Figure 3. General scheme of the minimal logistical cost function from seller k.

Before introducing the algorithm for constructing $g_k(Q)$ we formulate a few lemmas underpinning the method.

Lemma 6.1.1.

If $j, l \in \{1,...,L\}$ $j \neq l$ and $\exists\, b^l \in B_L^{\,l}$, $b^j \in B_L^{\,j}$ logistical bids from seller k, $(F_k^l, V_k^l) \subset b^l$ $(F_k^j, V_k^j) \subset b^j$ such that $F_k^l \geq F_k^j \wedge V_k^l \geq V_k^j$, then $g_k^l(Q) \geq g_k^j(Q)$ if $Q \geq 0$.

Remark: This means if both fix and variable costs are smaller in one of the two bids than the same relation holds for the total costs always.

Proof

Matching the definition of costs and the conditions of the lemma we get:

$g_k^l(Q) = F_k^l \lceil Q/Z \rceil + V_k^l\, Q$ and $g_k^j(Q) = F_k^j \lceil Q/Z \rceil + V_k^j\, Q \Rightarrow$

$g_k^l(Q) - g_k^j(Q) = (F_k^l - F_k^j)\lceil Q/Z \rceil + (V_k^l - V_k^j)Q \geq 0$. Thus the lemma is proved.

Consequently we can eliminate a logistical bid if we find another one with smaller fix and variable costs. In case of equality we have to use business rules based on previous experiences to find out which 3PL should be kept.

Lemma 6.1.2.

If $Z>0$ integer, $Q \geq 0$ real number, then $Q / \lceil Q/Z \rceil \leq Z$.

Proof

According to divisibility by Z we can express $Q = nZ + r$, $0 \leq r < Z$, $n \geq 0$ integer

If $r=0$, then $Q=nZ$, so $Q/\lceil Q/Z \rceil = nZ/n = Z$

If $r>0$, then $\lceil Q/Z \rceil = n+1$

$Q/(n+1) = n/(n+1)Z + r/(n+1) = Z - (Z-r)/(n+1) < Z$ since $r<Z$.

Lemma 6.1.3.

If the logistical bids of 3PLs j and l are such that $F_k^l \geq F_k^j \wedge V_k^l < V_k^j$

and $(F_k^l - F_k^j) / (V_k^j - V_k^l) \geq Z$, then $g_k^l(Q) \geq g_k^j(Q)$ for $Q \geq 0$.

Proof

Using Lemma 6.1.2. we rearrange the outer sides of the following inequality to get the statement.

$(F_k^l - F_k^j) / (V_k^j - V_k^l) \geq Z \geq Q / \lceil Q/Z \rceil$ hence $F_k^l \lceil Q/Z \rceil + V_k^l\, Q \geq F_k^j \lceil Q/Z \rceil + V_k^j\, Q$

From this lemma it follows that in such a case we can eliminate the bid of the 3PL l from the offers referring to seller k since we have a better choice.

In all other cases the cost functions will intersect resulting in alternating minimal offers. The following lemma shows that we do not have to compute the intersections of all pairs of functions since those that can not be eliminated can be sorted in a way that only adjacent functions do intersect each other.

Lemma 6.1.4.

If $S_k=\{ g_k^l(Q),\ l=1,2,\dots,L_k \}$ is a set of logistical cost functions related to seller k that cannot be eliminated from seeking the minimal function, i.e. $\neg\exists j,j'\ j\neq j'$, such that $g_k^{j'}(Q) \geq g_k^j(Q),\ Q\geq 0$, then the elements of S_k can be sorted in a way that $F_k^1\leq F_k^2\leq\dots\leq F_k^{Lk}$ and $V_k^1 \geq V_k^2\geq\dots\geq V_k^{Lk}$

Proof

Let us suppose that the lemma is not true. Then $\exists\ j,j'\in\{1,\dots,L_k\},\ j\neq j',\ F_k^{j'}\geq F_k^j \wedge V_k^{j'}\geq V_k^j$ but then lemma 6.1.1. causes $g_k^{j'}(Q) \geq g_k^j(Q)$, which contradicts the conditions of the lemma. Hence the lemma is proved.

We introduce Q^T for the total quantity required by the buyer:

$$Q^T \quad \sum_{i=1}^{N} Q_i \tag{15}$$

Now we describe the algorithm for the linearisation of the objective function (11).

Algorithm 6.1.

BEGIN

1. Let $S_k=\{ g_k^l(Q),\ l=1,2,\dots,L \}$
2. $\forall\ l=1,\dots,L-1$

   IF $\exists\ j\in\{1,\dots,L\},\ j\neq l,\ F_k^l\geq F_k^j \wedge V_k^l\geq V_k^j$

   THEN we eliminate the function from the set based on Lemma 6.1.1.

   $S_k := S_k \setminus g_k^l(Q)$

   ENDIF
3. $\forall\ g_k^l(Q)\in S_k$

   IF $\exists\ g_k^j(Q)\in S_k,\ j\neq l,\ (F_k^l - F_k^j)\,/\,(V_k^j - V_k^l)\geq Z$

   THEN we eliminate the function from the set based on Lemma 6.1.3.

   $S_k := S_k \setminus g_k^l(Q)$

   ENDIF
4. Sort the remaining elements of S_k by the fix costs in ascending order. Then based on Lemma 6.1.4 we get

   $F_k^1\leq F_k^2\leq\dots\leq F_k^{Lk}$ and $V_k^1\geq V_k^2\geq\dots\geq V_k^{Lk}$

5. Determine the intersection points of the consecutive functions by solving (16). Denote the number of intersections by A_k . The intersection points will form a sequence of increasing real numbers. $0<q_1^k<q_2^k<...<q_{A_k}^k \leq Q^T$ We also include the multiple values of Z if they are not in the sequence. Taking the lower cost values on each interval we get the minimal cost function $g_k(Q)$. Denote $l(q_j^k)$ the identifier of the best 3PL at q_j^k

$$F_k^j \lceil Q/Z \rceil + V_k^j Q = F_k^{j+1} \lceil Q/Z \rceil + V_k^{j+1} Q \qquad (16)$$
$$0 < Q \leq Q^T$$

6. Associate an F_k^l, V_k^l value pair with each q_l^k intersection point denoting the fix and variable cost valid from that point until the next one in the sequence. Calculate $C_k^l = F_k^l \lceil q_l^k /Z \rceil$ for each intersection point.

7. Let $q_0^k = 0$ and $q_{A_k+1}^k = Q^T$ closing elements to the sequence with $C_k^1 = 0$.

END

After this process we can transform (12) to (17) achieving the goal of linearisation. We have Q_i^{kl} , integer, and y_k^l binary variables where y_k^l decides weather the total quantity purchased from seller k lays in the lth. interval of the aggregate minimal cost function when buying Q_i^{kl} quantity of product i from seller k. This problem with the constraints given in (18)-(23) can be solved by any commercial integer programming packages.

$$Min \sum_{k=1}^{M} \sum_{l=1}^{A_k} \left(\sum_{i=1}^{N} P_i^k Q_i^{kl} + C_k^l y_k^l + V_k^l \sum_{i=1}^{N} Q_i^{kl} \right) \qquad (17)$$

$$y_k^l \in \{0,1\} \qquad (18)$$

$$Q_i^{kl} \geq 0 \quad \forall i \in \{1,...,N\} \qquad (19)$$

$$Q_i^{kl} \leq Q_i^{kh} \quad \forall i \in \{1,...,N\} \qquad (20)$$

$$\sum_{i=1}^{N} Q_i^{kl} - q_l^k y_k^l \geq 0 \quad \forall l \in \{1,...,A_k\}, k \in \{1,...,M\} \qquad (21)$$

$$\sum_{i=1}^{N} Q_i^{kl} - q_{l-1}^k y_k^l \leq 0 \quad \forall l \in \{1,...,A_k\}, k \in \{1,...,M\} \qquad (22)$$

$$\sum_{k=1}^{M} \sum_{l=1}^{A_k} Q_i^{kl} = Q_i \qquad (23)$$

Theorem 6.1.

If minimising the objective function (17) with constraints (18)- (23) results in $\{Q_i^{kl'}, Y_k^{l'}\ k\in A,\ i\in\{1,...,N\},\ l'\in\{1,...,A_k\}\}$, then this solution can be transformed into $\{Q_i^k, X_k^l\ k\in A,\ i\in\{1,...,N\},\ l\in\{1,...,L\}\}$, the solution of (11) with constraints (5), (6), (12).

Proof

Based on lemmas 6.1.1. - 6.1.4. and linearisation algorithm 6.1. the two solutions can be converted into each other as it (24) and (25) show:

$$Q_i^k = \sum_{l'=1}^{A_k} Q_i^{kl'} \tag{24}$$

$$X_k^l \quad \begin{cases} 1, & \text{if } \exists\, l'\in\{1,2\ldots,A_k\},\ Y_k^{l'}=1 \ \wedge l(q_{l'}^{k}) = l \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

The constraints match as:

(23) $\Rightarrow$ (5)

(19) $\wedge$ (20) $\Rightarrow$ (6) as from excluding quantity discounts $Q_i^{k1}=0\ \forall\ i\in\{1,...,N\},\ k\in A$

(21) $\wedge$ (22) $\Rightarrow$ (12)

Hence the theorem is proved.

The number of variables of the linear optimisation problem is expressed in (26) and it is rather high comparing to the original $M(N+L)$ variables. We give an estimate to A_k, the number of linear pieces of the logistical cost function relevant to seller k in order to keep control on the complexity of the problem.

$$(N+1)\sum_{k=1}^{M} A_k \leq M\cdot(N+1)\cdot Max_k(A_k) \tag{26}$$

The number of shipments from a seller cannot be higher then $\lceil Q^T/Z \rceil$ which, taking lemma 6.1.4. into consideration, results in the estimate (27):

$$Max_k(A_k) \leq \left\lceil Q^T/Z \right\rceil L. \tag{27}$$

Theorem 6.2. will show that in case of normally distributed parameters L can be replaced by a constant and hence A_k has an upper limit given in (28):

$$Max_k(A_k) \leq 6\left\lceil Q^T/Z \right\rceil. \tag{28}$$

Theorem 6.2.

If the variable part of the logistical costs follows a normal distribution then the mean of the maximum number of 3PLs contributing to the minimal cost function in an interval [$Q1$, $Q2$] is not more than 6, regardless of the number of bidding 3PLs.

Proof

Let ξ be the random variable of the fix logistical costs and η the random variable of the variable logistical costs where η follows a normal distribution. We sort the 3PLs according to their specified fix costs in ascending order this way determining a random sort on their variable costs. (Figure 4) For any ϑ random variable of normal distribution it is true with confidence of. 0.99 that $|\vartheta^{max} - \vartheta^{min}| < 6\sigma(\vartheta)$ where σ is the standard deviation of ϑ. Furthermore the mean of the distance between any two values of this distribution can be estimated by $\sigma(\vartheta)$, hence the mean of the length of the longest monotone sequence cannot be larger than 6. Along with Lemma 6.1.4. this gives the proof to the theorem.

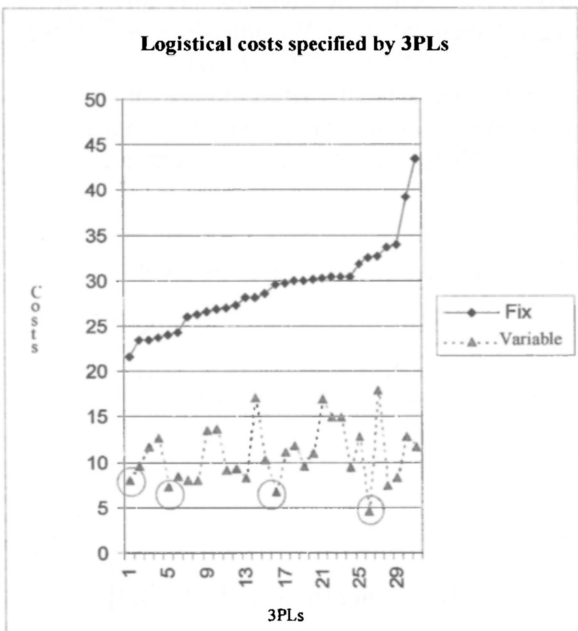

Figure 4. Monotone sequence in a random seguence of variable logistical costs

6.2. Line-haul Models with Quantity Discounts.

This scenario allows individual volume discounts on items but 3PLs can bid for line-hauls only. Here the same solution can be applied as above but new variables and constraints for discount categories should be added. Due to the limited space we only give some hints instead of the proper formulas. The linear optimisation problem can be formulated similarly to (17) adding decision variables showing if the quantity of product i purchased from seller k falls into discount category j. It is also needed a set of a pre-computable constant showing the cost of the quantity equal to the lower limit of this category j.

6.3 Models Allowing Consolidation Delivery

If we allow combinatorial bids on transportation then the linearisation might result in an unmanageable high number of variables. To overcome this problem we created a set of branch and bound solutions for different scenarios. Here we describe a heuristic algorithm for the case when all suppliers have enough quantities of all products, i.e. we release constraint (6). In the algorithm we investigate the costs consequences of purchasing from different combination of sellers exploiting constraint (10).

Let A be the set of identifiers of sellers $A=\{1,2,\ldots,M\}$. Denote $\boldsymbol{P(A)}$ the power set of A. An $S\in \boldsymbol{P(A)}$ is linearly ordered since the elements are different natural numbers. Denote Max (S) the highest element in S. We introduce $A^{\{m\}} \subseteq \boldsymbol{P(A)}$, $m \in A$, for equal size subsets of $\boldsymbol{P(A)}$. Denote $A^{\{m\}}=\{S \mid (S\in \boldsymbol{P(A)}) \wedge (|S|=m)\}$ We order each $A^{\{m\}}$ lexicographically. We order sets of different sizes naturally, if $S\in A^{\{m\}}$ and $S'\in A^{\{n\}}$ then $S'>S \Leftrightarrow n>m$. We introduce Parent(S) = S \ Max(S). It can be seen that if $|S|=1$ then Parent(S)= Ø. Denote Gen(S) the tree generated by S using the rule Gen(S)=$\{S': S' \in \boldsymbol{P(A)}, S'>S\}$.

This way $\boldsymbol{P(A)}$ consists of M disjoint trees as it is illustrated in Figure 6. The ordering we introduced goes across the trees level by level.

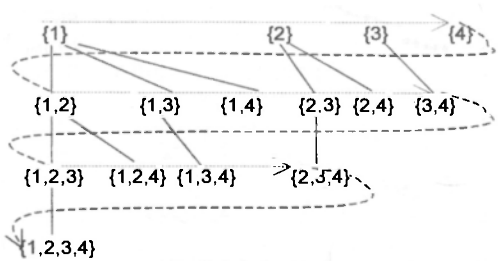

Figure 6. Traversing trees in P({1,2,3,4})

Denote Opt(S) the optimal solution of (4) in $S\in \boldsymbol{P(A)}$ with $C^{opt}_{\Sigma}(S)$ total cost of procurement from sellers in S, buying something from all of them:

$\text{Opt}(S)=\{[Q^k],[x_j^l]\},\ i\in\{1,\ldots,N\},\ k\in S,\ l\in\{1\ldots,L\},\ \Gamma^l_j\subseteq S$

Denote $w(s)$ the highest price vector for all item in Opt(S), w(S)=($w_1(S)$, $w_2(S),\ldots,.w_N(S)$) and F^k_{opt}, V^k_{opt} the logistical cost parameters in Opt(S) for $k\in S$. Let P_i^k the best unit price of product i at seller k.

Lemma 6.3.1.

If S', $S \in P(A)$ where $S'=S\cup\{k'\}$ and $\forall\ i\in\{1,\ldots,N\}$ $P_i^{k'}>= w_i(S)$ and $F^{k'}{}_{opt}>= F^{k}{}_{opt}$ $V^{k'}{}_{opt}>= V^{k}{}_{opt}$ $\forall\ k\in S$ then $C^{opt}{}_{\Sigma}(S')> C^{opt}{}_{\Sigma}(S)$.

Proof

The purchase cost of goods cannot be reduced since all prices are higher than in the previous combination. We have to add an extra location where the transportation costs are also higher according to the assumption. The only way of reducing the cost could be replacing the transport of two or more partly loaded trucks from the original sellers by one from the new seller but if we could do this then Opt(*S*) would not be an optimal solution either. Hence the lemma is proved.

In the **winner determination algorithm** we create a domain set *D* first by pruning the ordered set *P(A)* according to the constraints limiting the number of suppliers. Then we traverse the nodes of *D* in the defined order and create a set of candidate best solutions using a linearised model for the limited set of sellers. During this process we keep pruning subtrees that cannot contain candidate solutions based on Lemma 5.3.1. At the end we compare the candidate solutions at the remaining nodes of *D* and choose a final solution with minimal total cost.

The speed of the algorithm depends on the size of subtrees that can be pruned. Better result can be achieved if we order the suppliers first in descending order by the total cost that would occur if we purchased everything from one supplier. We can also pre-calculate a lower limit of the total cost and exit the algorithm if a candidate solution reaches it within a given tolerance.

7. Summary and Further Research Plans

EMMIL E-Marketplace Model Integrated with Logistics is a triangular marketplace based on the contribution of buyers, sellers and logistics service providers in the same transaction. It aims at finding partners with minimising the total purchase cost or maximising the revenue depending on the type of the marketplace. This paper focused on the buyer-oriented model (EMMIL/BM) where a buyer carries out procurement from many sellers. A composite reverse auction mechanism was defined with discrete rounds of open bids that alternate between sellers and 3PLs. The suggested bid structure was justified by cost considerations. A few scenarios were examined from algorithmic point of view for line-haul and consolidation deliveries.

The implementation process has been just started. Algorithms for line-haul models were tested so far by a prototype implementation using the Solver program of MS EXCEL [13]. Unfortunately this program can handle only 140 variables, but the tests have underpinned the theoretical results perfectly. A full scale implementation is currently being carried out. The problem is NP-complete but we believe that applying realistic business constraints in the number of contracted sellers will keep the computational time within acceptable limits even for high number of variables. A further stage of this research is the development of the distributed optimisation algorithms according to principles already set in [12].

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