Production Systems and Information Engineering
Volume 2 (2004), pp. 177-192

OBSERVER DESIGN VIA TP MODEL TRANSFORMATION

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[Received October 2004 and accepted February 2005]

Abstract. This paper presents a case study how to apply the recently proposed TP model transformation technique, that has been introduced for nonlinear state-feedback control design, to nonlinear observer design. The study is conducted through an example. This example treats the question of observer design to the prototypical aeroelastic wing section with structural nonlinearity. This type of model has been traditionally used for the theoretical as well as experimental analysis of two-dimensional aeroelastic behavior. The model investigated in the paper describes the nonlinear plunge and pitch motion of a wing, and exhibits complex nonlinear behavior. In preliminary works this prototypical aeroelastic wing section was stabilized by a state-feedback controller designed via TP model transformation and linear matrix inequalities. Extending this control strategy with the observer derived in this paper an output feedback strategy can be determined. Numerical simulations are used to provide empirical validation of the resulting observer.

Keywords: nonlinear control, linear parameter varying model, TP model transformation, parallel distributed compensation, linear matrix inequality

1. INTRODUCTION

The main goal of the paper is to study how to apply the TP (Tensor Product) model transformation to observer design. The motivation of this goal is that the TP model transformation was proposed under the Parallel Distributed Compensation (PDC) design framework [22] for nonlinear state feedback controller design [1, 5]. The TP model transformation is capable of transforming a given time varying (parameter dependent, where the parameters may include state variables) linear state-space model into time varying convex combination of finite number of linear time invariant models. The resulting linear time invariant models can then be readily substituted into Linear Matrix Inequalities (LMI), available under the PDC design framework, to determine a time varying (parameter dependent, where the parameters may include state variables) nonlinear controller according to given control specifications. The whole above design can be executed numerically by computers and hence the controller can be determined without analytical derivations in acceptable time. In most cases not all of the state variables are available, but only some of them. This paper studies how to apply the result of the TP model transformation to observer design under the PDC design framework similarly to the controller design. The resulting observer can then be applied to estimate the unavailable state variables.

A few papers were printed in last years dealing with the state-feedback control design of the prototypical aeroelastic wing section via TP model transformation, for instance see [2–4]. This paper focuses attention on the observer design to the prototypical aeroelastic wing section since not all of the state variables of the prototypical aeroelastic wing section are available in reality. The combination of the state-feedback controller and the observer leads to the output feedback control of the prototypical aeroelastic wing section.

2. BASIC NOTATION

This section is devoted to introduce the notations being used in this paper: $\{a, b, ..\}$: scalar values. $\{\mathbf{a}, \mathbf{b}, \ldots\}$: vectors. $\{\mathbf{A}, \mathbf{B}, \ldots\}$: matrices. $\{\mathcal{A}, \mathcal{B}, ..\}$: tensors. The $\mathcal{R}^{I_1 \times \cdots \times I_N}$: vector space of real valued $(I_1 \times I_2 \times \quad \times I_N)$-tensors. Subscript defines lower order: for example, an element of matrix $\mathbf{A}$ at row-column number i, j is symbolized as $(\mathbf{A})_{i,j} = a_{i,}$ Systematically, the ith column vector of $\mathbf{A}$ is denoted as $\mathbf{a}_i$, i.e. $\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots \end{bmatrix}$. $\diamond_{i,j,n}, \ldots$: are indices. $\diamond_{I,J,N}, \ldots$: index upper bound: for example: $i = 1..I$, $j = 1..J$, $n = 1..N$ or $i_n = 1..I_n$. $\mathbf{A}_{(n)}$: n-mode matrix of tensor $\mathcal{A} \in \mathcal{R}^{I_1 \times I_2 \times \cdots \times I_N}$ $\mathcal{A} \times_n \mathbf{U}$: n-mode matrix-tensor product. $\mathcal{A} \otimes_n \mathbf{U}_n$: multiple product as $\mathcal{A} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \times_3 \quad \times_N \mathbf{U}_N$. Detailed discussion of tensor notations and operations is given in [16].

3. BASIC CONCEPTS

The detailed description of the TP model transformation and PDC design framework is beyond the scope of this paper and can be found in [1, 2, 5, 22]. In the followings a few concepts are presented being used in this paper, for more details see [1, 2, 5, 22].

3.1. Linear parameter-varying state-space model

Consider parameter-varying state-space model:

$$sx(t) = A(p(t))x(t) + B(p(t))u(t) \tag{1}$$
$$y(t) = C(p(t))x(t) + D(p(t))u(t),$$

with input $\mathbf{u}(t)$, output $\mathbf{y}(t)$ and state vector $\mathbf{x}(t)$. The system matrix

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t)) \end{pmatrix} \in \mathcal{R}^{O\times I} \tag{2}$$

is a parameter-varying object, where $\mathbf{p}(t) \in \Omega$ is time varying $N-$dimensional parameter vector, where $\Omega = [a_1, b_1] \times [a_2, b_2] \times \quad \times [a_N, b_N] \subset \mathcal{R}^N$ is a closed hypercube. $\mathbf{p}(t)$ can also include some (or all) elements of $\mathbf{x}(t)$. Further, for a continuous-time system $s\mathbf{x}(t) = \dot{\mathbf{x}}(t)$; and for a discrete-time system $s\mathbf{x}(k) = \mathbf{x}(k+1)$ holds.

3.2. Convex state-space TP model

Equ. (2) can be approximated for any parameter $\mathbf{p}(t)$ as a convex combination of the R LTI system matrices $\mathbf{S}_r$, $r = 1..R$. Matrices $\mathbf{S}_r$ are also termed as vertex system matrices. Therefore, one can define basis functions $w_r(\mathbf{p}(t)) \in [0,1] \subset \mathcal{R}$ such that matrix $\mathbf{S}(\mathbf{p}(t))$ belongs to the convex hull of $\mathbf{S}_r$ as $\mathbf{S}(\mathbf{p}(t)) = co\{\mathbf{S}_1, \mathbf{S}_2, .., \mathbf{S}_R\}_{\mathbf{w}(\mathbf{p}(t))}$, where vector $\mathbf{w}(\mathbf{p}(t))$ contains the basis functions $w_r(\mathbf{p}(t))$ of the convex combination. The control design methodology, to be applied in this paper, uses univariate basis functions. Thus, the explicit form of the convex combination in terms of tensor product becomes:

$$\begin{pmatrix} s\mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix} \approx \tag{3}$$
$$\left(\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \quad \sum_{i_N=1}^{I_N} \prod_{n=1}^{N} w_{n,i_n}(p_n(t)) \mathbf{S}_{i_1,i_2,..,i_N} \right) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}$$

(3) is called as TP model in this paper. Function $w_{n,j}(p_n(t)) \in [0,1]$ is the j-th univariate basis function defined on the n-th dimension of Ω, and $p_n(t)$ is the n-th element of vector $\mathbf{p}(t)$. I_n (n=1,...,N) is the number of univariate basis functions used in the n-th dimension of the parameter vector $\mathbf{p}(t)$. The multiple index $(i_1, i_2, ..., i_N)$ refers to the LTI system corresponding to the i_n-th basis function in the n-th dimension. Hence, the number of LTI vertex systems $\mathbf{S}_{i_1,i_2,..,i_N}$ is obviously $R = \prod_n I_n$. One can rewrite (3) in the concise TP form as:

$$\begin{pmatrix} s\mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix} \approx \mathcal{S} \bigotimes_{n=1}^{N} \mathbf{w}_n(p_n(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}, \tag{4}$$

that is

$$\mathbf{S}(\mathbf{p}(t)) \underset{\varepsilon}{\approx} \mathcal{S} \overset{N}{\underset{n=1}{\otimes}} \mathbf{w}_n(p_n(t)).$$

Here, ε represents the approximation error, and row vector $\mathbf{w}_n(p_n) \in \mathcal{R}^{I_n}$ contains the basis functions $w_{n,i_n}(p_n)$, the $N+2$ -dimensional coefficient tensor $\mathcal{S} \in \mathcal{R}^{I_1 \times \cdots \times I_N \times O \times I}$ is constructed from the LTI vertex system matrices $\mathbf{S}_{i_1,i_2,\ldots,i_N} \in \mathcal{R}^{O \times I}$ The first N dimensions of $\mathcal{S}$ are assigned to the dimensions of Ω. The convex combination of the LTI vertex systems is ensured by the conditions:

Definition 1. The TP model (4) is convex if:

$$\forall n, i, p_n(t) \quad w_{n,i}(p_n(t)) \in [0,1]; \tag{5}$$

$$\forall n, p_n(t) \quad \sum_{i=1}^{I_n} w_{n,i}(p_n(t)) = 1. \tag{6}$$

This simply means that $\mathbf{S}(\mathbf{p}(t))$ is within the convex hull of LTI vertex systems $\mathbf{S}_{i_1,i_2,\ldots,i_N}$ for any $\mathbf{p}(t) \in \Omega$.

Remark 1. $\mathbf{S}(\mathbf{p}(t))$ has finite TP model representation in many cases ($\varepsilon = 0$ in (4)). However, one should face that exact finite element TP model representation does not exist in general ($\varepsilon > 0$ in (4)), see [25,26]. In this case $\varepsilon \mapsto 0$, when the number of LTI systems involved in the TP model goes to ∞. In the present observer design, the state-space dynamic model of the prototypical aeroelastic wing section can be exactly represented by a finite convex TP model.

4. Model of the prototypical aeroelastic wing section

In the last few years various studies of aeroelastic systems have emerged. [14] presents a detailed background and refers to a number of papers dealing with the modelling and control of aeroelastic systems. The following provides a brief summary of this background.

Regarding the properties of aeroelastic systems one can find the study of free-play non-linearity by Tang and Dowell in [23,24], by Price et al. in [21] and [20], by Lee et al. in [17], and a complete study of a class of non-linearities is in [28], [20]. O'Neil et al. [18] examined the continuous structural non-linearity of aeroelastic systems. These papers conclude that an aerolesatic system may exhibit a variety of control phenomena such as *limit cycle oscillation, flutter* and even *chaotic vibrations.*

Control strategies have also been derived for aeroelastic systems. [6] shows that controllers, capable of stabilizing structural non-linearity over flow regimes, can be derived via classical multivariable control methods. However, while several authors have investigated the effectiveness of linear control strategies for aeroelastic systems, experimental evidence has shown that linear control methods may not be reliable when non-linear effects predominate. For example in the case of large amplitude limit cycle oscillation behaviour the linear control methodologies [6] do not stabilize aeroelastic systems consistently. [12] and [6] proposed non-linear feedback control methodologies for a class of non-linear structural effects of the wing section [18]. Papers [12, 14, 15] develop a controller, capable of ensuring local asymptotic stability, via partial feedback linearization. It has been shown that by applying two control surfaces global stabilization can be achieved. For instance, adaptive feedback linearization [13] and the global feedback linearization technique were introduced for two control actuators in the work of [14]. TP model transformation based control design was introduced in [2–4]. This control design ensures global asymptotic stability with one control surface and is capable of involving various control specification beyond stability.

4.1. Equations of Motion

In this paper, we consider the problem of flutter suppression for the prototypical aeroelastic wing section as shown in Figure 1. The aerofoil is constrained to have two degrees of freedom, the plunge h and pitch α. The equations of motion of the system have been derived in many references (for example, see [10], and [9]), and can be written as

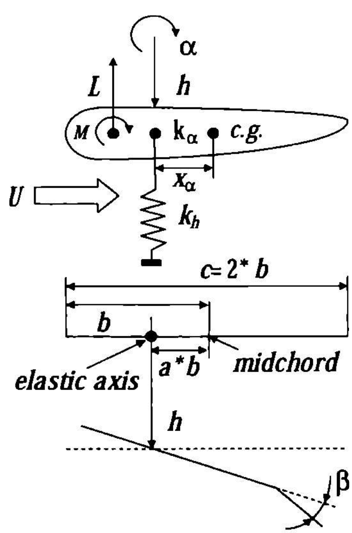

Figure 1: Aeroelastic model

$$\begin{pmatrix} m & mx_\alpha b \\ mx_\alpha b & I_a lpha \end{pmatrix} \begin{pmatrix} \ddot{h} \\ \ddot{\alpha} \end{pmatrix} + \begin{pmatrix} c_h & 0 \\ 0 & c_\alpha \end{pmatrix} \begin{pmatrix} \dot{h} \\ \dot{\alpha} \end{pmatrix} + \\ + \begin{pmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{pmatrix} \begin{pmatrix} h \\ \alpha \end{pmatrix} = \begin{pmatrix} -L \\ M \end{pmatrix}, \tag{7}$$

where

$$L = \rho U^2 b c_{l_\alpha} \left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a \right) b \frac{\dot{\alpha}}{U} \right) + \rho U^2 b c_{l_\beta} \beta \tag{8}$$

$$M = \rho U^2 b^2 c_{m_\alpha} \left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a \right) b \frac{\dot{\alpha}}{U} \right) + \rho U^2 b c_{m_\beta} \beta,$$

and where x_α is the non-dimensional distance between elastic axis and the centre of mass; m is the mass of the wing; I_α is the mass moment of inertia; b is semi-chord of the wing, and c_α and c_h respectively are the pitch and plunge structural damping coefficients, and k_h is the plunge structural spring constant. Traditionally, there have been many ways to represent the aerodynamic force L and moment M, including steady, quasi-steady, unsteady and non-linear aerodynamic models. In this paper we assume the quasi-steady aerodynamic force and moment, see work [10]. It is assumed that L and M are accurate for the class of low velocities concerned. Wind tunnel experiments are carried out in [6]. In the above equation ρ is the air density, U is the free stream velocity, c_{l_α} and c_{m_α} respectively, are lift and moment coefficients per angle of attack, and c_{l_β} and c_{m_β}, respectively are lift and moment coefficients per control surface deflection, and a is non-dimensional distance from the mid-chord to the elastic axis. β is the control surface deflection.

Several classes of non-linear stiffness contributions $k_\alpha(\alpha)$ have been studied in papers treating the open-loop dynamics of aeroelastic systems [8,23,27,28]. For the purpose of illustration, we now introduce the use of polynomial non-linearities. The non-linear stiffness term $k_\alpha(\alpha)$ is obtained by curve-fitting the measured displacement-moment data for non-linear spring as [19]:

$$k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 + 8580\alpha^3 + 17289.7\alpha^4).$$

The equations of motion derived above exhibit limit cycle oscillation, as well as other non-linear response regimes including chaotic response [8, 19, 28]. The system parameters to be used in this paper are given in [1] and are obtained from experimental models described in full detail in works [14, 19].

With the flow velocity $u = 15(m/s)$ and the initial conditions of $\alpha = 0.1(rad)$ and $h = 0.01(m)$, the resulting time response of the non-linear system exhibits limit cycle oscillation, in good qualitative agreement with the behaviour expected in this class

of systems. Papers [18, 19] have shown the relations between limit cycle oscillation, magnitudes and initial conditions or flow velocities.
Let the equations (7) and (8) be combined and reformulated into state-space model form:

$$\mathbf{x}(t) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} h \\ \alpha \\ \dot{h} \\ \dot{\alpha} \end{pmatrix} \quad \text{and} \quad \mathbf{u}(t) = \beta.$$

Then we have:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) = \mathbf{S}(\mathbf{p}(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}, \tag{9}$$

where

$$\mathbf{A}(\mathbf{p}(t)) = \begin{pmatrix} x_3 \\ x_4 \\ -k_1x_1 - (k_2U^2 + p(x_2))x_2 - c_1x_3 - c_2x_4 \\ -k_3x_1 - (k_4U^2 + q(x_2))x_2 - c_3x_3 - c_4x_4 \end{pmatrix}$$

$$\mathbf{B}(\mathbf{p}(t)) = \begin{pmatrix} 0 \\ 0 \\ g_3U^2 \\ g_4U^2 \end{pmatrix}$$

where $\mathbf{p}(t) \in \mathcal{R}^{N=2}$ contains values x_2 and U. The new variables are tabulated in Table 1. One should note that the equations of motion are also dependent upon the elastic axis location a.

5. OBSERVER DESIGN

The recently proposed very powerful numerical methods (and associated theory) for *convex optimization* involving Linear Matrix Inequalities (LMI) help us with the analysis and the design issues of dynamic systems models in acceptable computational time [7, 11]. One direction of these analysis and design methods is based on LMI's under the PDC design framework [22]. In this paper we apply the TP model transformation in combination with the PDC based observer design technique to derive viable observer methodologies for the prototypical aeroelastic wing section defined in the previous section. The key idea of the proposed design method is that the TP model transformation is utilized to represent the model (9) in convex TP model form with specific characteristics, whereupon PDC controller design techniques can immediately be executed.

Table 1: System variables

$d = m(I_\alpha - mx_\alpha^2 b^2)$
$k_1 = \frac{I_\alpha k_h}{d}$
$k_2 = \frac{I_\alpha \rho b c_{l_\alpha} + m x_\alpha b^3 \rho c_{m_\alpha}}{d}$
$k_3 = \frac{-m x_\alpha b k_h}{d}$
$k_4 = \frac{-m x_\alpha b^2 \rho c_{l_\alpha} - m \rho b^2 c_{m_\alpha}}{d}$
$p(\alpha) = \frac{-m x_\alpha b}{d} k_\alpha(\alpha)$
$q(\alpha) = \frac{m}{d} k_\alpha(\alpha)$
$c_1(U) = \frac{I_\alpha(c_h + \rho U b c_{l_\alpha}) + m x_\alpha \rho U^3 c_{m_\alpha}}{d}$
$c_2(U) = \frac{I_\alpha \rho U b^2 c_{l_\alpha}(\frac{1}{2} - a) - m x_\alpha b c_\alpha + m x_\alpha \rho U b^4 c_{m_\alpha}(\frac{1}{2} - a)}{d}$
$c_3(U) = \frac{-m x_\alpha b c_h - m x_\alpha \rho U b^2 c_{l_\alpha} - m \rho U b^2 c_{m_\alpha}}{d}$
$c_4(U) = \frac{m c_\alpha - m x_\alpha \rho U b^3 c_{l_\alpha}(\frac{1}{2} - a) - m \rho U b^3 c_{m_\alpha}(\frac{1}{2} - a)}{d}$
$g_3 = \frac{1}{d}(-I_\alpha \rho b c_{l_\beta} - m x_\alpha b^3 \rho c_{m_\beta})$
$g_4 = \frac{1}{d}(m x_\alpha b^2 \rho c_{l_\beta} + m \rho b^2 c_{m_\beta})$

5.1. TP model form of the prototypical aeroelastic wing section

5.1.1. TP model transformation

The goal of the TP model transformation is to transform a given state-space model (1) into convex TP model [1,2,5], in which the LTI systems form a tight convex hull. Namely, the TP model transformation results in (4) with conditions (5) and (6), and searches the LTI systems as a points of a tight convex hull of $\mathbf{S}(\mathbf{p}(t))$.

The detailed description of the TP model transformation is discussed in [1,2,5]. In the followings only the main steps are briefly presented. The TP model transformation is a numerical method and has three key steps. The first step is the discreatisation of the given $\mathbf{S}(\mathbf{p}(t))$ via the sampling of $\mathbf{S}(\mathbf{p}(t))$ over a huge number of points $\mathbf{p} \in \Omega$, where Ω is the transformation space. The sampling points are defined by a dense hyper rectangular grid. In order to loose minimal information during the discretisation we apply as dense grid as possible. The second step extracts the LTI vertex systems from the sampled systems. This step is specialized to find the minimal number of LTI vertex systems as the vertex points of the tight convex hull of the sampled systems. The third step constructs the TP model based on the LTI vertex systems obtained in the second step. It defines the continuous basis functions to the LTI vertex systems.

5.1.2. Determination of the convex TP model form of the aeroelastic model

We execute the TP model transformation on the model (9). We used the following parameters: $b = 0.135m$; $span = 0.6m$; $k_h = 2844.4N/m$; $c_h = 27.43Ns/m$; $c_\alpha = 0.036Ns$; $\rho = 1.225kg/m^3$; $c_{l_\alpha} = 6.28$; $c_{l_\beta} = 3.358$; $c_{m_\alpha} = (0.5+a)c_{l_\alpha}$; $c_{m_\beta} = -0.635$; $m = 12.387kg$; $x_\alpha = -0.3533 - a$; $I_\alpha = 0.065kgm^2$; $c_\alpha = 0.036$.
First of all, according to the three steps of the TP model transformation, let us define the transformation space Ω. We are interested in the interval $U \in [14,25](m/s)$ and we presume that the interval $\alpha \in [-0.2,0.2](rad)$ is sufficiently large enough. Therefore, let: $\Omega\quad [14,25] \times [-0.1,0.1]$ in the present example (note that these intervals can arbitrarily be defined). Let the grid density be defined as $M_1 \times M_2$, $M_1 = 300$ and $M_2 = 300$. Step 2 of the TP model transformation yields 6 vertex LTI systems:

$$\mathbf{A}_{1,1} = 10^3 \begin{pmatrix} 0 & 0 & 0.0010 & 0 \\ 0 & 0 & 0 & 0.0010 \\ -0.2314 & -0.0095 & -0.0034 & -0.0001 \\ 0.2780 & -1.1036 & 0.0071 & -0.0000 \end{pmatrix} \quad \mathbf{B}_{1,1} = \begin{pmatrix} 0 \\ 0 \\ -8.5825 \\ -32.4370 \end{pmatrix}$$

$$\mathbf{A}_{2,1} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ -231.3804 & -46.3063 & -4.3776 & -0.2573 \\ 277.9906 & -966.7931 & 10.6520 & 0.4104 \end{pmatrix} \quad \mathbf{B}_{2,1} = \begin{pmatrix} 0 \\ 0 \\ -27.3677 \\ -103.4344 \end{pmatrix}$$

$$\mathbf{A}_{3,1} = 10^3 \begin{pmatrix} 0 & 0 & 0.0010 & 0 \\ 0 & 0 & 0 & 0.0010 \\ -0.2314 & -0.0227 & -0.0039 & -0.0002 \\ 0.2780 & -1.0543 & 0.0089 & 0.0002 \end{pmatrix} \quad \mathbf{B}_{3,1} = 10^3 \begin{pmatrix} 0 \\ 0 \\ -0.0154 \\ -0.0580 \end{pmatrix}$$

$$\mathbf{A}_{1,2} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ -231.3804 & -16.5786 & -3.4333 & -0.1425 \\ 277.9906 & 23.0842 & 7.1447 & -0.0157 \end{pmatrix} \quad \mathbf{B}_{1,2} = \begin{pmatrix} 0 \\ 0 \\ -8.5825 \\ -32.4370 \end{pmatrix}$$

$$\mathbf{A}_{2,2} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ -231.3804 & -53.4094 & -4.3776 & -0.2573 \\ 277.9906 & 159.8695 & 10.6520 & 0.4104 \end{pmatrix} \quad \mathbf{B}_{2,2} = \begin{pmatrix} 0 \\ 0 \\ -27.3677 \\ -103.4344 \end{pmatrix}$$

$$\mathbf{A}_{3,2} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ -231.3804 & -29.8524 & -3.9054 & -0.1999 \\ 277.9906 & 72.3823 & 8.8983 & 0.1974 \end{pmatrix} \quad \mathbf{B}_{3,2} = \begin{pmatrix} 0 \\ 0 \\ -15.3526 \\ -58.0244 \end{pmatrix}$$

The third steps results in basis functions $w_{1,i}(U)$ and $w_{2,j}(\alpha)$ depicted in Figure 2. When we numerically check the error between the model (9) and the resulting TP model, we find that the error is about 10^{-11} that is caused by the numerical computation.

In conclusion, the aeroelastic model (9) can be described exactly in finite convex TP form of 6 vertex LTI models, also see [2]. Note that, one may try to derive the basis functions analytically from (9). The basis functions of α can be extracted from $k_\alpha(\alpha)$. Finding the basis functions of U, however, seems to be rather complicated. In spite of this, the computation of the TP model transformation takes a few seconds.

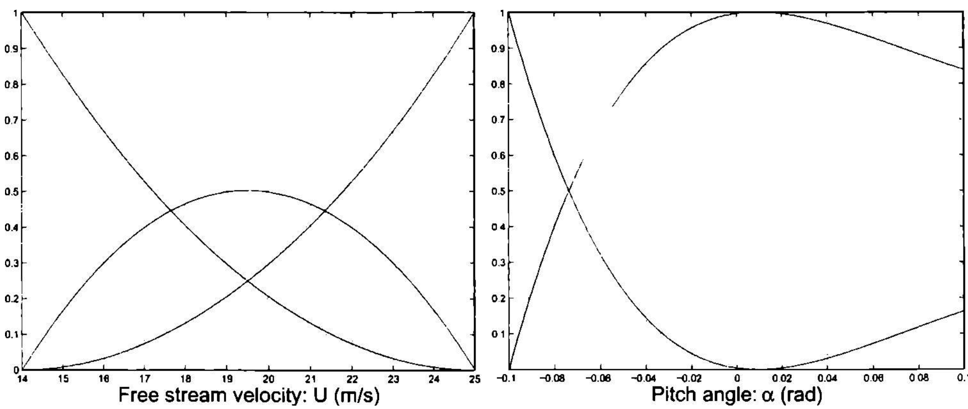

Figure 2: Basis functions on the dimensions U and α.

5.2. Observer design to the prototypical aeroelastic wing section

5.2.1. Method for observer design under PDC framework

In reality not all the state variables are readily available in most cases. Unavailable state variables should be estimated in the case of state-feedback control strategy. Under this circumstances, the question arises whether it is possible to determine the state from the system response to some input over some period of time. Namely, the observer is required to satisfy:

$$\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0 \quad \text{as} \quad t \to \infty,$$

where $\hat{\mathbf{x}}(t)$ denotes the state vector estimated by the observer. This condition guaranties that the steady-state error between $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ converges to 0. We use the following observer structure:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) + \mathbf{K}(\mathbf{p}(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t))$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}(\mathbf{p}(t))\hat{\mathbf{x}}(t),$$

That is in TP model form:

$$\dot{\hat{\mathbf{x}}}(t) = \mathcal{A} \underset{n}{\otimes} \mathbf{w}(p_n(t))\hat{\mathbf{x}}(t) + \mathcal{B} \underset{n}{\otimes} \mathbf{w}_n(p_n(t))\mathbf{u}(t) + \qquad (10)$$

$$+ \mathcal{K} \underset{n}{\otimes} \mathbf{w}(p_n(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t))$$

$$\hat{\mathbf{y}}(t) = \mathcal{C} \underset{n}{\otimes} \mathbf{w}(p_n(t))\hat{\mathbf{x}}(t).$$

At this point we should emphasize that in our example the vector $\mathbf{p}(t)$ does not contain values form the estimated state-vector $\hat{\mathbf{x}}(t)$, since $p_1(t)$ equals U and $p_2(t)$ equals the pitch angle ($x_2(t)$). These variables are observable. We estimate only state-values $x_3(t)$ and $x_4(t)$. Consequently the goal, in the present case, is to determine gains in tensor $\mathcal{K}$ for (5.1). For this goal the following LMI theorem can be find in [22]. Before dealing with this LMI theorem, we introduce a simple indexing technique, in order, to have direct link between the TP model form (3.4) and the typical form of LMI formulations:

Method 1. (Index transformation) Let

$$\mathbf{S}_r = \begin{pmatrix} \mathbf{A}_r & \mathbf{B}_r \\ \mathbf{C}_r & \mathbf{D}_r \end{pmatrix} = \mathbf{S}_{i_1, i_2, .., i_N},$$

where $r = ordering(i_1, i_2, .., i_N)$ ($r = 1..R = \prod_n I_n$). The function "ordering" results in the linear index equivalent of an N dimensional array's index $i_1, i_2, .., i_N$, when the size of the array is $I_1 \times I_2 \times \quad \times I_N$. Let the basis functions be defined according to the sequence of r:

$$w_r(\mathbf{p}(t)) = \prod_n w_{n,i_n}(p_n(t)).$$

Theorem 1. (Globally and asymptotically stable observer)
In order to ensure

$$\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0 \quad \text{as} \quad t \to \infty,$$

in the observer strategy (5.1), find $\mathbf{P} > 0$ and $\mathbf{N}_r$ satisfying equ.

$$-\mathbf{A}_r^T\mathbf{P} - \mathbf{P}\mathbf{A}_r + \mathbf{C}_r^T\mathbf{N}_r^T + \mathbf{N}_r\mathbf{C}_r > 0 \tag{11}$$

for all r and

$$-\mathbf{A}_r^T\mathbf{P} - \mathbf{P}\mathbf{A}_r - \mathbf{A}_s^T\mathbf{P} - \mathbf{P}\mathbf{A}_s + \tag{12}$$

$$+\mathbf{C}_r^T\mathbf{N}_s^T + \mathbf{N}_s\mathbf{C}_r + \mathbf{C}_s^T\mathbf{N}_r^T + \mathbf{N}_r\mathbf{C}_s > 0.$$

for $r < s \leq R$, except the pairs (r,s) such that $w_r(\mathbf{p}(t))w_s(\mathbf{p}(t)) = 0, \forall\mathbf{p}(t)$.

Since the above equations are LMI's with respect to variables $\mathbf{P}$ and $\mathbf{N}_r$, we can find a positive definite matrix $\mathbf{P}$ and matrix $\mathbf{N}_r$ or determine that no such matrices exist. This is a convex feasibility problem. Numerically, this problem can be solved very efficiently by means of the most powerful tools available in the mathematical programming literature e.g. MATLAB-LMI toolbox [11].
The observer gains can then be obtained as:

$$\mathbf{K}_r = \mathbf{P}^{-1}\mathbf{N}_r. \tag{13}$$

Finally, by the help of $r = ordering(i_1, i_2, .., i_N)$ in Method 1 one can define $\mathbf{K}_{i_1,i_2,..,i_N}$ from $\mathbf{K}_r$ obtained in (5.4) and store into tensor $\mathcal{K}$ of (5.1).

5.2.2. Observer design to the prototypical aeroelastic wing section

This section applies Theorem 1 to the TP model of the aeroelastic wing section. We define matrix $\mathbf{C}$ for all r from:

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t),$$

that is in present case:

$$\mathbf{C}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The LMIs of Theorem 1, applied to result of the TP model transformation, are feasible and yields 6 observer feedbacks:

$$\mathbf{K}_{1,1} = 10^3 \begin{pmatrix} 0.0000 & 0.0001 \\ 0.0001 & 0.0008 \\ -0.0432 & 0.0001 \\ 0.5674 & -3.8791 \end{pmatrix}$$

$$\mathbf{K}_{2,1} = 10^3 \begin{pmatrix} 0.0000 & 0.0002 \\ 0.0001 & 0.0008 \\ -0.0429 & -0.0356 \\ 0.5670 & -3.7432 \end{pmatrix}$$

$$\mathbf{K}_{3,1} = 10^3 \begin{pmatrix} 0.0000 & 0.0001 \\ 0.0001 & 0.0008 \\ -0.0430 & -0.0127 \\ 0.5672 & -3.8302 \end{pmatrix}$$

$$\mathbf{K}_{1,2} = 10^3 \begin{pmatrix} 0.0000 & -0.0001 \\ 0.0002 & 0.0010 \\ -0.0430 & -0.0442 \\ 0.5677 & 2.8271 \end{pmatrix}$$

$$\mathbf{K}_{2,2} = 10^3 \begin{pmatrix} 0.0000 & -0.0001 \\ 0.0002 & 0.0010 \\ -0.0431 & -0.0785 \\ 0.5672 & 2.9650 \end{pmatrix}$$

$$\mathbf{K}_{3,2} = 10^3 \begin{pmatrix} 0.0000 & -0.0001 \\ 0.0002 & 0.0010 \\ -0.0430 & -0.0566 \\ 0.5675 & 2.8768 \end{pmatrix}$$

In conclusion the state values $x_3(t)$ and $x_4(t)$ are estimated by (5.1) as:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))u(t) +$$

$$\left(\sum_{i=1}^{3} \sum_{j=1}^{2} w_{1,i}(U) w_{2,j}(\alpha) \mathbf{k}_{i,j} \right) (\mathbf{y}(t) - \hat{\mathbf{y}}(t)),$$

where

$$\mathbf{y}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{y}}(t) = \begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{pmatrix} \quad \text{and} \quad \mathbf{p}(t) = \begin{pmatrix} U \\ \alpha \end{pmatrix},$$

($x_1(t) = h$, plunge, and $x_2(t) = \alpha$, pitch). In order to demonstrate the accuracy of the observer, numerical experiments are presented in the next section.

5.2.3. Simulation results

We simulate the observer for initials $\mathbf{x}(0) = \begin{pmatrix} 0.01 & 0.1 & 0.1 & 0.1 \end{pmatrix}^T$ and $\hat{\mathbf{x}}(0) = \begin{pmatrix} -0.01 & -0.1 & -0.1 & -0.1 \end{pmatrix}^T$, for the open loop case. Figure 3 shows how the observer is capable of converging to the unmeasurable state values $x_3(t)$ and $x_4(t)$.

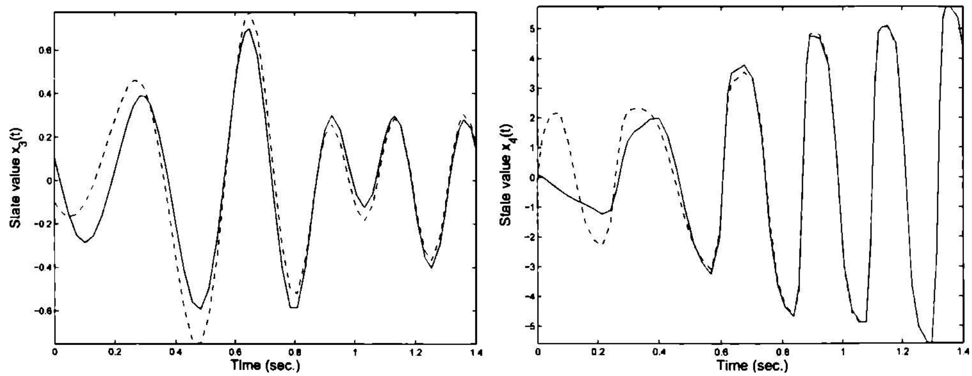

Figure 3: State values $x_3(t)$, $x_4(t)$ (solid line) and estimated values $\hat{x}_3(t)$, $\hat{x}_4(t)$ (dashed line) for open loop response.
($U = 20m/s$, $a = -0.4$, initials: $\mathbf{x}(0) = \begin{pmatrix} 0.01 & 0.1 & 0.1 & 0.1 \end{pmatrix}^T$
$\hat{\mathbf{x}}(0) = \begin{pmatrix} -0.01 & -0.1 & -0.1 & -0.1 \end{pmatrix}^T$)

6. CONCLUSION

First message of the paper is that the TP model transformation method under the PDC design framework can be used for observer design in the same way as for controller design. The second message is that the paper shows how to determine observer for the prototypical aeroelastic wing section.

REFERENCES

[1] BARANYI, P.: TP *model transformation as a way to* LMI *based controller design*, IEEE Transaction on Industrial Electronics, **51** (2004), No. 2.

[2] BARANYI, P., KORONDI, P., PATTON, R., AND HASHIMOTO, H.: *Global asymptotic stabilisation of the prototypical aeroelastic wing section via tp model transfromation*, Asian Journal of Control, **(to be printed in Vol. 7, No. 2, 2004)**.

[3] BARANYI, P., MICHEKBERGER, P., AND VÁRKONYI-KÓCZY, A.: *Numerical control design for aeroelastic systems*, in: *2nd Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence (SAMI 2004)*, Herl'any, Slovakia, 16-17 January, 2004 pp. 43–50.

[4] BARANYI, P. AND PATTON, R.: *A numerical control design method for prototypical aeroelastic wing section with structural non-linearity*, in: *European Control Conference (ECC'03)*, University of Cambridge, UK, 2003

[5] BARANYI, P., TIKK, D., YAM, Y., AND PATTON, R. J.: *From differential equations to* PDC *controller design via numerical transformation*, Computers in Industry, Elsevier Science, **51** (2003), 281–297.

[6] BLOCK, J. J. AND STRGANAC, T. W.: *Applied active control for nonlinear aeroelastic structure*, Journal of Guidance, Control, and Dynamics, **21** (1998), No. 6, 838–845.

[7] BOYD, S. GHAOUI, L. E. FERON, E., AND BALAKRISHNAN, V.: *Linear matrix inequalities in system and control theory*, Philadelphia PA:SIAM, (1994).

[8] DOWELL, E. H.: *Nonlinear aeroelasticity*, Proc. of the 31th AIAA Structures, Structural Dynamics, and Matirials Conference, AIAA Paper 97-1024, (1990), 1497–1509.

[9] (EDITOR), E. H. D., CURTISS, H. C. J., SCANLAN, R. H., AND SISTO, F.: *A Modern Course in Aeroelasticity*, Stifthoff and Noordhoff, Alpen aan den Rijn, The Netherlands, 1978.

[10] FUNG, Y. C.: *An Introduction to the Theory of Aeroelasticity*, John Wiley and Sons, New York, 1955.

[11] GAHINET, P., NEMIROVSKI, A., A.J.LAUB, AND M.CHILALI: LMI *Control Toolbox*, The MathWorks, Inc., 1995.

[12] KO, J., KRIDULA, A. J. AND STRGANAC, T. W.: *Nonlinear control theory for a class of structural nonlinearities in a prototypical wing section*, Proc. of the 35th AIAA Aerospace Scinece Meeting and Exhibit, AIAA paper 97-0580, (1997).

[13] KO, J., KURDILA, A. J., AND STRGANAC, T. W.: *Adaptive feedback linearization for the control of a typical wing section with structural nonlinearity*, Nonlinear Dynamics, (1997).

[14] KO, J., KURDILA, A. J., AND STRGANAC, T. W.: *Nonlinear control theory for a class of structural nonlinearites in a prototypical wing section*, AIAA Journal of Guidance, Control, and Dynamics, **20** (1997), No. 6, 1181–1189.

[15] KO, J., KURDILA, A. J., AND STRGANAC, T. W.: *Nonlinear dynamics and control for a structurally nonlinear aeroelastic system*, Proc. of the 38th AIAA Structures, Structural Dynamics, and Matirials Conference, AIAA Paper 97-1024, (1997).

[16] LATHAUWER, L. D., MOOR, B. D., AND VANDEWALLE, J.: *A multi linear singular value decomposition*, SIAM Journal on Matrix Analysis and Applications, **21** (2000), No. 4, 1253–1278.

[17] LEE, B. H. K. AND LEBLANC, P.: *Flutter analysis of two dimensional airfoil with cubic nonlinear restoring force*, National Aeronautical Istbalishment, Aeronautical Note-36, National Research Council, Ottava, Canada, (1986), No. 25438.

[18] O'NEIL, T. GILLIAT, H. C., AND STRGANAC, T. W.: *Investigatiosn of aeroelsatic response for a system with continuous structural nonlinearities*, Proc. of the 37th AIAA Structures, Structural Dynamics, and Matirials Conference, AIAA Paper 96-1390, (1996).

[19] O'NEIL, T. AND STRGANAC, T. W.: *An experimental investigation of nonlinear aeroelastic respons*, AIAA Journal of Aircraft (accepted).

[20] PRICE, S. J., ALIGHANBARI, H., AND LEE, B. H. K.: *The aeroeloasric response of a two dimensional airfoil with bilinear and cubic structural nonlinearities*, Proc. of the 35th AIAA Structures, Structural Dynamics, and Matirials Conference, AIAA Paper 94-1646, (1994), 1771–1780.

[21] PRICE, S. J., ALIGHANBARI, H., AND LEE, B. H. K.: *Postinstability behavior of a two dimensional airfoil with a structural nonlinearity of aircraft*, Journal of Aircraft, **31** (1994), No. 6, 1395–1401.

[22] TANAKA, K. AND WANG, H. O.: *Fuzzy Control Systems Design and Analysis - A Linear Matrix Inequality Approach*, Hohn Wiley and Sons, Inc., 2001, 2001.

[23] TANG, D. M. AND DOWELL, E. H.: *Flutter and stall response of a helicopter blade with structural nonlinearity*, Journal of Aircraft, **29** (1992), 953–960.

[24] TANG, D. M. AND DOWELL, E. H.: *Comparison of theory and experiment for nonlinear flutter and stall response of a helicopter blade*, Journal of Sound and Vibration, **165** (1993), No. 2, 251–276.

[25] TIKK, D., BARANYI, P., PATTON, R., AND TAR, J.: *Approximation capability of* TP *model forms*, Australian Journal of Intelligent Information Processing Systems (accepted for publication), (2004).

[26] TIKK, D., BARANYI, P., AND R.J.PATTON: *Polytopic and* TS *models are nowere dense in the approximation model space*, IEEE Int. Conf. System Man and Cybernetics (SMC'02), (2002), proc. on CD.

[27] YANG, Z. C. AND ZHAO, L. C.: *Analysis of limit cycle flutter of an airfoil in incompressible flow*, Journal of Sound and Vibration, **123** (1988), No. 1, 1–13.

[28] ZHAO, L. C. AND YANG, Z. C.: *Chaotic motions of an airfoil with nonlinear stiffness in incompressible flow*, Journal of Sound and Vibration, **138** (1990), No. 2, 245–254.