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OPTIMIZATION OF FACE-MILLING CONDITIONS ON THE BASE OF MATERIAL REMOVAL RATE (MRR)

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Abstract. There is a significant body of literature related to optimization of cutting processes. This paper shows a new approach to face-milling optimization based upon the intensity of stock removal. The state variable Q {cm^3/min} expresses both technological and production management aspects at the same time. An investigation of the effect of relative cost-deviation will also be shown.

Keywords: optimization of cutting conditions, milling, material removal rate (MRR)

1. Introduction

Determination of optimum cutting conditions is a classic task of technology process planning. In a Computer-Aided Process Planning (CAPP) system, one of the important steps is the selection of those machining parameters which yield optimum results. The success of the machining operation will depend on the selection of machining parameters. We may say that technological parameters have most important roles as they control the economical aspects. Determination of the optimum cutting parameters is of great importance especially for NC/CNC machine tools. The paper shows a new approach to face-milling optimization based upon the intensity of stock removal. The state variable Q {cm^3/min} suggested by us expresses both technological and production management aspects at the same time. The presented method is based on the previous results of several authors [1,2,3,5]. Similar suggestion was initiated from *E. Kiliç* [6] and his colleagues. They introduced the $x = v \cdot f$ state variable that made it possible to find the optimized cutting conditions.

2. Intensity parameters

The mathematical model of the optimization task can be formulated as follows: 1]

$$U = \{u_i\}, \quad i = 1,\dots, I \tag{1}$$

$$S = \{s_j(\mathbf{u})\}, \quad j = 1,\dots, J, \quad \mathbf{u} \subset U \tag{2}$$

$$C = \{c_k(\mathbf{u},\mathbf{s})\}, \quad k = 1,\dots, K, \quad \mathbf{s} \subset S \tag{3}$$

$$E = \{e_j(\mathbf{u})\}, \tag{4}$$

$$\Phi = \{\varphi_m(\mathbf{u},\mathbf{s})\}, \quad m = 1,\dots, M \tag{5}$$

where

U - set of technological parameters,
S - set of state variables,
C - set of production objective functions,
E - set of state equations
Φ - set of constraining relations.

The relationship (1)-(5), in the case of milling, can be written as follows (Figure 1):

$u_1 \Rightarrow d$ [mm] - depth of cut
$u_2 \Rightarrow w$ [mm] - width of cut,
$u_3 \Rightarrow f$ [m/min] - feed rate

Φ set of constraints, for example: maximum allowable feedrate; the cutting power allowed;
C the objective function depending on the parameters to be optimized and state variables (e.g.: cost or productivity of a given operation element);
S : the set of state variables (e.g.: $s_1 \Rightarrow T$, $s_2 \Rightarrow Q$ where T is the tool life [min], Q is material removal rate [cm^3/min]);
E the set of state equations (e.g.: the extended Taylor tool life equation, $T=T(\mathbf{u})$, material removal rate $Q=Q(\mathbf{u})$;).

Depth of cut and width of cut are depending on the current configuration of the allowance to be removed and its quasi-optimum value can be determined with the aid of geometric modelling.

By means of Q state variable both machining time and operation element cost can easily be expressed in case of face milling. For a given milling operation element we have:

$$Q = d\,w\,f = d\,w\frac{f_z\,z\,n}{1000} = d\,w\frac{f_z \cdot z}{1000}\frac{1000\,v}{\pi \cdot D_c} = d\,w\,f_z\,z\frac{v}{\pi\,D_c}, \tag{6}$$

$$t_m = \frac{V}{Q}, \tag{7}$$

$$\frac{1}{T}=\left[\frac{d^{x_v} f_z^{y_v} w^{z_v} z^{p_v} v}{C_v D_c^{w_v}}\right]^{\frac{1}{m}}=\left[\frac{Q\pi}{C_v d^{1-x_v} f_z^{1-y_v} w^{1-z_v} z^{1-p_v} D_c^{w_v-1}}\right]^{\frac{1}{m}} \tag{8}$$

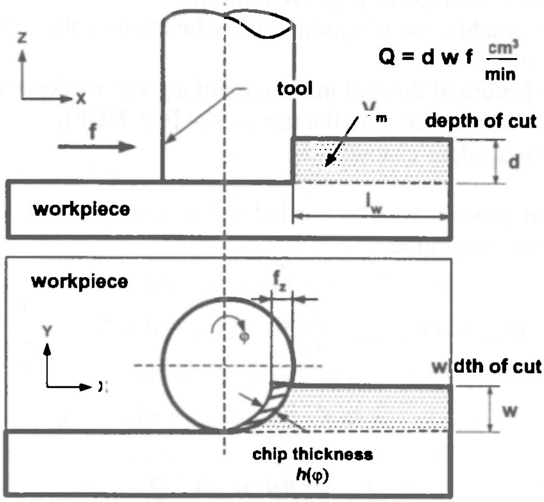

Fig. 1.: End milling cutting geometry

$$\tau=\frac{C_\Sigma}{c_w V_m}=\frac{1}{Q}+\frac{Q^{q-1}}{R^q}, \tag{9}$$

$$R=C_T f_z^{1-y_v} \tag{10}$$

$$C_T=\frac{C_v d^{1-x_v} w^{1-z_v} z^{1-p_v} D_c^{w_v-1}}{\pi\left(\frac{C^*_T}{c_w N_e}+t_c\right)^m}, \tag{11}$$

$$q=\frac{1}{m}, \tag{12}$$

where:

t_m - machining time [min],
f_z - feed per tooth [mm],
z - number of teeth,
D_c - tool diameter [mm],
V_m - the material volume to be removed [cm^3],

T - tool life [min],
C_v, x_v y_v z_v p_v w_v - empirical quasi-constants of Taylor equation which are constants for a given workpiece-tool pair;
τ - the specific time of removal (the so-called cost equivalent time) [min/cm^3];
C_Σ - the total cost of the operation [e.g. HUF]
c_w - specific cost of workpiece [e.g. HUF/min];
$R=R(d,w,f_z)$ - a state variable that is a monotonous function of the cutting parameters d, w, f_z [cm^3/min];
C_T - a complex feature of the tool in the case of a given workpiece;
C^*_t - the total cost connected with the use of tool [e.g. HUF],
N_e - number of tool edges changeable.

The optimization process will be carried out in the space of state (Q, R) with the following characteristic constraints:

$$Q_{\min} = d\, w\, f_{z\min}\, z \frac{v_{\min}}{\pi D_c}, \quad Q_{\max} = d\, w\, f_{z\max}\, z \frac{v_{\max}}{\pi D_c},$$

$$R_{\min} = R(f_{z\min}), \quad R_{\max} = R(f_{z\max}),$$

$$Q_{\min} \le Q(d, w, f) \le Q_{\max}, \tag{13}$$

where d and w are previously fixed. Other constraints have similar shape in the space of state (Q,R), but in case of face milling the power constraint has the following form:

$$\begin{aligned} P_a &= F_c\, v, \quad F_c = k_c\, d\, w\, f_z \frac{z}{D_c \pi}, \\ P_a &= \frac{k_c\, d\, w\, f_z\, z}{D_c\, \pi\, 60 \cdot 10^3}\, v = K\, Q \quad [kW], \end{aligned} \tag{14}$$

where K is a tool-workpiece specific constant and its value can be found in any tool catalogue [3], P_a is the allowable cutting power, which is represented as a vertical straight line.

Similarly, the *tool deflection* δ_f can be expressed in the plane (Q,R) in the following form:

E - *Young's* modulus of the tool material,
L - tool length,
D_c - cutter diameter,

$$\delta_f = \frac{F_c\, L^3}{3\, E\, I}, \quad I = \frac{\pi D_c^4}{64}, \tag{15}$$

$$\delta_f = \frac{64}{3\pi E}\frac{F_c L^3}{D_c^4} = \frac{64}{3\pi^2}\frac{E L^3 k_c d w f_z z}{D_c^3}, \tag{16}$$

where f_z can be expressed as:

$$f_z = \left(\frac{R}{C_s}\right)^{\left(\frac{-1}{y_v - 1}\right)} \tag{17}$$

then let us substitute f_z into (16):

$$R = C_T\left(\frac{3\delta_a D_c^3 \pi^2}{64 E L^3 k_c d w z}\right)^{1-y_v} = C_s C_\delta \delta_f^{1-y_v} \tag{18}$$

which can be represented in the plane (Q,R) as a horizontal line.

3. Typical decision making processes in the space of state (*Q, R, τ*)

In the case of face milling the same typical decision making processes can be carried out as in the case of rough turning. The shapes of the allowable power and the tool deflection constraints are vertical and horizontal straight lines. According to definition, the optimization domain is considered regular if the straight lines named valley line and optimum line are within the domain. The Figure 2 shows a regular optimization domain with the main constraints.

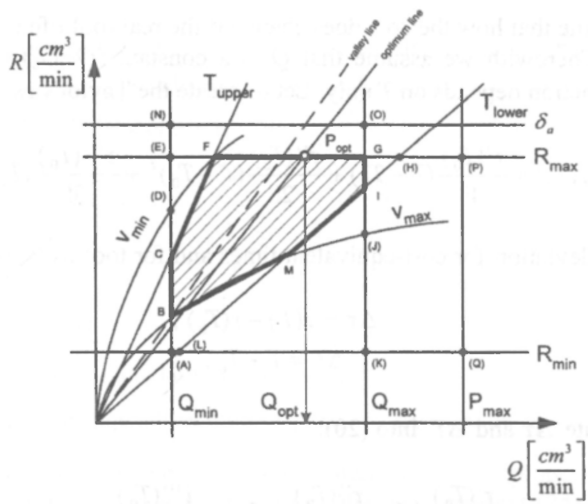

Fig. 2.: The constraints and the domains having a chance to be optimal in the space of state (τ, Q, R)

The direct task of operation element planning can be characterized by the following algorithm:
(1) Tool selection;
(2) Determination of the objective function and the constraints (fixing the depth of cut and width of cut in a heuristic way is also included);
(3) Optimization (Computing Q_{opt}, R_{opt});
(4) Planning the tool motions;
(5) Time computations;
(6) Post-processing (determining f_z).

4. Cost influence of tool-life's dispersion

On account of inhomogeneity of workpiece materials, in practice tool life has some dispersion. We prescribed an optimal tool life, but shop floor practice may experience difference between the prescribed and real tool-life. This dispersion may effect more or less cost.

Let us check that how this dispersion influences the total cost function (9). Let us express the total cost-equivalent time function as a function of Q and T:

$$\tau(Q,T) = \frac{1}{Q} + \frac{t_t^* + t_c}{TQ}, \tag{19}$$

where,

t_t^* - the so-called "tool cost equivalent time" [min],

t_c - the average time necessary for changing a tool edge [min].

We will examine that how the cost does change if the real tool life is not the prescribed T_0 but different. Therewith we assume that Q is a constant ($Q=Q_0$) therefore the cost-equivalent time function depends on T only. Let us create the Taylor's expansion of τ at T_0:

$$\tau(T) = \tau(T_0) + \frac{\tau'(T_0)}{1!}(T - T_0) + \frac{\tau''(T_0)}{2!}(T - T_0)^2 + \frac{\tau'''(T_0)}{3!}(T - T_0)^3 + \dots \tag{20}$$

The absolute deviation for cost-equivalent times and for tool lives, respectively, are as follows:

$$\Delta\tau = \tau(T) - \tau(T_0), \tag{21}$$

$$\Delta T = T - T_0 \tag{22}$$

Let us substitute $\Delta\tau$ and ΔT into (20):

$$\Delta\tau = \frac{\tau'(T_0)}{1!}\Delta T + \frac{\tau''(T_0)}{2!}(\Delta T)^2 + \frac{\tau'''(T_0)}{3!}(\Delta T)^3 + \dots . \tag{23}$$

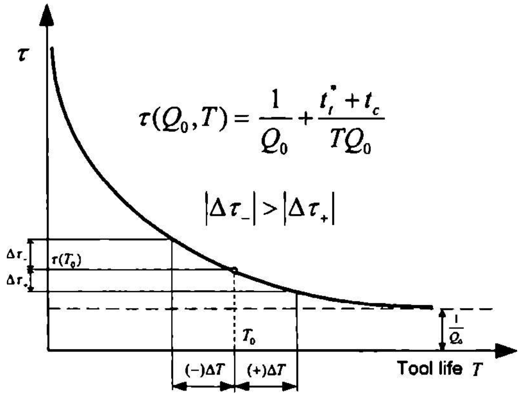

Fig. 3.: The cost equivalent time function as a one variable function of T (tool life)

We are finding $\frac{\Delta\tau}{\tau}$ relative difference of cost-equvivalent times as a function of $\frac{\Delta T}{T_0}$
After executing derivations, (22) can be rearranged as follows: (the mathemetical details can be found in the Appendix A):

$$\frac{\Delta\tau}{\tau} = -\frac{1}{1+\frac{T_0}{t_t^* + t_c}} \frac{\frac{\Delta T}{T_0}}{1+\frac{\Delta T}{T_0}}. \tag{24}$$

If the optimization domain is regular, we can do further reduction, because in this case $T_0 = \left(t_t^* + t_c\right)\left(q-1\right)$. After the substitution we will get the following: (the mathemetical details can be found in the Appendix B)

$$\frac{\Delta\tau}{\tau} = -\frac{1}{q} \frac{\frac{\Delta T}{T_0}}{1+\frac{\Delta T}{T_0}}. \tag{25}$$

According to (24) we can claim that if the optimization domain is regular then relative cost deviation does not depend on t^*_t and t_c but it depends on $q = \frac{1}{m}$ (empirical quasi-constants of Taylor equation) only. Therefore there is a direct proportionality between *m* and relative difference of cost-equivalent time. We can represent (24) easily by means of MatLab software package. Fig. 4 shows that, if the relative tool life dispersion 40% then cost-equivalent time difference is 18% only! (10% tool life dispersion causes 2% cost-equivalent time difference only.)

The main objective of this investigation is that if we have an allowable relative cost deviation domain, we can also define a relative tool life deviation domain. If at shop floor level we experienced more tool life deviation than the prescribed one during machining, we can not use any static optimization, we must use dynamic optimization strategies.

5. Conclusions

A new approach to face milling optimization based upon the intensity of stock removal has been presented. The applicability limits of the new method were described with mathematical formulas considering the stochastical characteristic of the tool life. Continuing work will focus on finishing and semi-finishing of the end milling operation and determination of the cutting force.

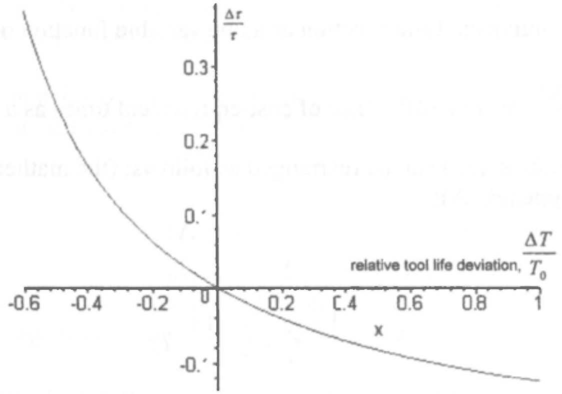

Fig. 4.: Relative cost equivalent time function difference ($\Delta\tau/\tau$) versus relative tool life deviation ($\Delta T/T$), where m= 0.25

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APPENDIX A

Mathematical details of relative cost difference in general case

Let us execute the derivation in (22) then we will have the following:

$$\alpha = \frac{t_l^* + t_c}{Q_0},$$

$$\Delta\tau = -\frac{\alpha}{T_0^2}\Delta T + \frac{\alpha}{T_0^3}\Delta T^2 - \frac{\alpha}{T_0^4}\Delta T^3 + \ldots + \frac{\alpha}{T_0^{n+1}}\Delta T^n, \tag{26}$$

$$\Delta\tau = \frac{\alpha}{T_0}\left[-\left(\frac{\Delta T}{T_0}\right) + \left(\frac{\Delta T}{T_0}\right)^2 - \left(\frac{\Delta T}{T_0}\right)^3 + \ldots + (-1)^n\left(\frac{\Delta T}{T_0}\right)^n\right],$$

$$\Delta\tau = \frac{\alpha}{T_0}\left[\sum_{i=1}^{n}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n\right].$$

Let us determine the relative cost difference, we have:

$$\frac{\Delta\tau}{\tau} = \frac{\frac{\alpha}{T_0}}{\frac{1}{Q_0} + \frac{\alpha}{Q_0}}\left[\sum_{i=1}^{n}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n\right] = \frac{t_l^* + t_c}{T_0 + t_l^* + t_c}\left[\sum_{i=1}^{n}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n\right], \tag{27}$$

which can be simplified to the next form:

$$\frac{\Delta\tau}{\tau} = \frac{1}{\frac{T_0}{t_l^* + t_c} + 1}\left[\sum_{i=1}^{n}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n\right]. \tag{28}$$

We will limit our further investigations that case where $\left|\frac{\Delta T}{T_0}\right| < 1$, which means $-1 < \frac{\Delta T}{T_0} < 1$. The summation in (27) is a convergent, alternating geometrical series therefore, if n goes to infinity we will get:

$$\sum_{i=1}^{n}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n = -\frac{\Delta T}{T_0} + \left(\frac{\Delta T}{T_0}\right)^2 - \left(\frac{\Delta T}{T_0}\right)^3 + \left(\frac{\Delta T}{T_0}\right)^4. \tag{29}$$

According to the summation rule of geometrical series $a_1 = -\frac{\Delta T}{T_0}$ and $q = -\frac{\Delta T}{T_0}$, therefore the addition formula will be as follows:

$$\sum_{i=1}^{n\to\infty}(-1)^n\left(\frac{\Delta T}{T_0}\right)^n = \frac{a}{1-q} = \frac{-\dfrac{\Delta T}{T_0}}{1+\dfrac{\Delta T}{T_0}}. \tag{30}$$

Let us substitute (29) into 27:

$$\frac{\Delta\tau}{\tau} = -\frac{1}{1+\dfrac{T_0}{t_t^* + t_c}}\,\frac{\dfrac{\Delta T}{T_0}}{1+\dfrac{\Delta T}{T_0}}. \tag{31}$$

APPENDIX B

Reduction possibilities in case of regular optimization domain

The optimization domain is regular, if the straight lines of valley line and optimum line are within the regular domain. In this case the optimum tool life can be expressed as:

$$R = MIN(\partial\tau/\partial Q)$$

$$\frac{\partial\tau}{\partial Q} = \frac{1}{Q^2} + \frac{(q-1)Q^{q-2}}{R^q} = 0; \tag{32}$$

from which:

$$R = (q-1)^{\frac{1}{q}}\,Q \tag{33}$$

The gradient of optimum line is $(q-1)^{\frac{1}{q}}$ and $C_T = \left(\frac{T_0}{t_t^* + t_c}\right)^{\frac{1}{q}}$ for tool life. In case of regular optimization domain this two gradient will be equal:

$$\left(\frac{T_0}{t_t^* + t_c}\right)^{\frac{1}{q}} = (q-1)^{\frac{1}{q}}, \tag{34}$$

and we can express T_0 for it:

$$T_0 = \left(t_i^* + t_c\right)\left(q - 1\right) \tag{35}$$

Let us substitute it into (23):

$$\frac{\Delta\tau}{\tau} = -\frac{1}{1 + \frac{\left(t_i^* + t_c\right)\left(q - 1\right)}{\left(t_i^* + t_c\right)}} \frac{\frac{\Delta T}{T_0}}{1 + \frac{\Delta T}{T_0}}, \tag{36}$$

$$\frac{\Delta\tau}{\tau} = -\frac{1}{q} \frac{\frac{\Delta T}{T_0}}{1 + \frac{\Delta T}{T_0}}. \tag{37}$$