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SYMBOLIC ALGEBRAIC COMPUTATION AS A WAY FOR MODELLING KINEMATICAL TASKS THEMSELVES

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Abstract The paper introduces the symbolic algebraic computational capability of Surface Constructor, a software tool for gear investigation. Use of symbolic expressions in the kinematical model gives a higher freedom in constructing and modifying the model. Modelling in this manner we can model the kinematical model itself and can make a kinematical modelling shell. The advantages of applying symbolic algebraic computation are discussed and an example on determining the working surfaces of a hypoid pair and analysing the pattern of connection is presented.

Keywords: Symbolic algebraic computation, Kinematical modelling, Design of hypoid gear

1. Introduction

Modelling complex kinematical systems such as multi degrees of freedom robots or multi axes machine tools needs a freedom in definition of the relative motions and positions of parts. This task becomes especially interesting at simulating work of hobbing or teething machines. It would be the best to have a general modelling tool that gives the maximal possibilities to define the relative kinematical relations and the applied tool-surfaces, too. The well-known CAD and CAM applications apply numerical representations of required kinematical transformation matrices and spline surfaces. This method gives fairly good resolution at modelling fixed architectures with simple translations and rotations. However, in more complex situations there is a need for applying algebraic expressions as transformation matrix elements or in the parametric surface vector. This approach is valid when the task is the modelling of contacting of mating hypoid gear surfaces or producing the machined surface cut by a conical hob. To covering all these situations the best method is to apply symbolic representation and symbolic algebraic computation. Symbolic computation is known as one of the areas of applied artificial intelligence. Use of symbolic expressions in the model gives a higher freedom in constructing and modifying the model. Modelling in this manner we can model the kinematical model itself and can make a kinematical modelling shell. The paper presents a developed design tool that fulfils the above theory and an example for modelling hypoid gear surfaces generated by the same intermediary generating cone.

2. Symbolic algebraic computation in the Surface Constructor software

One of the requirements of today gear-connection investigator software is a powerful geometric modeller, which can give the engineer the maximum freedom to build the kinematical model of the analysed cutting machine or the gearing. The Surface Constructor (SC) software developed by the author has these characteristics. SC applies the 'Reaching Model', a hybrid theory that can produce the enveloped surface and can detect all the types of local undercuts and global cuts and applies symbolic algebraic representation and computation to give the maximum freedom in kinematical modelling [1]. The use of computers for symbolic algebraic computation is not as well known as using for numerical computations.

What is symbolic computation? Using symbolic computation we can enter not only numbers but algebraic expressions, too. The computer manipulates the expressions, simulates hand algebra, uses algebraic identities and so on and gives the result: another expression or group of expressions. We may read articles or books discussing general symbolic software, for example REDUCE, MACSYMA, CAMAL, LAM, FORMAC, SYMBOL, and so on [2], and articles about special software application including symbolic computation module [3]. SC is the member of the last mentioned group.

The set of algebraic manipulations performed by a general symbolic software is big: basic operations: +,-,*,/; manipulation of parentheses; cancelling common factors; factorisation; manipulating functions and expressions, variables and simple numbers; performing sequence of assignments; the use of trigonometric and logarithmic identities; derivation and integration of expressions; simplifying and so on.

SC can perform a suitable selected subset of above mentioned manipulations only: basic operations, manipulation of parentheses, manipulation of SIN, COS, TAN, SQRT functions, manipulation of expressions, variables and numeric values, performing sequence of assignments, simplifying expressions and using trigonometric identities. The symbolic module in SC can compute multiplied matrices and vectors, can generate the inverse matrix in symbolic form because these manipulations and the use of trigonometric identities are very frequent in problems resolved by SC.

The symbolic computations are needed in the following circumstances:

- to generate the transformation matrix and inverse transformation matrix of two neighbouring systems of co-ordinates,
- to produce the resultant matrix from one system of co-ordinates to another using sequence of matrix multiplication,
- to produce the generating surface using matrix-vector multiplication (given the matrix of the generating motion and vector-scalar function of space curve),
- to produce the resultant matrix from the selected system of co-ordinates to the world system of co-ordinates (to give an animated, moving picture of the modelled problem).

3. An example of realisation of a symbolic operation

For example let us examine one of the most frequently used symbolic operation. We have SIN and COS function for sum of angles and we want to factorise it. We have to use one of the well-known trigonometric identities, for example SIN(A+B) SIN(A)*COS(B)+SIN(B)*COS(A). In a more complicated situation A or B may be a complex symbolic expression involving terms and factors of constants, variables and functions. So we have to give a general and exact resolution. For this purpose it is excellent to give the rules of transformation using a sequence of syntax diagrams. In order to spare memory and running time a one byte coding had been applied. A constant, a variable, a function or an expression may substitute for a one byte code. In this manner we can get a very quick program: it can produce a 20 line matrix expression in an imperceptible amount of time. The meanings of the symbols +,-,* and / are obvious. Hereafter S will be the code for SIN, C for COS.

Unfortunately we can only give here a very short overview of realisation because of page limit of the paper.

The steps of the used process are:

1. to find an occurrence of a function of angle-sum in the given symbolic expression, if there is,
2. to produce the transformed function of sum with its environment,
3. to substitute the function of sum and its environment for transformed function of sum with its environment,
4. to repeat the above steps.

Fig.1. on the next page shows a few syntax diagrams related to the earlier discussion.

4. Demonstration of symbolic computation in SC

In the next the fist example will demonstrate the symbolic data handling, manipulation and calculation in the SC program. Then, in the second example, a fairly complex geometrical problem will be resolved by the SC: determining the two connecting surfaces of a hypoid gears using an intermediary generating cone surface. By this example we may get some impression about modelling and visualisation power of SC. Among features not discussed here there are visualisation of path of moving, space of relative speed and acceleration, curvature parameters, animation of the enveloping process, visualisation of R-Φ functions that are special features of SC. The software is capable to determine undercuts and calculate the unknown enveloped surface using local or global calculation.

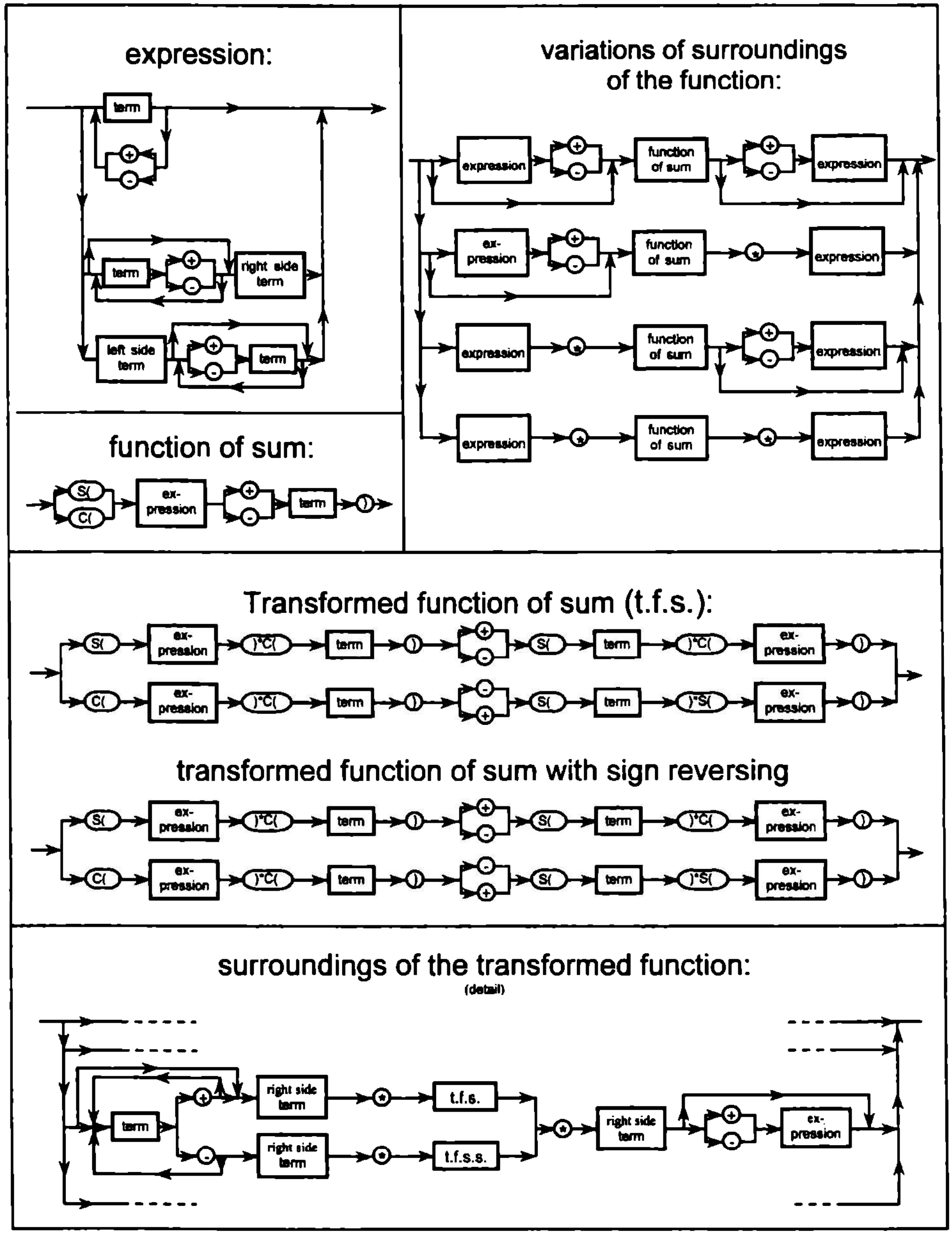

Fig. 1. Syntax diagrams

The symbolic computation starts with entering the generating curves or surfaces by parametric expressions. The input panel for the generating surface is shown in Fig. 2.

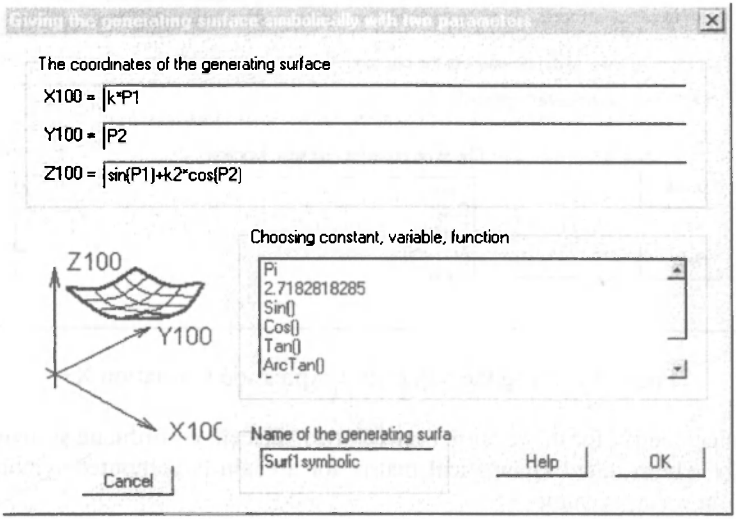

Fig.2. Entering a two-parametric symbolic generating surface

The kinematical relations can be defined by a chain of co-ordinate systems. The relation between two adjacent systems can be entered on a panel like shown in Fig.3.

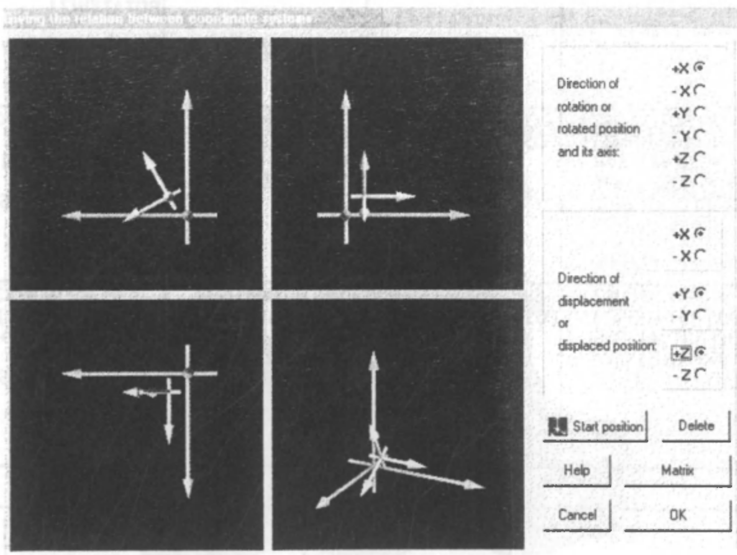

Fig.3. Selecting the directions of translation and rotation

The symbolic expression for position or time-parameter dependent motion have to be assigned in every selected direction using a window similar to the next shown in Fig.4.

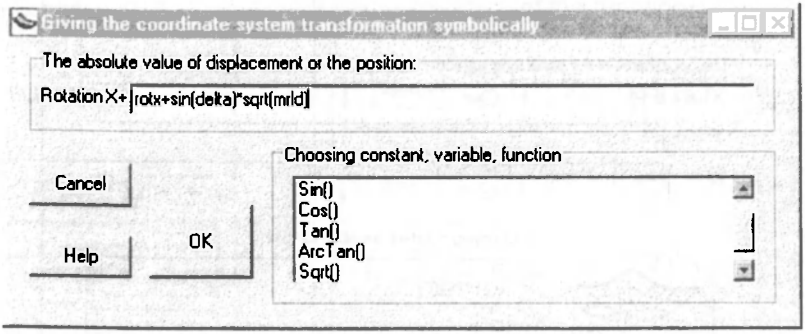

Fig.4. Assigning the parametric expression to rotation X+

The kinematical matrix for the relation between two adjacent co-ordinate systems generated automatically. The resulted kinematical matrix for a chain is computed symbolically in a snap. Fig.5. shows an example.

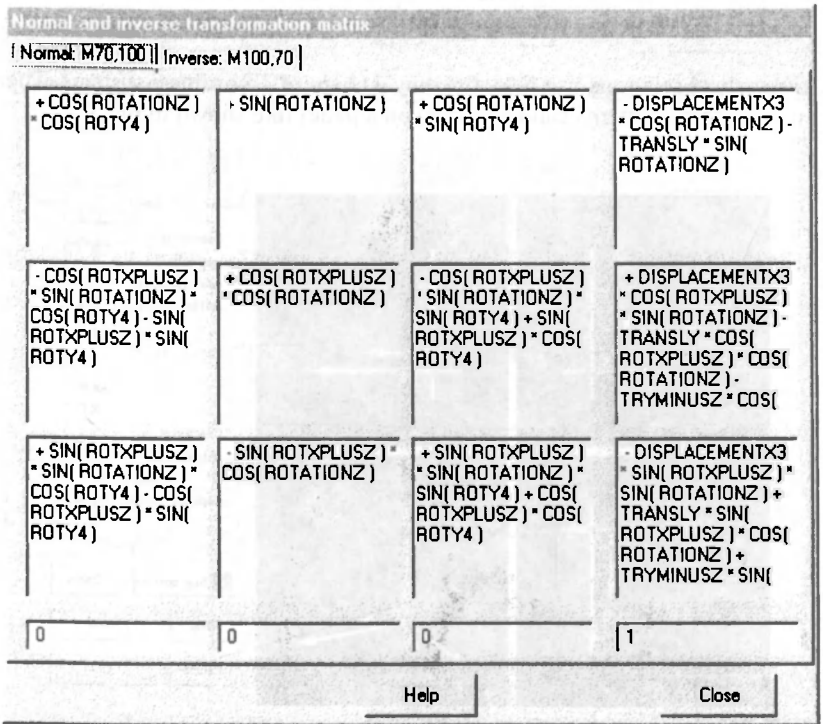

Fig.5. Result of a kinematical matrix concatenation

After building up the kinematical model symbolically the value entering follows. This is important for numerical calculations, to determine the second enveloped surface, or visualise the model. A comfortable panel helps this task, see below in Fig.6.

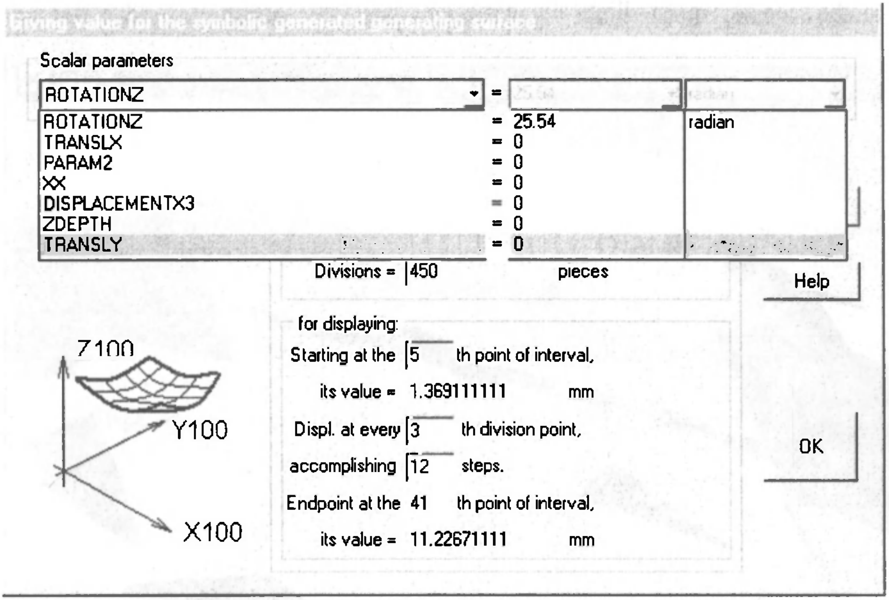

Fig.6. Assigning numerical values to parameters

In the next the result of modelling a hypoid gearing will be presented. The generated connecting surface parts are drawn in Fig.7.

Fig.7. Motion of the hypoid gears can be animated

The generation of the two hypoid gears was made using an intermediary generating cone. This cone envelops the hypoid gears. The cone contacts to one of the hypoid gears in a curve as shown in Fig.8. The two contacting lines intersect each other on the cone. This intersection point will give the centre of the point-like connection of the two hypoid gears. This can be seen as a quasi-elliptical pattern.

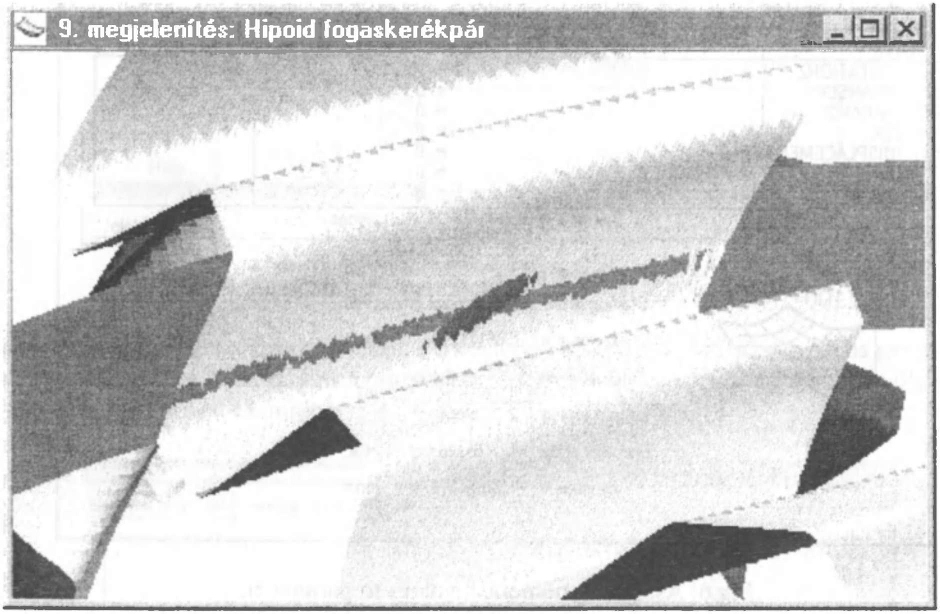

Fig.8. Contacting curves and patterns can be analysed

5. Conclusion

The paper gave a short overview of symbolic computational ability of Surface Constructor, a kinematical modelling software. Using SC, the designer can generate the conjugate surface of a given surface. In this process the relative motion has to be entered and represented by transformation matrices. Both the given surface and the relative motion are used in symbolic algebraic form. The main calculations concerning on kinematical relations are performed automatically in symbolic way. Having substituted concrete parameter values the user can get the kinematical model of a real problem, i.e. contacting gear surfaces or manufacturing worm using a grinding wheel. The task can be analysed changing values of parameters, or the model can be changed itself entering new symbolic expressions for the generating surface and/or the kinematical relations.

Thanks to symbolic algebraic representation and computation it is possible to model the kinematical modelling process itself.

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