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OpTol: Spatial Tolerance Analysis Application

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Abstract. The analysis of manufacturing and assembly dimension chains is indispensable for performing up-to-date part manufacturing and assembly. It will both reduce the manufacturing and assembly costs and will result in a well-grounded body of knowledge and improved level of design. The paper deals with mathematical models for calculating spatial dimension chains and introduces the OpTol Tolerance Calculator software. This application is capable of calculating planar (2D) and spatial (3D) dimensional chains by using the classical worst-case and statistical methods, as well as applying the modern six-sigma tolerancing method. OpTol system also contains a CAD module in order to support engineers in analysing their existing assemblies.

Keywords: 3D tolerance analysis, tolerance calculation, dimension chains, direct linearization method

1. Introduction

For performing up-to-date part manufacturing and assembly, the analysis of dimension chains for manufacturing and assembly is essential. Such an analysis will reduce the manufacturing and assembly costs, on the one hand, and will result in a well-grounded body of knowledge and improved level of design, on the other hand. The build-up and analysis of dimension and tolerance chains play an important role in periods of design, production planning and execution of the manufacturing process. The designer provides information by part drawings giving dimensions and tolerances for planning tasks for technology and material processing.

Beyond determining the geometry of parts, the manufacturing dimension and tolerance chains give feasible manufacturing methods and the possible order of manufacturing processes, as well as the costs of production of the part. The task of assembling dimension and tolerance chains is to determine the relative position of parts needing to be assembled to fulfil the requirements of operation (the function).

2. Fundamentals of dimension and tolerance chains

The dimension and tolerance chain – or simply dimension chain – consists of at least two toleranced dimensions connected together and the resultant dimensions derived from them. The chain used for tolerance calculation is always closed, i.e. comprises the open dimension chain in the dimension chain of the drawing and the resultant dimension. The dimension chain expresses: the chain of dimensions needed to define a part; the relation of a pair of toleranced dimensions; and the operational or assembly location produced by a series of toleranced dimensions. Dimensions occurring in dimension chains are called components**.** The closing or resultant dimension is the term that is worked out last. There can be only one resultant in each tolerance chain.

2.1. Chain types

Dimension chains can be: linear dimension chains, where all of the dimensions are parallel to each other; planar dimension chains, where the dimensions are partially or fully non-parallel but all of them lie in one or more parallel planes; spatial dimension chains, where the dimensions are partially or fully non-parallel and do not lie in one or more parallel planes; and angular dimension chains, where the dimensions are angular and the number of angle legs meet in one corner (see Figure 1).

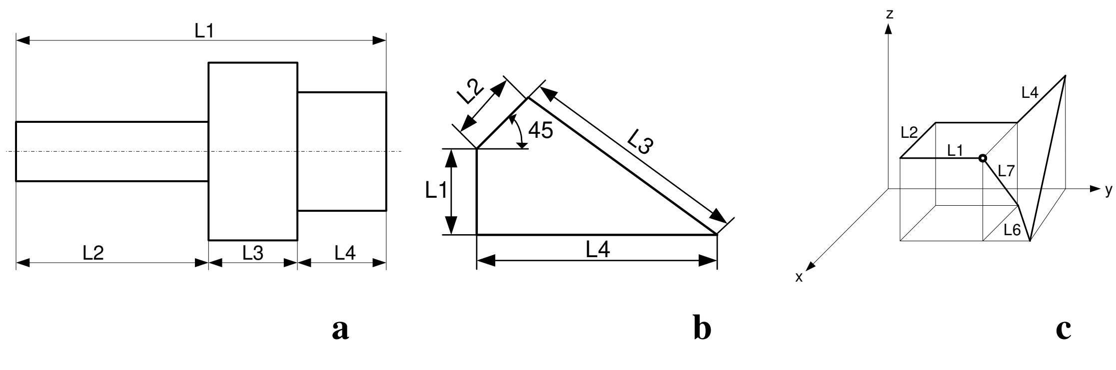


Figure 1. **a**: linear dimension chain, **b**: planar dimension chain, **c**: spatial dimension chain

In different assemblies, several different types of dimension chains can be found and these can connect to each other in different ways. The main characteristic of the serial type of connection is that if one link of the dimension chain changes, then the basis of the next chain will be changed (see Figure 2a). It follows from this that a serial type of chains has a common basis.

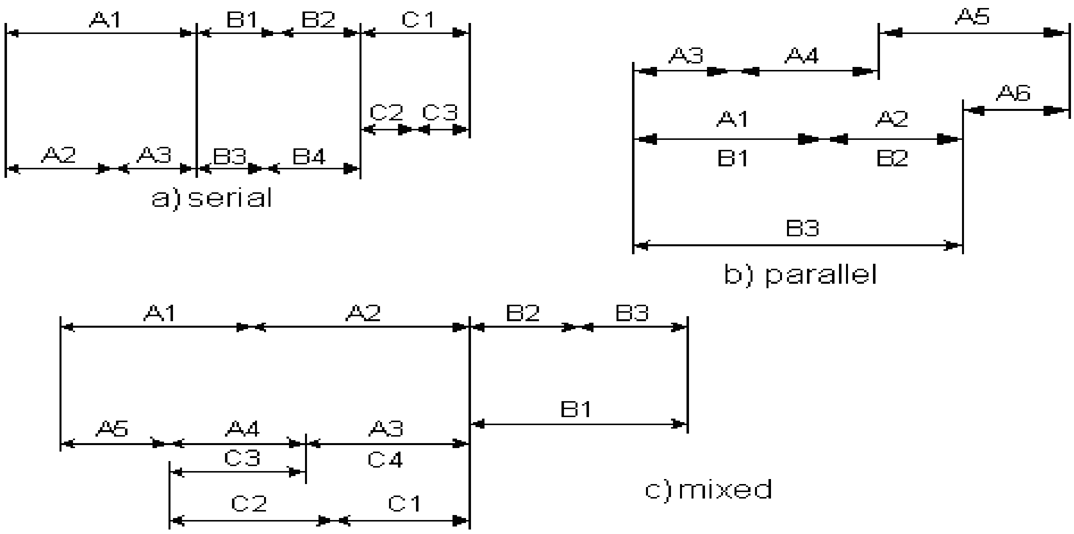


Figure 2. Connection types of dimension chains

2.2. Assembly dimension and tolerance chains

An assembly includes the joining of the connected components, the controlling of their allocation after fitting the corresponding basic surfaces and – if necessary – the correction of the allocation error. An assembly dimension chain is a sequence of dimensions which returns to itself in a determined order. This chain connects the surfaces of the components whose mutual positions are to be determined. The components of the dimension chain are characterized by their nominal values and permissible variations.

2.3. Tolerance analysis and allocation

In tolerance analysis all the component tolerances are known or prescribed and we have to calculate the resulting tolerance. In the case of tolerance allocation, construction requirements determine the assembly tolerance and the unknown component tolerances are to be calculated. We distribute the actual assembly tolerance corresponding to the appropriate components. The design application for tolerance analysis is based on analytical models, which take into account the stack up of tolerances in the assembled components.

3. Engineering calculation methods for assembly tolerances

If the process of part manufacturing is known, the tolerances can be chosen from tables of standard tolerances according to the process elements. In addition to this, the industrial standards often provide useful data for our calculations. We introduce briefly the two current models (see [6]).

3.1. Worst-case model

This model is often called the model of total changeability or calculation of maximum-minimum. The purpose of this method is to determine the assembly tolerance (T_Δ) by means of the summarization of the component tolerances. Each component is assumed to be at its greatest or least dimension; hereby we have the worst assembly limits.

In the case of a *one-dimensional* (linear) dimension chain we have:

$$\mathrm{T}_\Delta = \sum_{i=1}^{n-1} T_i \, , \tag{1}$$

For a *multidimensional* (nonlinear) dimension chain:

$$T_\Delta = \sum_{i=1}^{n-1} \left| \frac{\partial f}{\partial X_i} \right| T_i \, , \tag{2}$$

where X_i means the nominal component dimension, $f(X_i)$ is the assembly function describing the resulting dimension of the given assembly and T_i denotes the width of the tolerance zone for the i-th dimension. The partial derivatives represent the sensitivity of the assembly tolerance regarding the changes in the independent component dimensions.

Equation (2) is not obvious at all. First observe that there is a well defined, analytical connection between the nominal component dimensions and the resulting dimension (closing component):

$$X_n = L_\Delta = f(X_1, X_2, ..., X_i ... X_{n-1}) \, . \tag{3}$$

Components $X_1, X_2, ..., X_i ... X_{n-1}$ are made with tolerances T_1, T_2, ..., T_i..., T_{n-1} , so the resulting dimension L_Δ will have tolerance T_Δ :

$$L_\Delta + T_\Delta = f(X_1 + T_1, X_2 + T_2, ..., X_i + T_i ... X_{n-1} + T_{n-1}). \quad (4)$$

The *(n-1)* variables function L_Δ in Eq. (4) is assumed to have an expansion into the Taylor series, so it is differentiable at any time with respect to each independent variable in the neighbourhood of the point $X_1, X_2, ..., X_i ... X_{n-1}$:

$$\begin{aligned} L_\Delta + T_\Delta = f(X_1, X_2, ..., X_i, ..., X_{n-1}) + \frac{\partial f}{\partial X_1} T_1 + \frac{\partial f}{\partial X_2} T_2 + ... + \frac{\partial f}{\partial X_i} T_i + ... + \frac{\partial f}{\partial X_{n-1}} T_{n-1} + \\ + \frac{1}{2!} \frac{\partial^2 f}{\partial X_1^2} T_1^2 + \frac{1}{2!} \frac{\partial^2 f}{\partial X_2^2} T_2^2 + ... + \frac{1}{2!} \frac{\partial^2 f}{\partial X_i^2} T_i^2 + ... + \frac{1}{2!} \frac{\partial^2 f}{\partial X_{n-1}^2} T_{n-1}^2 + ... \end{aligned} \quad (5)$$

In Eq. (5) the members of second, third, etc. order in the Taylor series can be neglected, because the tolerances T_i are small, and their squares and higher powers are smaller. Subtracting both sides of Equation (5) from (3) we obtain:

$$L_\Delta - (L_\Delta + T_\Delta) = -T_\Delta = -\frac{\partial f}{\partial X_1} T_1 + \frac{\partial f}{\partial X_2} T_2 + ... + \frac{\partial f}{\partial X_i} T_i + ... + \frac{\partial f}{\partial X_{n-1}} T_{n-1} = -\sum_{i=1}^{n-1} \frac{\partial f}{\partial X_i} T_i . (6)$$

Since tolerances T_i are the width of the tolerance zone for the component X_i and T_Δ is the width of the resulting tolerance zone, these numbers must be positive by definition. Multiplying both sides of Equation (6) by (-1) we have to use the modulus of partial derivatives because they can take negative numbers. In this way we have the equation:

$$T_\Delta = \sum_{i=1}^{n-1} \left| \frac{\partial f}{\partial X_i} \right| T_i$$

which is the same as Equation (2).

3.2. Statistical tolerance analysis

In this case the stack up of tolerances shows an analogy with random variations. The measured values X_i belonging to the function $y = f(X_i)$ (this function was described above) contain random errors δX_i. These errors have unknown signs and they vary their dimensions in given bounds. The linear addition of the greatest values of the errors δX_i would result in a too-high stack up. It is quite unlikely that the errors have the same signs and that they take their greatest value at the same time. Deviations can compensate for each other in the summation. Due to this observation we calculate the uncertainty factor δy in terms of Gauss summation law of random errors (instead of linear addition):

$$\delta y = \sqrt{\sum_{i=1}^{n-1} \left(\frac{\partial f}{\partial X_i} \delta X_i \right)^2} . \tag{7}$$

For application of the law errors must be independent, and within their bounds the partial derivatives $\frac{\partial f}{\partial X_i}$ can be considered to be constant values. The practical tolerance limit T_Δ assumes that the components of the dimension chain join each other with the value of the greatest probability within their tolerance area. If the extreme tolerance limits meet, the tolerance limit can be exceeded and we have a rejected assembly.

In the discrete processes of the machine industry the errors of measurements follow a typical discrete distribution. This is *binomial distribution* [2]. Adding numerous independent random variables where the variances of the components are negligible compared with the variance of the sum, we always get a variable of *normal distribution* independently of the distribution of the components.

From the viewpoint of the machine industry the most important distribution is the normal or Gaussian distribution. Although it is a continuous distribution, it is suitable for building a mathematical model of the *variation* of measurements (instead of using binomial distribution). The most typical example of normal distribution arises by measuring, in the case of random errors [2]. The general form of the density function of Gaussian normal distribution is the following:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-u)^2}{2\sigma^2} \right), \tag{8}$$

where x denotes expected value (mean of an infinite number of measured data) and σ is the standard deviation. The graph of the density function (8) is shown in Figure 3.

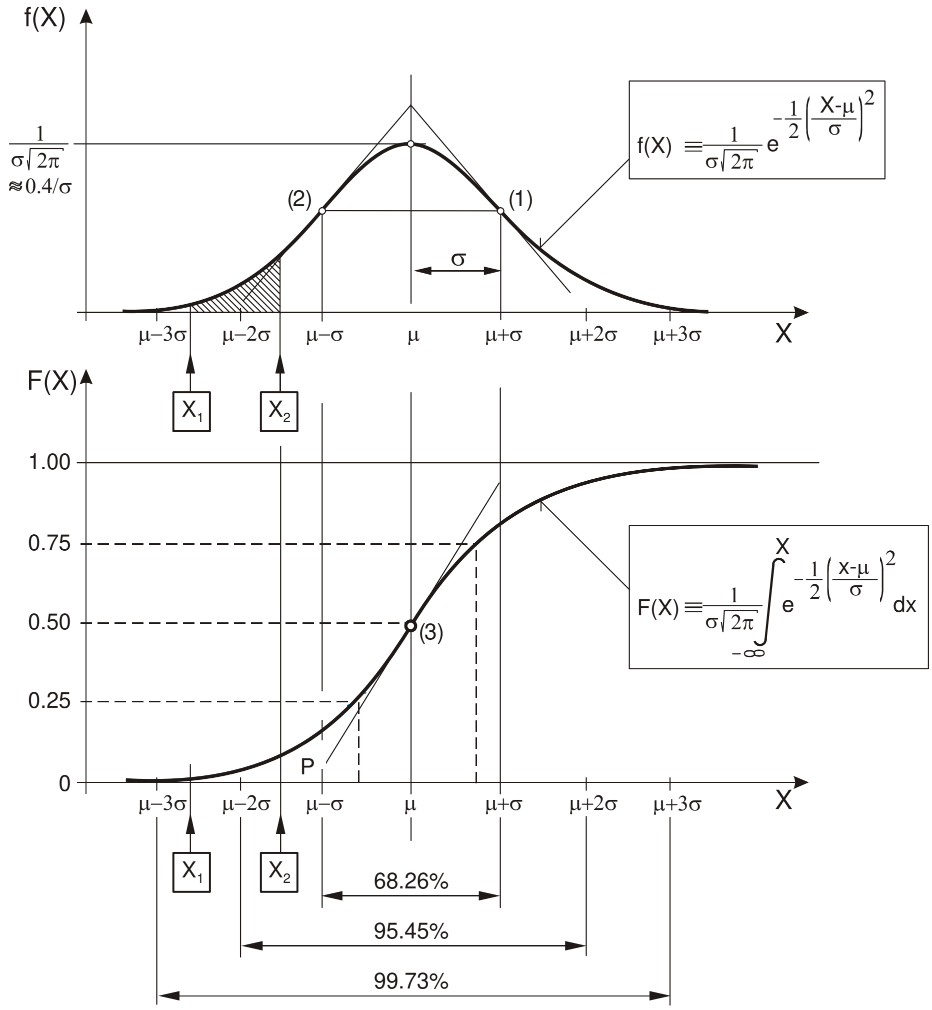


Figure 3. Density function *f(X)* and distribution function *F(X)* in the case of normal distribution. The points denoted by (1), (2) and (3) are inflections [9]

The domain of a normal distributed variable is the real line so an ideal Gauss-curve is situated above the interval [-∞, +∞]. In practice the normal distribution can be considered final and the outer part of the interval [μ-3σ, μ+3σ] is not significant so we can neglect it. It is shown in Figure 3 that 68.26 percent of all possible values of a normal distribution variable lie in the interval [μ-σ, μ+σ], there is 95.45 percent between μ-2σ and μ+2σ and nearly the whole mass (99.73 percent) is settled in the interval [μ-3σ, μ+3σ]. The latter interval width is usually considered to be the 'technological 100 percent'. The normal distribution is completely determined by its two parameters: the expected value (μ) and the standard deviation (σ). We cannot calculate the exact expected value we use; instead we use the most possible value of the measured data given by the mean of measure sequences achieved in sufficiently large number. Similarly we consider empirical deviation instead of theoretical standard deviation.

Following statistical laws, component tolerances are accumulated in a square-root form. We allow for the lowest probability of the worst-case combinations, assuming that the variations of the components are normally distributed. In general, tolerances are supposed to suit a 6σ deviation of the normal distribution. Tolerance of the closing component in an assembly dimension chain is given by the following formulas.

In the one-dimensional case:

$$T_\Delta = \sqrt{\sum_{i=1}^{n-1} T_i^2} \ . \tag{9}$$

In the case of multi-dimensional chains:

$$T_\Delta = \sqrt{\sum_{i=1}^{n-1} \left(\frac{\partial f}{\partial X_i} \right)^2 T_i^2} \ . \tag{10}$$

In a more general case when tolerance distribution differs from ±3σ:

$$T_\Delta = C_f Z \sqrt{\sum_{i=1}^{n-1} \left(\frac{\partial f}{\partial X_i} \right)^2 \left(\frac{T_i}{Z_i} \right)^2} \ , \tag{11}$$

where Z is the required number of standard deviation according to the described assembly tolerance and Z_i denotes the expected deviations of the component tolerances. The correction factor C_f is often taken into account when circumstances differ from the ideal case. Typical values of C_f are 1.4 and 1.5.

3.3. Bounds of common assembly models

In statistical models we assume manufacturing variants which are normally distributed symmetrically about the centre of tolerance limits. These models do not consider possible asymmetry or deformation. Figure 4 illustrates the occurrence of unexpected rejects if we do not take asymmetry into account.

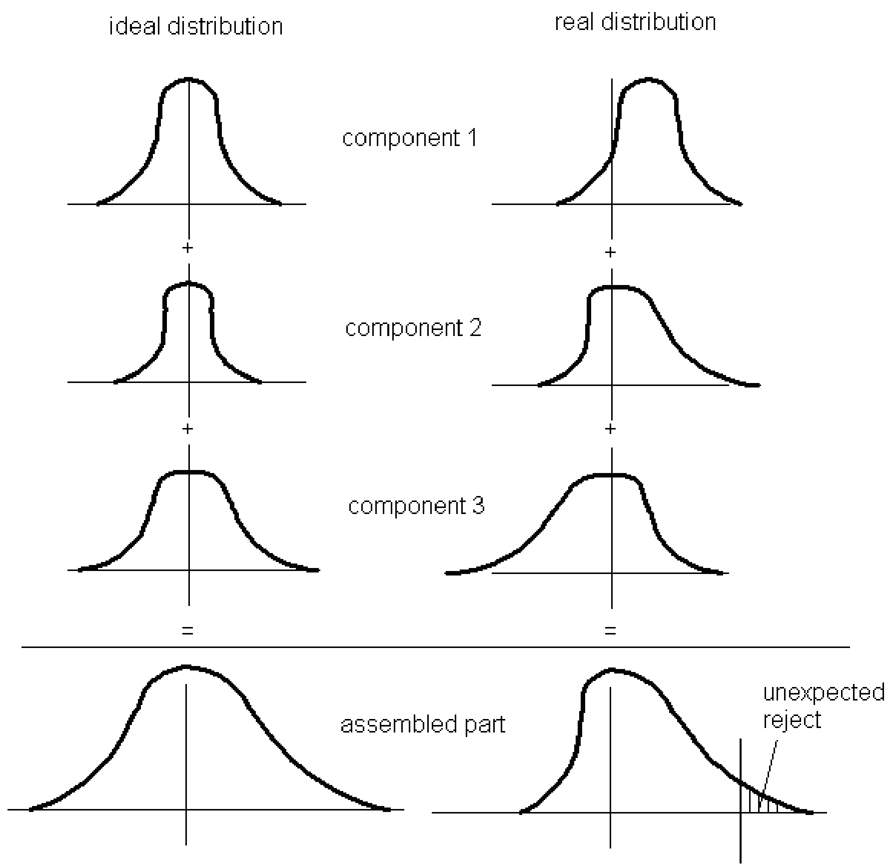


Figure 4. Ideal and real distribution for a three-component assembly

Asymmetric deformation is expressed in shift from the real measurement. It is very dangerous because the variations can stack in the given assembly resulting in an unexpectedly high percentage of rejects. All manufacturing processes show asymmetry, although some processes produce larger variations than others. Asymmetric deformation can come from setup errors, tool wear, etc. Asymmetric deformation occurs in a natural way in some processes, for instance by thermal contraction of the assembly parts cast in die. Deformation in an assembly model is just as critical as the capacity or variance of the process. Furthermore, statistical approaches or genetic algorithms can be applied in case of non-ideal (real) probability distribution of component tolerances [15, 9].

3.4. Motorola 6 sigma model

The more we refine the process control, the more we have decreasing spread of operations and if the distribution of variations is symmetrical, fewer and fewer rejects will occur [12]. Figure 5 shows that if the lower limit (LL) and upper limit (UL) of the dimension are within the ±6σ limits, then we achieved the so-called 'six sigma quality'. If the UL and LL are set at the ±3σ limits, then we have 0.27% reject assemblies. This number does not seem very high, but it means that for one million products we can reject 2700 of them. Setting the UL and LL at ±4.5σ will yield 3.4 ppm rejects (products per million). In the case of ±6σ this ratio is nearly 100%, with only 2 rejected products per billion.

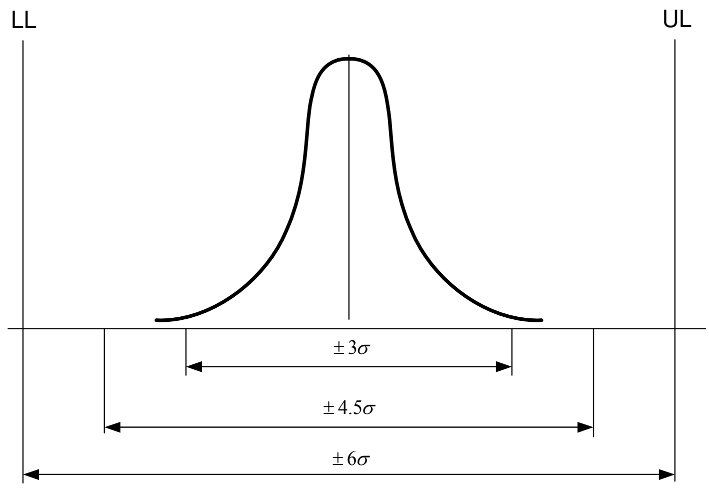


Figure 5. Density function of normal distribution in ±6σ model

It may sound surprising, but 'Six Sigma' is actually the target quality level of today's major manufacturing corporations. It seems easy to achieve this quality level by increasing UL and LL up to the ±6σ limits. But this solution cannot be successful because the UL and LL limits are not arbitrarily chosen; they must meet exacting requirements in the planning and working processes.

3.5. Estimated mean shift

Chase and Greenwood offer a new model for describing assembly tolerance stackup which contains the estimation of the expected asymmetric deformation (see [3]). We call this method 'Estimated Mean Shift Model', since the constructor has to estimate the deformation of each component of the given assembly unit. This is done in the following way: surrounding symmetrically the centre of the tolerance area we define a zone (see Figure 5) which gives the possible position of some dimension of a typical component sequence.

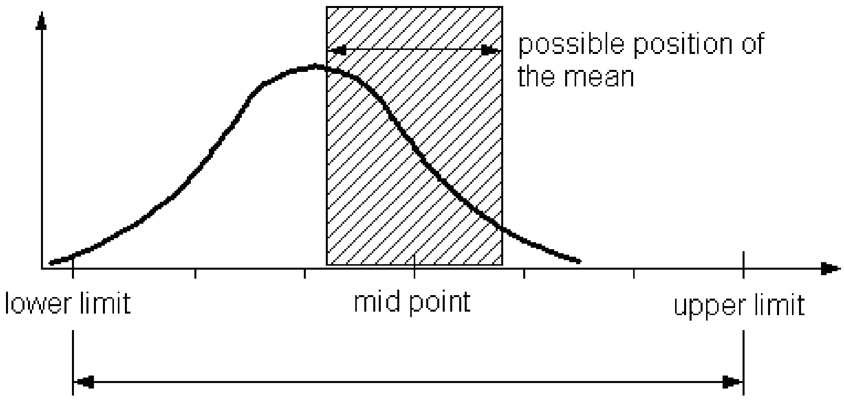


Figure 6. Position of the mean is not exactly known

The centred tolerance zone is given by a proportion of the tolerance area described for the actual component dimension. This number is between 0 and 1. In strictly controlled producing processes it is sufficient to choose a low mean shift factor, e.g. between 0.1 and 0.2. If the process is less known, e.g. in the case of a component supplied by a new business partner, we choose 0.7 or 0.8 in order to allow for some uncertainity.

After estimating the mean shift zone regarding each of the components we calculate the assembly tolerance in terms of the following mathematical model:

$$T_\Delta = \sum_{i=1}^{n-1} \left| m_i \frac{\partial f}{\partial X_i} T_i \right| + \sqrt{\sum_{i=1}^{n-1} ((1-m_i)^2 \left(\frac{\partial f}{\partial X_i} \right)^2 T_i^2)}, \tag{12}$$

where m_i denotes the mean shift factor for the i-th component. The assembly tolerance in Equation (2.12) consists of two parts. The first expression is the sum of the mean shifts which are given as the worst limits. The second part of the formula is the sum of the component tolerances calculated in a statistical way. So we obtain the contributions in the closing assembly tolerance according to the mean shift or deformation and component tolerance or deviation, respectively [3, 4].

Choosing every mean shift factor to be zero, Equation (12) can be reduced to the simple statistical model. In addition we obtain the Worst Case Model if all of the mean shifts are chosen to be 1.

We should mention further advantages of the Estimated Mean Shift Model. Mixed application of the factors assures proper flexibility in a given assembly. Some components may correspond to the worst limit while others may vary to a great extent in accordance with the statistical case. Because of a weakly controlled component we are not constrained to apply the worst case model for the whole assembly unit.

3.6. Effect of the mean shift

Figure 7 demonstrates clearly the effect of the mean shift. The values UL and LL were originally set at the ±6σ limits of the distribution. The mean shift of the arising dimension has been shifted 1.5σ to the right, leaving 4.5σ to remain. Since UL is 4.5σ from the mean, it will yield increasing rejects, i.e. 3.4/2 = 1.7 ppm. It is not a large number, but compared to it the ±6σ case (without mean shift the reject products are 2 per billion) we get nearly a factor of 1000 increase!

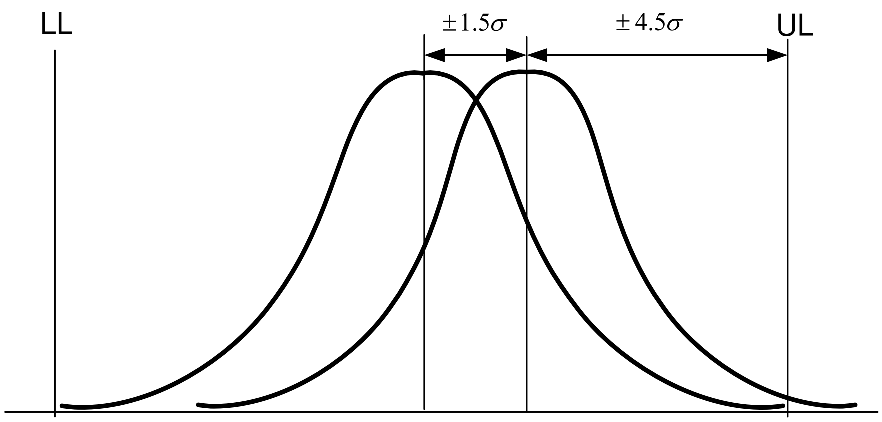


Figure 7. Effect of the mean shift

4. Other tolerance analysis methods

In certain cases other methods are applied in tolerance analysis, especially when the dimensions of the components are not normally distributed. We need to give full distribution as input in order to apply the assembly equation.

The Monte Carlo Simulation and the Method of Moments are useful tools for analysing tolerances if dimensions of the assemblies differ from the normal distribution [5]. The Monte Carlo Simulation generates pseudo-random numbers in order to describe a wide range of distribution curves. Each component receives an input dimension randomly for the assembly equation. After determining the value of the closing assembly variable, it is compared with the described assembly limit. This procedure is repeated again and again and the number of the assemblies proved rejects is divided by the number of the trials to estimate the proportion of the rejected assemblies [7], [8], [13].

The Method of Moments uses the empirical moments of the contributing distributions and the first and second derivatives of the assembly function to find the first four moments of the assembly distribution.

There is an alternative idea requiring a less sophisticated, quicker program, which can be regarded as a mixture of the methods mentioned. This hybrid method applies the Monte Carlo Simulation for generating assembly values in a smaller number. The sample size is usually between 1000 and 5000. The resultant assembly dimensions are used to compute the statistical moments of the assembly distribution and to estimate the percentage of rejected products. With the aid of this trick we avoid the greatest difficulties arising in the Method of Moments since we do not need numerical derivatives and summation of series in order to calculate assembly moments from the component moments. Considering that the sample is in the order of thousand the calculation is extremely simplified compared with the original Monte Carlo Simulation [11].

Constraint networks can also be applied for determining an optimum allocation of tolerances among components of an assembly and at the same time minimizing the total cost of manufacturing [20].

5. Classic methods of solving assembly dimension chains

The problems which can be solved on the basis of the theory of dimension chains are divided into three groups:

- calculation of closing tolerance based on the described tolerances of the components of the dimension chain;
- determination of the component tolerances using the described closing tolerance;
- determination of closing and component tolerances meeting general requirements.

These problems can be interpreted both for the component and the assembly dimension chains. The classical methods for assembly dimension chains are the following: (1) method of total changeability; (2) method of limited changeability; (3) method of selective coupling; (4) method of post fitting; (5) method of adjusting.

The first two methods have been already discussed to some extent previously. In this paper we give more details on total changeability.

5.1. Method of total changeability

In the case of total changeability, assembly can be carried out with randomly selected identical parts, and in this way the closing dimension will always be the prescribed value without adjusting the inserted parts.

If the dimension chain is solved with the method of total changeability, then it is not sufficient to calculate the tolerance values of all the parts, but each part has to be machined within these prescribed tolerance limits. Without this condition it is not possible to consider applying the method of total changeability.

Advantages of this method are:

- assembly is simple and economical because no adjustment or selecting of components is needed,
- the assembly process can be carried out with semi-skilled workers,
- by virtue of total changeability, assembly processes can be carried out in parallel factories,
- the assembly process can be carried out on an assembly line,
- greater ease in managing machining of spare parts: we can assemble each part onto the product without adjustment and fitting.

The main disadvantage of the method is that part machining requires high accuracy. The method of total changeability is the most economical if the dimensions of chain are very accurate, but the number of components is low [14]. It follows from this that the method of total changeability is used in mass production in the case of high precision and a low number of components.

5.2. Method of limited changeability

Increasing the precision of machining tolerances acts upon the production cost.

Therefore increasing the precision requirements would be limited by the costs and assembly precision.

When calculating the tolerances with the method of total changeability, the theoretical starting point is that particular components are machined with limit dimensions. Components with limit dimensions of opposite direction can be assembled and will meet the precision requirements. During product manufacturing a very small percentage of components are prepared to their limit dimensions. Therefore considering the variations of machined dimensions the part tolerances are extensible and in this way production can be more economical, except that a certain number of products will exceed the prescribed tolerance limits (a higher rejection rate) [14].

Using the method of limited changeability, it is not possible to ensure the resultant dimensions between the prescribed limits. Applying the theory of the probability calculation we can increase the tolerance values of certain components, but we risk that rejected products will pass the tolerance limits. Increasing the tolerance values leads to more economical part production but marginally increases the reject rate.

Taking these factors into account, we can generally say that the method of limited changeability can be applied if the dimension chain consists of several components and a tight tolerance is prescribed only for the closing dimension. As a consequence of choosing tight closing dimension, it is possible to increase the tolerances of the individual dimension chain components, which reduces the machining cost.

In the case of the method of limited changeability, dimension chain solutions work on the principle that dimensional deviations of chain links as well as the summation of these deviation values have a random character, therefore the rules of probability calculation have to be applied. According to these rules, the boundary values of the closing dimension can be calculated with the summation of the regular and random errors of the links [14].

5.3. Indices of process capability

To measure process capability, there are two indices used in modern industrial practice:

C_p process capability index,

C_{pk} C_p adjusted for mean shift.

Process Capability Index:

$$C_p = \frac{UL - LL}{6\sigma}$$

Cp adjusted for k:

$$C_{pk} = C_p(1-k)$$

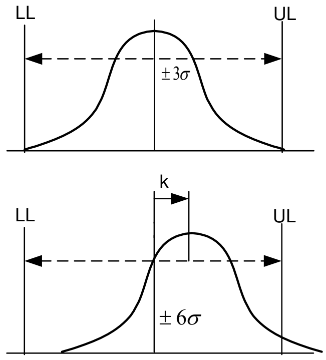


Figure 8. Indices of process capability

The value of capability index C_p is 1.0 only if the limits LL and UL are exactly on the 3σ boundaries of the standard deviation of dimensions. At this time, using the general assumptions of tolerance analysis, all the tolerances correspond exactly to 3σ. If LL and UL correspond to ±6σ, then C_p = 2.0, which matches the quality level 6σ. The previous explanation shows that C_p is a good indication of the quality level, but the mean shift is not taken into consideration.

C_{pk} adjusts the value of C_p, taking the mean shift into consideration. It can be seen from Figure 8 that C_{pk} is *(1-k)* times C_p, where *k=[0..1]*. If the mean shift is 25%, then *k=0.25,* i.e. the distance from the mean of UL and LL, thus the process capacity sinks to 75%.

C_p expresses how close the limits UL and LL are to the process capability ±3σ supposing symmetric distribution; while C_{pk} represents how close the nearest UL and LL limits are supposing non-symmetric distribution.

The model presented here is the 'Six sigma program' developed by Motorola Corporation. This model also takes the qualitative mean shift observed during the mass production of assemblies into account.

Instead of the relationship $T_i=3\sigma_i$, the resultant tolerance can be calculated as:

$$T_i = 3\, Cp_i\, \sigma_i\,, \tag{13}$$

which meets higher quality requirements. Taking the mean shift into account, the previous formula with the substitution of C_{pk} is as follows:

$$T_i = 3\, Cpk_i\, \sigma_i\,, \tag{14}$$

moreover:

$$\sigma_i = \frac{T_i}{3\,Cpk_i} \,. \tag{15}$$

Since *Cpk* is less than *Cp*, the estimated standard deviation σ_i will be greater.

In the case of mass production, the mean of the process can be shifted, e.g. as a consequence of tool wear or thermal expansion. In the long term, the aim of Motorola's Six Sigma principle is to achieve the quality level of 4.5σ. In order to realise it, the quality level of 6σ is to be aimed at in the short term:

$$\text{Short term: } T_i = 3\,Cp_i\,\sigma_i = 3\cdot 2.0\cdot\sigma_i = 6\sigma \,. \tag{16}$$

$$\text{Long term: } T_i = 3\,Cpk_i\,\sigma_i = 3\cdot 2.0\cdot(1-k)\,\sigma_i = 4.5\sigma \,. \tag{17}$$

If the mean shift is less than 0.25 ($k<0.25$), then in the long term, a quality level higher than 4.5σ can also be reached. If $k>0.25$, then the 4.5σ cannot be maintained.

6. Direct linearization method for analysing 3D mechanical assemblies

Kinematic tolerance analysis methods have had an extensive literature in the last five years. Kyung and Sack have successfully applied a nonlinear kinematic tolerance analysis algorithm for planar mechanical systems comprised of higher kinematic pairs [10], and additionally a combination of the direct linearization method and a kinematic error analysis was presented by Wittwer and Chase [19]. Joskowicz and Sacks introduced a new model of kinematic variation, called kinematic tolerance space, that generalizes the configuration space representation of nominal kinematic function [16]. Anselmetti et al. developed a new functional tolerancing method for analysing 3 dimensional variations of mechanical assemblies utilizing a solver implemented in Microsoft Excel [1].

In general, the kinematic constraints for a 3D mechanical assembly can be described by means of closed vector loops. The vector loop is traversed from the starting point to the end point of the mechanism, finally the cyclic translations and rotations will sum to zero. As a last step, the coordinate system of the end point has to be made congruent with the one at the beginning by means of a rotation. The method of vector loops derives from the 2D calculation method, as its spatial extension [5]. In the 3D case the equations of the system are much more complex. At this time it is highly practical to represent the rotation and translation constraints in matrix form. The closed vector chain can be expressed as a product of transformation matrices representing the constraints. To describe transformation from point *i-1* to point *i* of the mechanism, a combination of three rotation matrices and one translation matrix is necessary in the most general case. The problem can be simplified if we carry out translations always along the local *x*-axis. For 3D rotational transformations, the following matrices can be used:

$$[R_x]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0' & \cos\varphi_x & -\sin\varphi_x & 0 \\ 0' & \sin\varphi_x & \cos\varphi_x & 0 \\ 0' & 0 & 0 & 1 \end{bmatrix}, [R_y]=\begin{bmatrix} \cos\varphi_y & 0 & \sin\varphi_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi_y & 0 & \cos\varphi_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, [R_z]= \begin{bmatrix} \cos\varphi_z & -\sin\varphi_z & 0 & 0 \\ \sin\varphi_z & \cos\varphi_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{18}$$

For translation it is assumed that the translational vector is always parallel to the local *x*-axis:

$$[T]=\begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{19}$$

With these matrices, the kinematic constraints of the assembly can be written in form of the following equation:

$$[R_1][T_1][R_2][T_2]...[R_i][T_i]...[R_n][T_n][R_f] = [I], \tag{20}$$

where $[R_i]$ is the product of rotation matrices at joint *i*; $[T_i]$ is the translation matrix at joint *i*; $[R_f]$ is the rotation matrix required to bring the loop to be congruent at the last joint; and I is the identity matrix. Equation (20) is a series of rotations and translations to transform the local coordinates from vector-to-vector to the end point via the joints representing the mechanism. At each joint, the rotation matrix $[R_i]$ is a product of rotation matrices, which aligns the local *x*-axis with the direction of the next vector. Then the transformation matrix $[T_i]$ contains only one translation value *L* along the local x-axis, indicating the length of the current vector. Equation (20) can be decomposed into six independent non-linear equations. Since the nominal dimensions are much greater than their tolerances, the solution can be obtained through linearization. Six equations describe the loop variation in the global x,y,z and Θ_x, Θ_y, Θ_z directions, as follows:

$$\delta H_i = \sum_{j=1}^{n} \frac{\partial H_i}{\partial x_j} \delta x_j + \sum_{k=1}^{m} \frac{\partial H_i}{\partial x_k} \delta u_k \qquad \left(i = x, y, z, \Theta_x, \Theta_y, \Theta_z\right), \tag{21}$$

where δx_j are variations in the manufactured dimensions and angles (j = 1…n), δx_k are variations in the dependent assembly variables (k = 1…m) and δH_i is the resultant assembly variation in the corresponding global direction. For closed loops, δH_i is zero and δu_k means the kinematic adjustments bringing about closure. The applicable perturbation method can be found in [14], [6]. If derivation is needed with respect to translational and rotational variables then the actual variable has to be substituted into Equation (20) as L+ΔL in case of translational (L) and φ+Δφ in case of rotational (φ) variables. Due to the small perturbation the equation does not express a closed loop, but a small error vector will produced. The derivates can be expressed with numerical approximation. A detailed derivation of this method can be found in [6]. Based on this method, Equation (21) can be expressed as a linearized matrix form:

$$\{\delta H\}=[M]\{\delta X\}+[A]\{\delta U\}=\{\Theta\}\,. \tag{22}$$

where {δH} is vector of the clearance variations; {δX} is vector of the variations of the manufactured dimensions; {δU} is vector of the variations of the assembly dimensions; [M] is matrix of the first order partial derivatives of the manufactured variables, [A] is matrix of the first order partial derivatives of the assembly variables; and {Θ} is the zero vector.

Each element of [M] and [A] matrices can be determined with the perturbation method. The structure of both matrices will be as follows:

$$\left[A_i\right] = \left\{ \frac{\partial H_x}{\partial x_i}, \frac{\partial H_y}{\partial x_i}, \frac{\partial H_z}{\partial x_i}, \frac{\partial H_{\Theta x}}{\partial x_i}, \frac{\partial H_{\Theta y}}{\partial x_i}, \frac{\partial H_{\Theta z}}{\partial x_i} \right\}^T \tag{23}$$

where x_i is the i-th assembly variable. The structure of [M] matrix is the same, but u_i will be used instead of x_i. Equation (22) can be solved for [δU]:

$$\{\delta U\} = -[A]^{-1}\,[M]\{\delta X\}. \tag{24}$$

On the score of Equation (24), when [A] is a square matrix, {δU} vector can be determined. This matrix method is highly applicable in computerized implementations.

7. Introducing the Optol 3D tolerance calculation software

Developing a computerized algorithm and its integration into a CAD system is a difficult and complicated task. The research group has proposed a general 'CAD system' independent model. Our starting point is that CAD systems are able to export geometrical data of each design feature with an arbitrarily chosen coordinate system (in Pro/Engineer CAD system datum coordinate systems can be used for this purpose, in CATIA this export is also available). It is obvious that input data for our algorithm will be coordinates of vector end-points. Figure 9 depicts the functional diagram of our OpTol Software. The input data of the OpTol system is a special text-based Loop file that describes an assembly loop. The user is able to analyse an existing Pro/Engineer 2001 assembly by means of the OpTol Pro/Engineer module. In this case, the OpTol module creates the input Loop file for the OpTol System. This module was implemented by using Pro/JLink, which is an extension creator toolkit of the Pro/Engineer software. Additionally, the OpTol system can be used without Pro/Engineer, creating Loop files by using a simple text editor.

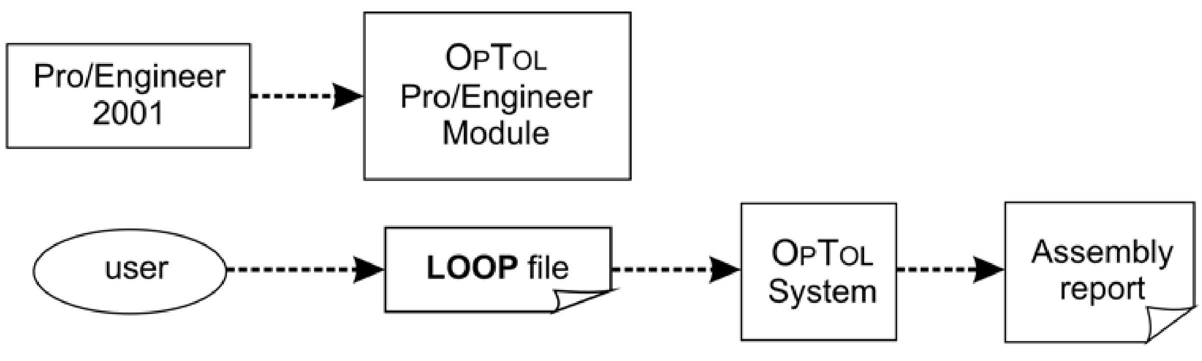


Figure 9. Functional diagram of the OpTol system

The OpTol system generates a detailed html assembly report as a result of tolerance calculations. The OpTol system also supports three dimensional tolerancing calculation based on worst-case, statistical and six sigma methods. The OpTol system does not yet support geometrical tolerances.

In further versions, our team is planning to extend the functionality of the system by supporting geometrical tolerances and tolerance allocation methods.

Our development strategy is based upon using only open-source software tools and components. All components of the OpTol software were developed in Java utilizing NetBeans IDE and Java Swing API. The OpTol system is basically platform independent but its installer and launcher application only runs under a Windows platform. In the following section a simple 2D example with multiple loops will be presented.

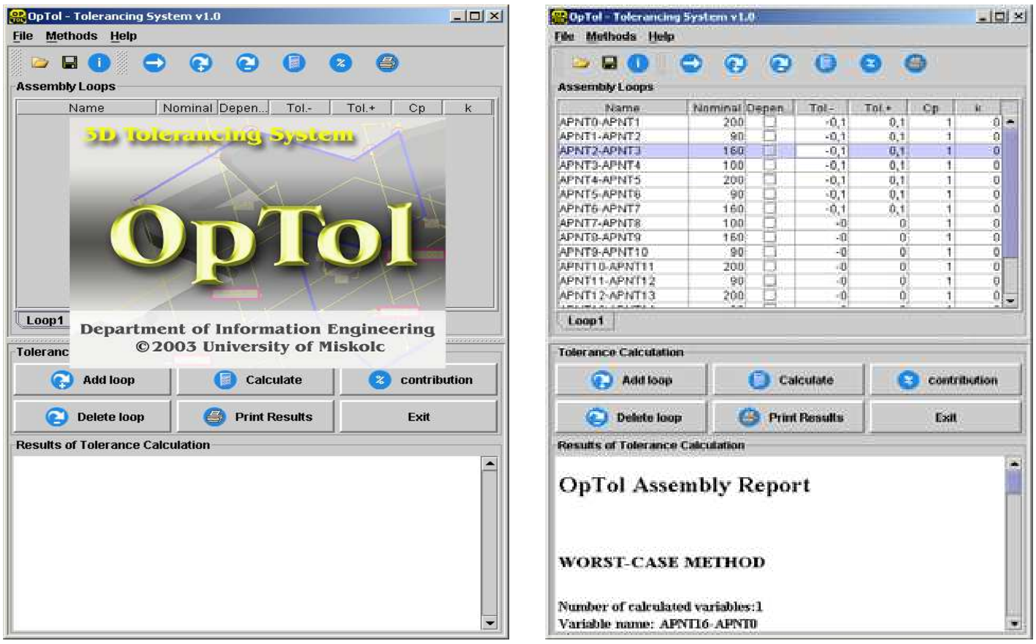
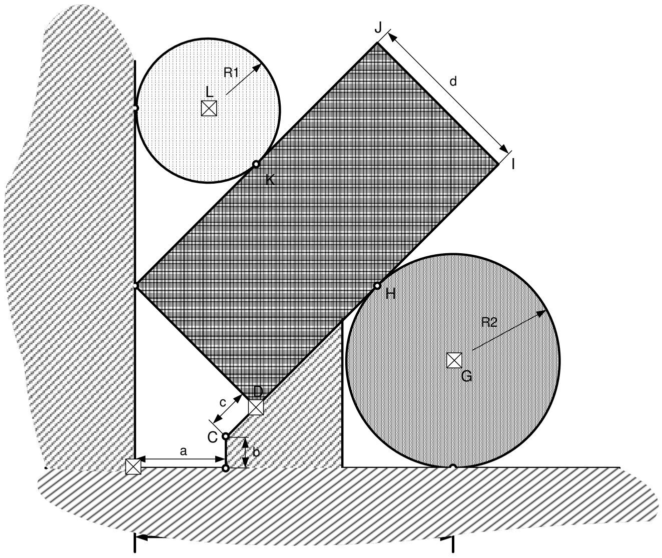


Figure 10. Screenshots of the OpTol Tolerancing System

7.1. 2D Tolerance calculation example (multiple-loops)

We tested the system on a relatively complicated industrial assembly, but publishing the results exceeds the limited length of this paper, at the same time the following 2D example demonstrates the fundamental functions of our application well.

Figure 11 represents a model assembly consisting of four parts: two cylinders, one block and a base. We are looking for the tolerance values of the dimensions X1, X2, X3. The following table shows the x and y coordinates of the points A, B, C… M (Point A is the origin). Assume that for the sake of convenience, all the tolerance values of each dimension (line segments e.g.: AB, BC, DE, … LM) are the same: ± 0.05 mm. A datum reference point must be defined for each part (⌧).



Point name	A	B	C	D	E	F	G	H	K	L	M
x-y coords	0, 0	3, 0	3, 1	4, 2	0, 6	10.5,0	10.5, 3.52	8, 6	4, 10	2.42, 11.8	0,11.8

Figure 11. 2D model example with four parts

The next step of the tolerance calculation is to determine the number of necessary assembly loops. The applicable relation is as follows: L = J – P + 1 where J is the number of joints, and P expresses the number of parts. For our example: J = 6, P = 4, thus L = 3.

7.2. Creating vector loops

A vector loop must fulfil some modelling rules when it passes through parts: [14]

- enter through a joint into a part,
- follow datum path to the datum reference point of the part,
- follow dimensions to another joint,
- leave part.

Figure 11shows this process.

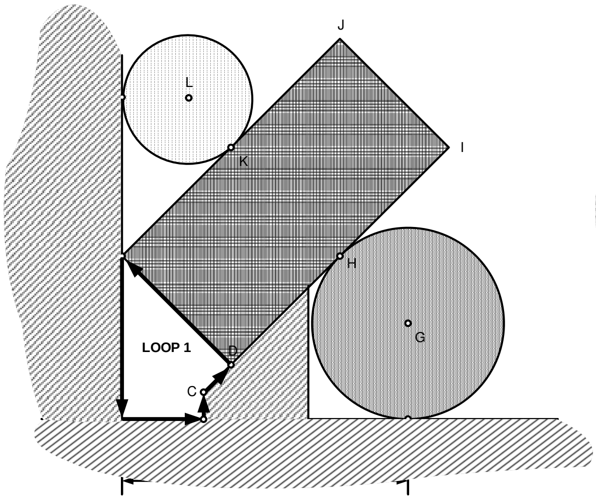


Figure 12a. Loop One

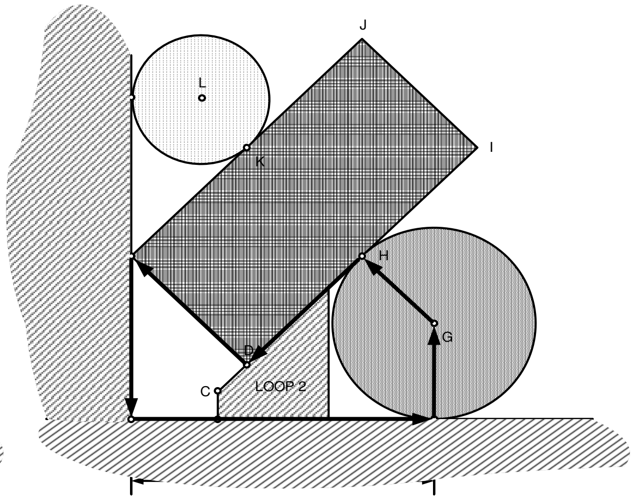


Figure 12b. Loop Two

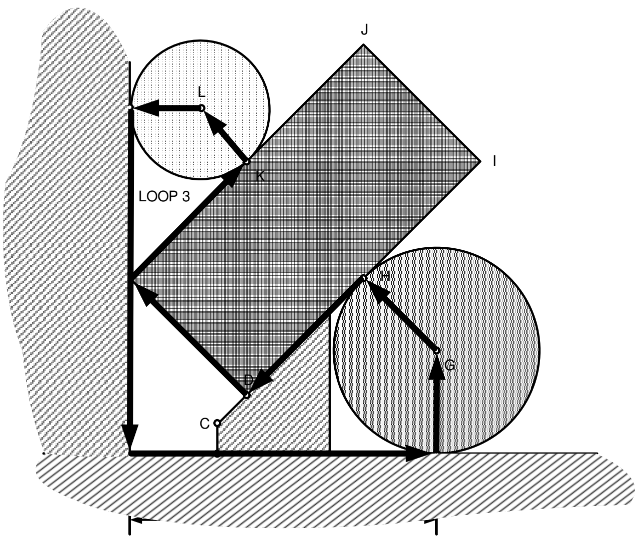


Figure 12c. Loop Three

The loops must go through every part and every joint in the assembly. The following table shows the Loop files, which are importable into the OpTol System (these source files can be found in the folder "*[Install dir]*/Tutorial/"):

Example2D_1.loop			Example2D_2.loop			Example2D_3.loop		
0.0, 0.0, 0.0, A			0.0, 0.0, 0.0, A			0.0, 0.0, 0.0, A		
3.0, 0.0, 0.0, B			10.5, 0.0, 0.0, F			10.5, 0.0, 0.0, F		
3.0, 1.0, 0.0, C			10.5, 3.52, 0.0, G			10.5, 3.52, 0.0, G		
4.0, 2.0, 0.0, D			8.0, 6.0 ,0.0, H			8.0, 6.0, 0.0, H		
0.0, 6.0, 0.0, E			4.0, 2.0, 0.0, D			4.0, 2.0, 0.0, D		
0.0, 0.0, 0.0, A			0.0, 6.0, 0.0, E			0.0, 6.0, 0.0, E		
			0.0, 0.0, 0.0, A			4.0, 10.0,0.0, K		
						2.42, 11.8,0.0, L		
						0.0, 11.8,0.0, M		
						0.0, 0.0, 0.0, A		
Name	Tol. -	Tol. +	Name	Tol.-	Tol. +	Name	Tol. -	Tol. +
A-B	-0.02	0.05	A-F	-0.02	0.02	A-F	-0.02	0.02
B-C	-0.02	0.01	F-G	-0.05	0	F-G	-0.05	0
C-D	-0.1	0.01	G-H	-0.01	0.01	G-H	-0.01	0.01
E-A	-0.03	0.04	H-D	-0.01	0.01	H-D	-0.01	0.01
			D-E	-0.05	0.05	D-E	-0.05	0.05
			E-A	-0.01	0.04	E-K	-0	0
						K-L	-0	0
						L-M	-0	0
						M-A	-0.02	0.05

Figure 13. Three loop files describing the example. The lower part of the table contains the sample tolerance values which have to be set in the application.

Restart OpTol system and import the entire three loops into the application:

- Import loop by pushing *"Ctrl+I"* and select Example2D_1.loop from the *[Install dir]*/Tutorial folder.

- Push the button 

to add a new loop to the table. Select the tab ‘Loop2’ and import the next loop by pushing *“Ctrl+I”* and select Example2D_2.loop.
- Push the button 

to add a new loop to the table. Select the tab ‘Loop3’ and import the next loop by pushing *“Ctrl+I”* and select Example2D_3.loop.

The next step is to edit the tolerance values. Figure 13 contains the tolerance values for each segment. In OpTol, you must set any parameter of a dimension only once. Having completed this procedure, check the checkbox ‘dependent variable’ for the following dimensions: EA, AF, MA (remember: these were X1, X2 and X3). Since we have not indicated the values *Cp* and *k*, we should turn off the six-sigma statistical method. After clicking on the ‘Calculate’ button, you will see the following result in the ‘Results of Tolerance Calculation’ tab.

OpTol Assembly Report

WORST-CASE METHOD

Number of calculated variables:3
Variable name: E-A

Tolerance	**+-0.08** [mm]
Calculated Upper limit	6.08 [mm]
Calculated Lower limit	5.92 [mm]

Variable name: A-F

Tolerance	**+-0.021** [mm]
Calculated Upper limit	10.521 [mm]
Calculated Lower limit	10.479 [mm]

Variable name: M-A

Tolerance	**+-0.046** [mm]
Calculated Upper limit	11.846 [mm]
Calculated Lower limit	11.754 [mm]

STATISTICAL (3-sigma) METHOD

Number of calculated variables:3

Variable name: E-A

Tolerance	**+-0.041** [mm]
Calculated Upper limit	6.041 [mm]
Calculated Lower limit	5.959 [mm]
Reject Calculation	
Design Upper limit	6.04 [mm]
Design Lower limit	5.99 [mm]
Predicted Rejects (UL)	1609.937 [ppm]
Predicted Rejects (LL)	230718.717 [ppm]
Predicted Rejects Total	232328.654 [ppm]

Variable name: A-F

Tolerance	**+-0.012** [mm]
Calculated Upper limit	10.512 [mm]
Calculated Lower limit	10.488 [mm]
Reject Calculation	
Design Upper limit	10.52 [mm]
Design Lower limit	10.48 [mm]
Predicted Rejects (UL)	0.498 [ppm]
Predicted Rejects (LL)	0.498 [ppm]
Predicted Rejects Total	0.996 [ppm]

Variable name: M-A

Tolerance	**+-0.028** [mm]
Calculated Upper limit	11.828 [mm]
Calculated Lower limit	11.772 [mm]
Reject Calculation	
Design Upper limit	11.85 [mm]
Design Lower limit	11.78 [mm]
Predicted Rejects (UL)	0.035 [ppm]
Predicted Rejects (LL)	15548.773 [ppm]
Predicted Rejects Total	15548.808 [ppm]

Figure 14. OpTol assembly report of the 2D example tolerance calculation

7.3. Calculating percentual contribution

This procedure is very useful if you want to see how each dimension contributes to a selected variable. If you press 

when the 'loop 1' tab is the selected tab on the pane, you will get the following.

According to Figure 15, BC is the principal contributor of the dimension EA, followed by the dimension CD. If the resultant tolerance is not desirable, we must change the dimension tolerance values. According to the percent contribution results we must reduce the tolerance values of the dimension BC.

Note: this percent contribution calculation is not a trivial task. We have three dimension loops and they affect the tolerance of the dimension EA simultaneously.

(Worst-case) Percent Contribution of variable: E-A		(Statistical) Percent Contribution of variable: E-A	
A-B	0.0%	**A-B**	0.0%
B-C	37.68%	**B-C**	54.24%
C-D	28.87%	**C-D**	31.83%
D-E	8.88%	**D-E**	3.01%
F-G	15.7%	**F-G**	9.42%
G-H	4.42%	**G-H**	0.75%
H-D	4.44%	**H-D**	0.75%
E-K	0.0%	**E-K**	0.0%
K-L	0.0%	**K-L**	0.0%
L-M	0.0%	**L-M**	0.0%

Figure 15. Percent contribution of dimension 'EA'

8. Conclusion

The OpTol 3D tolerancing software and its mathematical models have been presented. The software is utilizing a direct linearization method to solve tolerance calculations up to 3 dimensional cases. The OpTol System can work as a stand alone system or consists of a CAD module in order to support engineers to analyse their existing assemblies. The OpTol system installation package can be downloaded from the website alpha.iit.uni-miskolc.hu/OpTol/setup_trial.exe. The package includes a detailed user's guide with examples and a fully functional trial license.

Continuing work will focus on handling geometrical tolerances and implementing tolerance allocation methods and cost optimizations.

9. Acknowledgements

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References

[1] ANSELMETTI, B., MEJBRI, H. and MAWUSSI, K.: *Coupling experimental design – digital simulation of junctions for the development of complex tolerance chains*, Computers in Industry, Volume 50, Issue 3, pp. 277-292, 2003.

[2] BÁLINT, L. and GRIBOVSZKI, L.: *Fundamentals of Manufacturing Science and Technology.* Textbooks Publisher, Budapest, 1980. (in Hungarian).

[3] CHASE, K. W. and GREENWOOD, W. H.: *Design Issues in Mechanical Tolerance Analysis,* Manufacturing Review, Volume 11, No 1, pp. 50-59, 1988.

[4] CHASE, K. W.: *Tolerance Analysis of 2-D and 3-D Assemblies,* ADCATS Report No. 99-4, Department of Mechanical Engineering, Brigham Young University, Utah, 1999.

[5] DRAKE, P. J.: *Dimensioning and Tolerancing Handbook,* McGraw-Hill Professional ISBN: 0070181314, 1999.

[6] GAO, J. and CHASE, K. W.: *Generalized 3-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments,* IEEE Transactions, Volume 30, Number 4, pp. 367-377, 1998.

[7] GERTH, R. J. and HANCOCK, W. M.: *Computer aided tolerance analysis for improved process control,* Computers & Industrial Engineering, Volume 38, Issue 1, pp. 1-19, 2000.

[8] JOSKOWICZ, L., SACKS, E. and SRINIVASAN, V.: *Kinematic tolerance analysis,* Computer-Aided Design, Volume 29, Issue 2, pp. 147-157, 1997.

[9] KORN, A.G. and KORN, T.M.: *Mathematical Handbook for Scientists and Engineers.* Definitions, Theorems and Formulas for Reference and Review. Second, Enlarged and Revised Edition – McGraw-Hill Book Company, (in Hungarian: Technical Publisher, Budapest, 1975, p.567).

[10] KYUNGA, MIN-HO, SACKS, ELISHA: *Nonlinear kinematic tolerance analysis of planar mechanical systems,* Computer-Aided Design 35, pp. 901–911, 2003.

[11] LIN, CHIH-YOUNG, HUANG, WEI-HSIN, JENG, MING-CHANG, DOONG, JI-LIANG: *Study of an assembly tolerance allocation model based on Monte Carlo simulation,* Journal of Materials Processing Technology 70, pp. 9-16, 1997.

[12] *Motorola Six Sigma model*, http://www.isixsigma.com/me/six_sigma/, 2005.

[13] NIGAM, S. D. and TURNER, J. U.: *Review of statistical approaches to tolerance analysis,* Computer-Aided Design, Volume 27, Issue 1, pp. 6-15, 1995.

[14] ROBISON, R. H.: *A Practical Method for Three-Dimensional Tolerance Analysis Using a Solid Modeller*, M.S. Thesis, Mechanical Engineering Department, Brigham Young University, 1989.

[15] SHAN, A., ROTH, R. N. and WILSON, R. J.: *Genetic algorithms in statistical tolerancing,* Mathematical and Computer Modelling, Volume 38, Issues 11-13, pp. 1427-1436, 2003.

[16] SKOWRONSKI, V. J. and TURNER, J. U.**:** *Using Monte-Carlo variance reduction in statistical tolerance synthesis,* Computer-Aided Design, Volume 29, Issue 1, pp. 63-69, 1997.

[17] SOLTI, E.: *Tolerance computations of Economical Manufacturing.* Technical Publisher, Budapest, 1968. (in Hungarian).

[18] TÓTH, T.: *Interactive Programme System for Determining the Optimum Machining Tolerances Having Regard to Assembly Requirements,* Proceedings of the Twenty-Ninth International Matador Conference, Manchester, pp. 83-91, 1992.

[19] WITTWER, J. W., CHASE, K. W. and HOWELL, L. L.: *The direct linearization method applied to position error in kinematic linkages,* Mechanism and Machine Theory, Volume 39, Issue 7, pp. 681-693, 2004.

[20] YANG, C. C. and NAIKAN, A. V. N.: *Optimum design of component tolerances of assemblies using constraint networks,* International Journal of Production Economics, Volume 84, Issue 2, pp. 149-163, 2003.