Production Systems and Information Engineering
Volume 13 (2), pp. 1-15
doi: https://doi.org/10.32968/psaie.2025.2.1 1

EFFICIENCY ANALYSIS OF THE HILL CLIMBING AND ELITIST STRATEGY OF ANT SYSTEM IN THE APPLICATION OF FLOW SHOP SCHEDULING PROBLEMS

ANITA AGÁRDI[1]
University of Miskolc, Hungary
Institute of Information Technology
anita.agardi@uni-miskolc.hu

Abstract. This paper examines the efficiency of the Hill Climbing and Elitist Strategy of Ant System algorithms on a well-known production scheduling task, the Flow Shop Scheduling task. In the task, the properties of the machines and jobs are given and all jobs must be performed on each machine. The objective function is the makespan minimization. For this task, the researchers created benchmark datasets on which the efficiency of each algorithm can be proven. This research uses the Taillard dataset to demonstrate the efficiency of the algorithms.

Keywords: Hill Climbing, Elitist Strategy of Ant System, Flow Shop Scheduling Problem, Taillard benchmark

1. Introduction

This article solves a common production scheduling problem, the Flow Shop Scheduling [1] problem, using metaheuristics. In Flow Shop Scheduling, given jobs must be performed on a given number of machines. Each job must be performed exactly once. Only one job can be performed on a machine at a time. Jobs must not be interrupted, so if a given machine has already started a job, it cannot be left unfinished. Jobs are executed sequentially on each machine, the goal is to determine the order of the jobs. If the next job is still being performed by the previous machine, the given machine must wait until it is completed. The goal is makespan minimization.

[1] The author used ChatGPT (OpenAI) to assist in the textual formulation of several section of the manuscript. The author reviewed and edited the AI-generated content for clarity, accuracy and coherence, and take full responsibility for the final version.

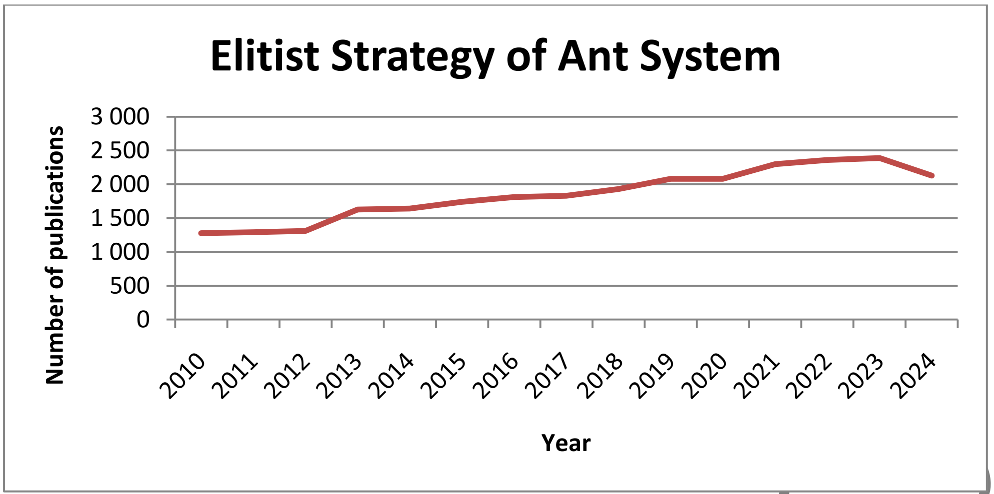

Figure 1. Number of publications:
Elitist Strategy of Ant System

The Elitist Strategy of Ant System [2] is an important development in the Ant System (AS) algorithm, introduced by Marco Dorigo et al. in the development of the original Ant System.
In the case of the Elitist Strategy of Ant System [3], we can also see a continuous increase in the number of publications. In 2010, less than 1,500 articles were published, but by 2013 the number of articles had already exceeded 1,500. In 2024, researchers had already published more than 2,000 articles.

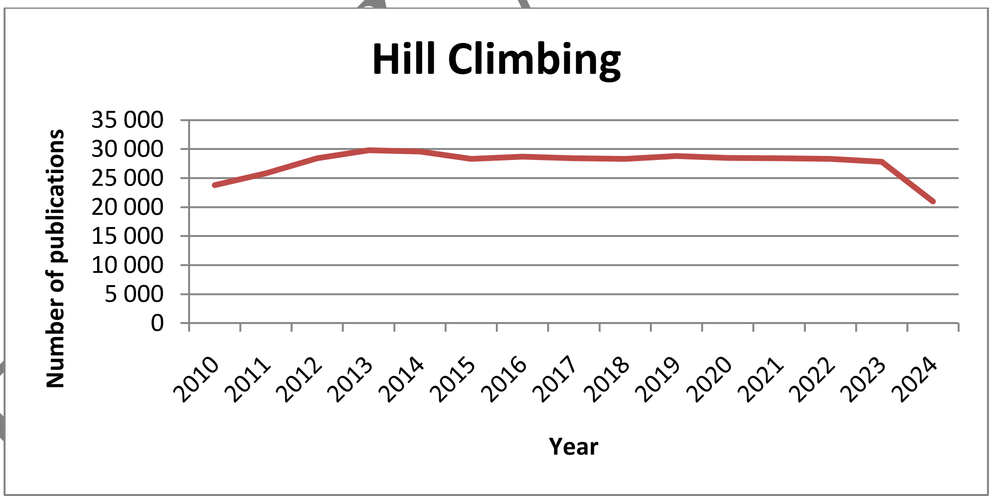

Figure 2. Number of publications:
Hill Climbing

The history of the Hill Climbing [4] algorithm is difficult to pin down to a single publication, as the principles emerged early in mathematical and operations research problems. However, the theoretical foundations and practical applications of the algorithm began to appear in the 1950s.
A similar number of articles were published on the Hill Climbing algorithm over the years between 2010 and 2024. This number has usually ranged between 25,000 and 35,000.

There are several papers in the literature that solve the Flow Shop Scheduling problem using Elitist Strategy of Ant System and Hill Climbing algorithm.
In [5] publication, the Flexible Job Shop Scheduling was solved using Genetic Algorithm and Random Restart Hill Climbing.
In [6] article, the authors solve Reentrant Flow Shop Scheduling, where Hill Climbing is also applied.
In [7] publication, the authors discuss Simultaneous Generalized Hill-Climbing Algorithm. The authors solve several optimization problems using this algorithm.
In [8] article, the authors solve Multi-Objective Two-Stage Flow Shop problem using Hybrid Ant Colony Optimisation.
In addition, the following papers solve Job Shop Scheduling problems using Ant Colony: [9-11]
The following sections of the article present the following: The second section presents the applied algorithms, such as Hill Climbing and Elitist Strategy of Ant System. The third section contains the test results. The last section presents the conclusion and further research directions.

2. Applied metaheuristics

This section presents the applied metaheuristic algorithms. These algorithms performs on a single solution (Hill Climbing) or a set of solutions (Elitist Strategy of Ant System).

2.1. Hill Climbing

Hill Climbing [7] is a local search algorithm. At each step, it selects the best neighboring solution. The algorithm works similar to mountain climbing, where the goal is to reach the highest point. The highest point in the metaheuristic can be the global or a local optimum.
Steps of the algorithm:

- **Initial solution:** An initial solution is taken as a basis, which can be a randomly generated solution or the result of a construction algorithm. Initially, this will be the current solution.
- **Evaluation of neighboring solutions:** This step means the neighbors of the current solution and evaluate them.
- **Selection of the best neighbor:** This step means the selection of the best neighboring solution, i.e. the solution that has the best fitness value and is better than the current solution.
- **Stopping condition:** This means repeating the above steps until finding a better neighboring solution, or reaching a certain iteration number, running time, or no longer getting a significant improvement.

2.2. Elitist Strategy of Ant System

The Elitist Strategy of Ant System (ESAS) [2] is also an improved version of Ant Colony Optimization (ACO) developed by Marco Doringo and his colleagues. During each iteration, the pheromone updates are adjusted to the goodness of the solutions. So, the better a given solution is, the more pheromone is received for that road section. Thus, the ants prefer to choose these road sections when building their road. So, when building a new solution, smaller road sections (older better solutions) are given more emphasis.
Steps of the algorithm:

- **Initialization:** Assigning an initial pheromone value to each edge.
- **Finding the best solution:** During each iteration, the best solutions are selected. These will receive a higher pheromone value.
- **Pheromone update:** The best solutions are provided with a larger amount of pheromone. The pheromone update is done with the following formula:

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \Delta\tau_{ij} + \Delta\tau^*_{ij}$$

$\Delta\tau^*_{ij}$: pheromone value of best solution

ρ: evaporation of the pheromone

$\Delta\tau_{ij}$: the traditional pheromone update value (belongs to the weaker solutions)

- Termination condition: the termination condition can be reaching a certain number of iterations, convergence, or reaching a certain running time.

3. Test results

In this section, the test results of the Hill Climbing and Elitist Strategy of Ant System are presented. The implementation of the algorithms was created in Java programming environment. The implemented software contains several components, such as the file reading component, the optimization components (in this paper, two algorithms were selected from a number of heuristic algorithms) and the result output component. During the test runs, a predictable time (just a few minutes) was given for how long the algorithm should run. A well-known benchmark dataset, the Taillard [14] dataset, was used to test the efficiency of the algorithms.

Test results of the Hill Climbing

Table 1. Test results: Hill Climbing

Instance	**HC**		
	Max	**Avg**	**Min**
Ta001	1377	1341	1297
Ta002	1417	1395.6	1383
Ta003	1258	1169.8	1100
Ta004	1470	1415.8	1362
Ta005	1346	1328.8	1298
Ta006	1281	1273	1250
Ta007	1317	1276.8	1257
Ta008	1391	1321	1240
Ta009	1387	1302	1255
Ta010	1243	1190.4	1161
Ta011	1878	1757.8	1649
Ta012	1867	1828	1810
Ta013	1723	1668.2	1627
Ta014	1635	1542	1487
Ta015	1616	1570.8	1536
Ta016	1592	1529.8	1466
Ta017	1649	1596.6	1555
Ta018	1786	1702.4	1651
Ta019	1737	1702.6	1676
Ta020	1726	1702.8	1686

Table 1 shows the results of the Hill Climbing algorithm on the Taillard dataset [14]. The first column shows the dataset identifier. The second column shows the maximum of the runs for each dataset element. The third column shows the average, and the fourth shows the minimum of the runs. Hill Climbing did not prove to be that efficient.
It can be seen that there is a large difference between the best and worst fitness values. In the case of the Ta001 dataset, the lowest fitness value was 1297, while the average value was 1341, and the highest was 1377. Based on the results, there is less fluctuation between the runs, and the performance of the Hill Climbing algorithm is balanced. In the case of Ta002, the lowest value was 1383, the average fitness value was 1395.6, and the highest was 1417. This dataset had higher fitness values than the previous one. This suggests that there is a smaller improvement in the search. For Ta003, the minimum was 1100, the average was 1169.8, and the maximum was 1258. For Ta004, the minimum fitness value was 1362, the average value was 1415.8, and the maximum value was 1470. The results of the dataset show that the differences between the runs are significant. This indicates that the algorithm is stuck in local optima. All three statistics of the algorithm runs – maximum, average, and minimum – result in relatively high fitness values. This indicates that Hill Climbing was unable to find optimal solutions to the problem under study. For each dataset, a significant difference can be observed between the best (minimum) and worst (maximum) runs. For example, for Ta011, the minimum is 1649, while the maximum is 1878. This indicates that there is a large variability in performance between runs of the algorithm. Such a fluctuation may indicate that the algorithm tends to get stuck in local optima.

Table 2. Comparision of the minimum test results: Hill Climbing

Instance	HC	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1297	14.57	11.72	14.57	5.94	2.08
Ta002	1383	10.48	5.57	10.48	1.81	4.27
Ta003	1100	32.73	26.00	32.73	16.36	-0.18
Ta004	1362	16.59	11.67	16.59	6.31	7.86
Ta005	1298	11.63	8.09	11.63	3.31	-0.54
Ta006	1250	18.48	14.40	18.48	9.04	11.28
Ta007	1257	17.98	16.23	17.98	9.86	3.34
Ta008	1240	19.52	15.56	19.52	11.21	4.19
Ta009	1255	17.05	11.39	17.05	9.40	4.06
Ta010	1161	18.60	14.04	18.60	10.51	6.20
Ta011	1649	23.95	18.56	21.95	2.97	3.88
Ta012	1810	19.67	17.29	19.67	1.27	-5.08
Ta013	1627	19.24	17.52	19.24	3.01	-4.43
Ta014	1487	21.79	19.84	21.79	3.97	1.95
Ta015	1536	25.85	25.85	25.85	5.27	2.41
Ta016	1466	29.06	24.62	29.06	8.46	-0.61
Ta017	1555	26.24	25.02	26.24	4.31	4.31
Ta018	1651	24.59	21.50	24.59	4.85	5.94
Ta019	1676	17.72	13.84	17.72	4.24	-3.10
Ta020	1686	21.65	18.68	21.65	5.69	2.14

Table 2 shows the minimum of the Hill Climbing algorithm run results. It compares this with the results of the algorithms published by the researchers [12-13]. The table shows that the results published by the researchers gave better

results than HGSA in some cases. Most of the time, this algorithm gave better results by 10-20%.
In the case of the Hill Climbing algorithm, the minimum fitness value is 1297 for Ta001. On this data set, the results of HMM-PFA and IIGA differed by 14.57%, the value of HGA differed by 11.72%, DSOMA differed by 5.94% and HGSA by 2.08%. This means that although better results can be achieved in some runs of HGSA, in most cases the minimum value achieved by Hill Climbing is better.
For the Ta002 dataset, the minimum HC value is 1383, which is 10.48% different from HMM-PFA, 5.57% different from HGA and 10.48% different from IIGA, 1.81% different from DSOMA and 4.27% different from HGSA. It can be seen that the Hill Climbing algorithm produced better results compared to the algorithms.
For Ta003, the HC fitness value is 1100. Here, the differences are very significant compared to the comparison algorithms: 32.73% (HMM-PFA and IIGA), 26.00% (HGA) and 16.36% (DSOMA), while for HGSA it is almost the same (-0.18%). This large difference shows that the Hill Climbing algorithm found a significantly better solution than most published results. For Ta004, the minimum HC value is 1362, while the HMM-PFA, HGA and IIGA values show a difference of 16.59% and 11.67%, and DSOMA and HGSA differ by 6.31% and 7.86%, respectively. Similarly, for Ta005, the differences for the fitness value of 1298 are 11.63% (HMM-PFA and IIGA), 8.09% (HGA) and 3.31%, respectively -0.54% (DSOMA, HGSA).
In the other rows of the table – from Ta006 to Ta020 – we also see that the minimum values achieved by the Hill Climbing algorithm are sometimes much lower (better) than those published by the researchers. For example, for Ta010, compared to the HC value of 1161, HMM-PFA gave a difference of 18.60%, HGA gave a difference of 14.04%, IIGA gave a difference of 18.60%, DSOMA gave a difference of 10.51% and HGSA gave a difference of 6.20%. Similarly, for other data sets (e.g. Ta016, Ta017, Ta018) we can see an improvement of 10–20%.
Table 2 shows that although HGSA can achieve better results than other algorithms on some datasets, Hill Climbing results are typically 10–20% better. This means that Hill Climbing is able to find solutions that give significantly better fitness values in most cases.

Table 3. Comparision of the average test results: Hill Climbing

Instance	HC	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1341	10.81	8.05	10.81	2.46	-1.27
Ta002	1395.6	9.49	4.61	9.49	0.89	3.32
Ta003	1169.8	24.81	18.48	24.81	9.42	-6.14
Ta004	1415.8	12.16	7.43	12.16	2.27	3.76
Ta005	1328.8	9.05	5.58	9.05	0.92	-2.84
Ta006	1273	16.34	12.33	16.34	7.07	9.27
Ta007	1276.8	16.15	14.43	16.15	8.16	1.74
Ta008	1321	12.19	8.48	12.19	4.39	-2.20
Ta009	1302	12.83	7.37	12.83	5.45	0.31
Ta010	1190.4	15.68	11.22	15.68	7.78	3.58
Ta011	1757.8	16.28	11.22	14.40	-3.40	-2.55
Ta012	1828	18.49	16.14	18.49	0.27	-6.02
Ta013	1668.2	16.29	14.61	16.29	0.47	-6.79
Ta014	1542	17.44	15.56	17.44	0.26	-1.69
Ta015	1570.8	23.06	23.06	23.06	2.94	0.14

Ta016	1529.8	23.68	19.43	23.68	3.94	-4.76
Ta017	1596.6	22.95	21.76	22.95	1.59	1.59
Ta018	1702.4	20.83	17.83	20.83	1.68	2.74
Ta019	1702.6	15.88	12.06	15.88	2.61	-4.62
Ta020	1702.8	20.45	17.51	20.45	4.65	1.13

Table 3 shows the average test values of the Hill Climbing algorithm. The results are compared with those published by researchers. Even here, in many cases, the HC algorithm showed a 10-20% better result compared to the researchers' results.
For the Ta001 dataset, the average result of the HC algorithm is 1341. The results of HMM-PFA and IIGA are 10.81% higher, and HGA is 8.05%. In the case of DSOMA, the difference is 2.46%, and for HGSA, the difference is -1.27%.
For Ta002, the average fitness value is 1395.6. The difference in the results for HMM-PFA and IIGA is 9.49%. HGA is 4.61% lower, and DSOMA is 0.89%. In the case of HGSA, a difference of 3.32% can be observed.
Ta003: Here the HC fitness value is 1169.8. The results of HMM-PFA and IIGA are 24.81% higher. In the case of HGA, there is a difference of 18.48%. In the case of DSOMA, the difference is 9.42%, while in the case of HGSA it is -6.14%.
Ta004: The average fitness value of HC is 1415.8. In the case of HMM-PFA and IIGA, these values are 12.16% higher, while in the case of HGA they are 7.43%. In the case of DSOMA and HGSA, the differences are 2.27% and 3.76%, respectively.
Ta005: The average HC fitness value is 1328.8. HMM-PFA and IIGA differ by 9.05%, HGA by 5.58%, while DSOMA is only 0.92%. In the case of HGSA, the difference is -2.84%.
In the middle range (Ta006–Ta010), the average fitness values of the HC algorithm (e.g. Ta006: 1273, Ta007: 1276.8, Ta008: 1321, Ta009: 1302, Ta010: 1190.4) are generally 12–16% higher for HMM-PFA and IIGA, while slightly smaller differences of 7–14% are observed for HGA. The difference is 4–8% for DSOMA.
For datasets with higher fitness values (Ta011–Ta020), the average HC values range from 1542 to 1828. The results for HMM-PFA and IIGA are generally 15–23% higher. The differences for HGA are also significant (about 11–19%). The DSOMA results have negative values in some cases (e.g. Ta011, Ta014). This indicates that DSOMA gave a better result. In the case of HGSA, the difference is negative in several cases (e.g. Ta012, Ta013, Ta016).
Based on the values for each TaXXX data set, it can be stated that the performance of the HC algorithm varies, but in many cases it produced 10–20% better results than the HMM-PFA, HGA and IIGA algorithms.
For several data sets, the results of DSOMA and HGSA show a smaller difference, sometimes better than HC.

Table 4. Comparision of the maximum test results: Hill Climbing

Instance	HC	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1377	7.92	5.23	7.92	-0.22	-3.85
Ta002	1417	7.83	3.03	7.83	-0.64	1.76
Ta003	1258	16.06	10.17	16.06	1.75	-12.72
Ta004	1470	8.03	3.47	8.03	-1.50	-0.07
Ta005	1346	7.65	4.23	7.65	-0.37	-4.09
Ta006	1281	15.61	11.63	15.61	6.40	8.59
Ta007	1317	12.60	10.93	12.60	4.86	-1.37
Ta008	1391	6.54	3.02	6.54	-0.86	-7.12

Ta009	1387	5.91	0.79	5.91	-1.01	-5.84
Ta010	1243	10.78	6.52	10.78	3.22	-0.80
Ta011	1878	8.84	4.10	7.08	-9.58	-8.79
Ta012	1867	16.01	13.71	16.01	-1.82	-7.98
Ta013	1723	12.59	10.97	12.59	-2.73	-9.75
Ta014	1635	10.76	8.99	10.76	-5.44	-7.28
Ta015	1616	19.62	19.62	19.62	0.06	-2.66
Ta016	1592	18.84	14.76	18.84	-0.13	-8.48
Ta017	1649	19.04	17.89	19.04	-1.64	-1.64
Ta018	1786	15.17	12.32	15.17	-3.08	-2.07
Ta019	1737	13.59	9.84	13.59	0.58	-6.51
Ta020	1726	18.83	15.93	18.83	3.24	-0.23

Table 4 shows the maximum values of the Hill Climbing algorithm and their relationship to the results published by researchers. Here, in several cases, the results published by researchers were better than the HC algorithm.
Ta001: The maximum value of the HC algorithm is 1377. In the case of HMM-PFA and IIGA, there is a difference of 7.92%. They give a slightly worse maximum value, while in the case of HGA, the difference is 5.23%. The DSOMA and HGSA algorithms show a difference of -0.22% and -3.85%.
Ta002: Here, the maximum of HC is 1417. HMM-PFA and IIGA show a difference of 7.83%, while HGA differs by 3.03%. In the case of DSOMA, the difference is -0.64%, while in the case of HGSA, the difference is 1.76%.
Ta003: The maximum value of the HC algorithm is 1258. A significant difference of 16.06% is observed in the case of HMM-PFA and IIGA, while the HGA method gives a difference of 10.17%. DSOMA shows a difference of 1.75%, however, in the case of HGSA the difference is -12.72%.
Ta004: The maximum value of HC for this data set is 1470. HMM-PFA and IIGA result in a value higher by 8.03%, and HGA by 3.47%. In the case of DSOMA, the difference is -1.50%, i.e. negative. In the case of HGSA, it is almost zero (-0.07%), so here the results of HC and the HGSA published by the researchers are almost identical.
Ta005: The value of HC is 1346. In the case of HMM-PFA and IIGA, the difference is 7.65%, while HGA is 4.23%. DSOMA shows a difference of -0.37%, and HGSA -4.09%.
Ta006: The maximum value of HC is 1281, while HMM-PFA and IIGA produce a difference of 15.61%, and HGA a difference of 11.63%. The DSOMA and HGSA results show a difference of 6.40% and 8.59%, respectively. This means that the researchers' methods resulted in a slightly worse maximum value for this data set.
Ta007: The HC value is 1317. The HMM-PFA and IIGA methods show a difference of 12.60%, and the HGA a difference of 10.93%. DSOMA has a positive 4.86%, and the HGSA has a difference of -1.37%.
Ta008: The HC maximum fitness value is 1391. The HMM-PFA and IIGA show a difference of 6.54%, and the HGA a difference of 3.02%. DSOMA resulted in a difference of -0.86% and HGSA -7.12%.
Ta009: The HC fitness value is 1387. The HMM-PFA shows a difference of 5.91%, HGA 0.79%, and IIGA 5.91%. The DSOMA and HGSA values result in a difference of -1.01% and -5.84%, respectively.
Ta010: The maximum fitness of HC is 1243. HMM-PFA and IIGA resulted in a difference of 10.78%, and HGA resulted in a difference of 6.52%. DSOMA showed a difference of 3.22%, and HGSA -0.80%.
In the cases of data sets with higher fitness values (Ta011–Ta020), the maximum values of HC are higher (range 1878–1726).
Based on the analysis of Table 4, it can be said that in addition to the maximum

values of the Hill Climbing algorithm, the methods published by the researchers produced varying results. In several cases, HMM-PFA, HGA, and IIGA yielded worse results with a difference of 5–8%. For some data sets, DSOMA and HGSA showed a negative, i.e. better result.

Test results of the Elitist Strategy of Ant System

Table 5. Test results: Elitist Strategy of Ant System

Instance	ESAS		
	Max	**Avg**	**Min**
Ta001	1297	1297	1297
Ta002	1368	1367.6	1367
Ta003	1162	1153.6	1138
Ta004	1388	1370.6	1362
Ta005	1296	1282.2	1273
Ta006	1252	1247.2	1236
Ta007	1271	1264	1260
Ta008	1277	1269	1257
Ta009	1298	1295.4	1292
Ta010	1178	1168.6	1161
Ta011	1725	1712.4	1692
Ta012	1795	1782	1760
Ta013	1618	1608.6	1602
Ta014	1509	1490.8	1476
Ta015	1573	1560.8	1554
Ta016	1519	1510	1492
Ta017	1595	1584	1568
Ta018	1671	1663.8	1649
Ta019	1695	1686	1671
Ta020	1714	1707	1697

Table 5 shows the results of the Elitist Strategy of Ant System. There are no large differences between the average, minimum and maximum of the test runs.

For data sets such as Ta001, Ta002 or Ta010, the values achieved by the ESAS algorithm are almost the same. The algorithm is able to converge quickly.

For higher value data sets, it can be observed that the differences between the maximum, average and minimum of the test runs are small. For example, for Ta011, the maximum is 1725, the average is 1712.4 and the minimum is 1692. For Ta020, these values are 1714, 1707 and 1697. The algorithm is able to operate stably and reliably even for higher values.

Table 6. Comparision of the minimum test results: Elitist Strategy of Ant System

Instance	ESAS	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1297	14.57	11.72	14.57	5.94	2.08
Ta002	1367	11.78	6.80	11.78	3.00	5.49
Ta003	1138	28.30	21.79	28.30	12.48	-3.51
Ta004	1362	16.59	11.67	16.59	6.31	7.86
Ta005	1273	13.83	10.21	13.83	5.34	1.41

Ta006	1236	19.82	15.70	19.82	10.28	12.54
Ta007	1260	17.70	15.95	17.70	9.60	3.10
Ta008	1257	17.90	14.00	17.90	9.71	2.78
Ta009	1292	13.70	8.20	13.70	6.27	1.08
Ta010	1161	18.60	14.04	18.60	10.51	6.20
Ta011	1692	20.80	15.54	18.85	0.35	1.24
Ta012	1760	23.07	20.63	23.07	4.15	-2.39
Ta013	1602	21.10	19.35	21.10	4.62	-2.93
Ta014	1476	22.70	20.73	22.70	4.74	2.71
Ta015	1554	24.39	24.39	24.39	4.05	1.22
Ta016	1492	26.81	22.45	26.81	6.57	-2.35
Ta017	1568	25.19	23.98	25.19	3.44	3.44
Ta018	1649	24.74	21.65	24.74	4.97	6.06
Ta019	1671	18.07	14.18	18.07	4.55	-2.81
Ta020	1697	20.86	17.91	20.86	5.01	1.47

Table 6 presents the maximum values of the Elitist Strategy of Ant System. Also some HGSA values were better than the implemented algorithm. However, in the case of HMM-PFA, HGA and IIGA algorithms, the ESAS algorithm outperformed by more than 20%.

Ta001: The maximum value of the ESAS algorithm is 1297. The HMM-PFA, HGA and IIGA results show a 14.57% higher value. This indicates that ESAS has achieved a significant improvement here. DSOMA shows a 5.94% difference. In the case of HGSA, It can be seen only a 2.08% difference.

Ta002: The maximum of ESAS is 1367. The HMM-PFA resulted 11.78% difference, the HGA resulted 6.80% difference, and the IIGA also resulted 11.78%. In the case of DSOMA and HGSA results, the differences are 3.00% and 5.49%, so the advantage of the ESAS algorithm is clearly visible here.

Ta003: The fitness value of ESAS is 1138. In the case of HMM-PFA and IIGA, the fitness is 28.30%. HGA differs from the ESAS fitness result by 21.79% and DSOMA by 12.48%. However, in the case of HGSA, the difference is -3.51%.

Ta004: The fitness maximum of the ESAS algorithm is 1362. The HMM-PFA, HGA and IIGA methods produce values of 16.59% higher, while the DSOMA and HGSA results show a difference of 6.31% and 7.86%, respectively.

Ta005: The ESAS fitness maximum is 1273. The HMM-PFA, HGA and IIGA produced a difference of 13.83%. The DSOMA shows a difference of 5.34% and the HGSA shows a difference of 1.41%.

Ta006: With a fitness value of 1236 for the ESAS algorithm, the HMM-PFA, HGA and IIGA results show a difference of 19.82%. The DSOMA results in a difference of 10.28%, while the HGSA results in a difference of 12.54%.

Ta007 and Ta008: ESAS fitness values are 1260 and 1257. The differences are 17.70–17.90% for HMM-PFA, HGA and IIGA, while DSOMA shows 9.60% and 9.71% differences and HGSA shows 3.10% and 2.78% differences.

Ta009 and Ta010: ESAS fitness values are 1292 (Ta009) and 1161 (Ta010). HMM-PFA, HGA and IIGA show 13.70% and 18.60% differences. DSOMA and HGSA results show a smaller difference (Ta009: 6.27% and 1.08%; Ta010: 10.51% and 6.20%).

Ta011 – Ta013: For these data sets, the ESAS values (Ta011: 1692, Ta012: 1760, Ta013: 1602) of the HMM-PFA, HGA and IIGA methods generally show a difference of 20% or more. This indicates that the ESAS algorithm achieved significantly better results here. However, smaller differences can be observed for the DSOMA and HGSA columns, sometimes with negative values (for example, in the case of Ta012, the HGSA is -2.39%).

Ta014 – Ta020: The maximum fitness values of ESAS range from 1476 to 1697.

The differences are 20% for the HMM-PFA, HGA and IIGA methods. Smaller differences are observed for the DSOMA and HGSA values. They show that although the ESAS algorithm largely outperforms the other methods. HGSA can achieve better performance on some datasets.

Based on the analysis of Table 6, it can be said that the Elitist Strategy of Ant System algorithm is significantly better in terms of maximum values, often by 10–20%. Although in some cases the results of HGSA outperform the performance of ESAS.

Table 7. Comparision of the average test results: Elitist Strategy of Ant System

Instance	ESAS	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1297	14.57	11.72	14.57	5.94	2.08
Ta002	1367.6	11.73	6.76	11.73	2.95	5.44
Ta003	1153.6	26.56	20.15	26.56	10.96	-4.82
Ta004	1370.6	15.86	10.97	15.86	5.65	7.18
Ta005	1282.2	13.01	9.42	13.01	4.59	0.69
Ta006	1247.2	18.75	14.66	18.75	9.28	11.53
Ta007	1264	17.33	15.59	17.33	9.26	2.77
Ta008	1269	16.78	12.92	16.78	8.67	1.81
Ta009	1295.4	13.40	7.92	13.40	5.99	0.82
Ta010	1168.6	17.83	13.30	17.83	9.79	5.51
Ta011	1712.4	19.36	14.17	17.44	-0.84	0.04
Ta012	1782	21.55	19.14	21.55	2.86	-3.59
Ta013	1608.6	20.60	18.86	20.60	4.19	-3.33
Ta014	1490.8	21.48	19.53	21.48	3.70	1.69
Ta015	1560.8	23.85	23.85	23.85	3.60	0.78
Ta016	1510	25.30	20.99	25.30	5.30	-3.51
Ta017	1584	23.93	22.73	23.93	2.40	2.40
Ta018	1663.8	23.63	20.57	23.63	4.04	5.12
Ta019	1686	17.02	13.17	17.02	3.62	-3.68
Ta020	1707	20.15	17.22	20.15	4.39	0.88

Table 7 compares the average test value of the Elitist Strategy of Ant System algorithm with the results published by researchers.

Ta001: The average running value of the ESAS algorithm is 1297. The HMM-PFA, HGA and IIGA methods gave 14.57% higher results. In the case of DSOMA, the difference is 5.94%, while HGSA is 2.08%.

Ta002: The ESAS value is 1367.6. The HMM-PFA shows a difference of 11.73%, HGA 6.76% and IIGA 11.73%. The differences between DSOMA and HGSA are 2.95% and 5.44%, respectively.

Ta003: The average fitness value of ESAS is 1153.6. In this case, HMM-PFA and IIGA produced a difference of 26.56%, HGA a difference of 20.15%. DSOMA has a difference of 10.96%. In the case of HGSA we can see a difference of -4.82%.

Ta004: The ESAS value is 1370.6. The HMM-PFA and IIGA differ by 15.86%, and the HGA by 10.97%. The DSOMA and HGSA show a difference of 5.65% and 7.18%, respectively.

Ta005: The ESAS algorithm has an average value of 1282.2. The HMM-PFA and IIGA have a difference of 13.01%, and the HGA has a difference of 9.42%. The DSOMA has a difference of 4.59% and the HGSA has a difference of 0.69%.

Ta006: The ESAS has an average value of 1247.2. The HMM-PFA has a difference of 18.75%, the HGA has a difference of 14.66%, and the IIGA has a

difference of 18.75%. DSOMA and HGSA result in a difference of 9.28% and 11.53%, respectively.
Ta007 – Ta008: ESAS values are 1264 fitness (Ta007) and 1269 fitness (Ta008). HMM-PFA and IIGA have 17.33% and 16.78% differences for these data sets. HGA shows 15.59% and 12.92% differences. DSOMA and HGSA values have 9.26% and 2.77% differences (Ta007) and 8.67% and 1.81% differences (Ta008).
Ta009 – Ta010: ESAS averages are 1295.4 (Ta009) and 1168.6 (Ta010). For Ta009, HMM-PFA and IIGA produce a difference of 13.40%, HGA a difference of 7.92%, while DSOMA and HGSA show a difference of 5.99% and 0.82%, respectively. For Ta010, HMM-PFA, HGA and IIGA produce a difference of 17.83%. DSOMA has a difference of 9.79%, while HGSA has a difference of 5.51%.
Ta011 – Ta013: The average ESAS values here are 1712.4 (Ta011), 1782 (Ta012) and 1608.6 (Ta013). In the case of the HMM-PFA, HGA and IIGA methods, we can see a difference of 19–21%. For the DSOMA and HGSA columns, smaller, sometimes negative differences also occur (for example, Ta012: HGSA -3.59%). These methods achieve better results in some cases.
Ta014 – Ta020: The ESAS fitness values are 1490.8 (Ta014) and 1707 (Ta020). For these data sets, the HMM-PFA, HGA and IIGA methods usually produce a difference of 20–25%, while for the DSOMA and HGSA columns the differences are smaller (usually around 2–5%).
The average values of the ESAS algorithm in the table range from 1168.6 to 1712.4, depending on which TaXXX data set is involved.

Table 7. Comparision of the maximum test results: Elitist Strategy of Ant System

Instance	ESAS	HMM-PFA %	HGA %	IIGA %	DSOMA %	HGSA %
Ta001	1297	14.57	11.72	14.57	5.94	2.08
Ta002	1368	11.70	6.73	11.70	2.92	5.41
Ta003	1162	25.65	19.28	25.65	10.15	-5.51
Ta004	1388	14.41	9.58	14.41	4.32	5.84
Ta005	1296	11.81	8.26	11.81	3.47	-0.39
Ta006	1252	18.29	14.22	18.29	8.87	11.10
Ta007	1271	16.68	14.95	16.68	8.65	2.20
Ta008	1277	16.05	12.22	16.05	7.99	1.17
Ta009	1298	13.17	7.70	13.17	5.78	0.62
Ta010	1178	16.89	12.39	16.89	8.91	4.67
Ta011	1725	18.49	13.33	16.58	-1.57	-0.70
Ta012	1795	20.67	18.27	20.67	2.12	-4.29
Ta013	1618	19.90	18.17	19.90	3.58	-3.89
Ta014	1509	20.01	18.09	20.01	2.45	0.46
Ta015	1573	22.89	22.89	22.89	2.80	0.00
Ta016	1519	24.56	20.28	24.56	4.67	-4.08
Ta017	1595	23.07	21.88	23.07	1.69	1.69
Ta018	1671	23.10	20.05	23.10	3.59	4.67
Ta019	1695	16.40	12.57	16.40	3.07	-4.19
Ta020	1714	19.66	16.74	19.66	3.97	0.47

Table 7 compares the maximum running values of the Elitist Strategy of Ant System with the results of the researchers.
Ta001: The maximum value of the ESAS algorithm is 1297. The HMM-PFA, HGA, and IIGA methods show a difference of 14.57%. These algorithms achieved

results that were about 14–15% worse. DSOMA has a difference of 5.94% and HGSA has a difference of 2.08%.

Ta002: The ESAS fitness value is 1368. The differences are 11.70%, 6.73%, and 11.70% for HMM-PFA, HGA, and IIGA. The DSOMA method shows a difference of 2.92%, while the HGSA method shows a difference of 5.41%.

Ta003: In this data set, the maximum value of ESAS is 1162, compared to which HMM-PFA and IIGA show a difference of 25.65%, and HGA a difference of 19.28%. DSOMA produces a difference of 10.15%, while HGSA has a value of -5.51%, which indicates that the HGSA method has achieved a better (lower) maximum value than the ESAS algorithm. Ta004: The maximum value of ESAS is 1388. HMM-PFA and IIGA methods show a difference of 14.41%, and HGA a difference of 9.58%. DSOMA shows a difference of 4.32%, while HGSA shows a difference of 5.84%, so here too the advantage of the ESAS algorithm is evident, although the differences are smaller in the case of DSOMA and HGSA.

Ta005: The ESAS value is 1296, and the HMM-PFA and IIGA methods show a difference of 11.81%. While the HGA method shows a difference of 8.26%. The DSOMA method results in a difference of 3.47%, while the HGSA method shows only a difference of -0.39%. The latter indicating that the HGSA method gave almost the same or slightly better results.

Ta006: The ESAS maximum fitness value is 1252, to which the HMM-PFA shows a difference of 18.29%, the HGA shows a difference of 14.22% and the IIGA also shows a difference of 18.29%. The DSOMA and HGSA methods result in a difference of 8.87% and 11.10%, respectively, which indicates that the ESAS shows a significant advantage in this data set.

Ta007 – Ta008: ESAS values were fitness 1271 (Ta007) and 1277 (Ta008). The differences of HMM-PFA, HGA and IIGA are 16.68%/16.05% and 14.95%/12.22%, respectively. For the DSOMA algorithm, the differences are 8.65% (Ta007) and 7.99% (Ta008). The HGSA values indicate a difference of 2.20% and 1.17%, respectively.

Ta009 – Ta010: For Ta009, the ESAS fitness maximum is 1298. HMM-PFA and IIGA result in a difference of 13.17%, HGA results in a difference of 7.70%, DSOMA results in a difference of 5.78%, while HGSA results in a difference of 0.62%. For Ta010, the ESAS fitness value is 1178. The deviations are of the order of 16.89% (HMM-PFA and IIGA), 12.39% (HGA), 8.91% (DSOMA) and 4.67% (HGSA).

Ta011 – Ta013: ESAS maximum values are 1725 for Ta011, 1795 for Ta012 and 1618 for Ta013. For HMM-PFA, HGA and IIGA, the deviations are generally in the order of 18–20%. However, for DSOMA and HGSA, smaller negative deviations are observed (e.g. Ta011: -1.57% and -0.70%).

Ta014 – Ta020: ESAS maximum values in this range range from 1509 to 1714. For HMM-PFA, HGA and IIGA, the deviations are generally in the order of 19–24%. For DSOMA and HGSA, the deviations are in the order of 2–4%.

Analytical conclusion

Based on the run tables of the Hill Climbing and Elitist Strategy of Ant System algorithms, the following conclusions can be drawn:

- General performance differences: The ESAS algorithm achieved lower or equal values compared to the HC algorithm for all tested instances, especially for the maximum and average solutions. This suggests that ESAS is more efficient in approximating the global optimum, since Hill Climbing tends to get stuck in local optima.
- Comparison of maximum values: For example, for Ta001, HC max = 1377,

while ESAS max = 1297, which is an 80-unit improvement in favor of ESAS. A similar trend is seen for most instances (e.g. Ta004, Ta005, Ta011), where ESAS consistently produces lower maximum values.

- Average performance: The average (Avg) values also show that ESAS is more stable and consistent in approaching better solutions. For the HC algorithm, a larger variance is observed between the average and minimum values, indicating instability of the solutions.
- Minimum values and stability: The minimum values of the ESAS algorithm are generally closer to the mean, indicating that the method is less sensitive to random deviations. For the HC algorithm, the minimum values often differ significantly from the maximum and average values, indicating the tendency of the method to local optimization.
- Summary conclusion: The ESAS algorithm provides better and more stable performance on the examined Taillard data sets, especially for complex problems. Hill Climbing is a fast and simple method, but it tends to get stuck in local optima and shows a larger variance in the results. Based on the comparison, ESAS is recommended for more efficient and reliable optimization for this type of flow shop problems.

4. Summary

In this article, I tested the efficiency of two metaheuristic algorithms, the Hill Climbing and the Elitist Strategy of Ant System, on a frequently used production scheduling task, the Flow Shop Scheduling task. The Hill Climbing algorithm operates on a single possible solution, while the Elitist Strategy of Ant System operates on a set of solutions. These algorithms start from one or more possible solutions, then continuously improve or create new solutions, and finally return with the best solution found if a stopping condition is met. The stopping condition can be reaching a certain number of iterations, running time, or convergence. The test results were run on the Taillard data set. The article reports the minimum, maximum, and average of the run results. In addition, these results are compared with results published by other researchers. The article shows that the Elitist Strategy of Ant System algorithm is efficient, and this algorithm is worth using in the case of Flow Shop Scheduling.

Future research areas include, on the one hand, examining the effectiveness of different metaheuristics and creating hybrid metaheuristics. On the other hand, examining production scheduling tasks that are much more complex than the aforementioned Flow Shop Scheduling task and better model real-world tasks.

Acknowledgement. Supported by the University Research Scholarship Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund.”

EKÖP Egyetemi Kutatói Ösztöndíj Program

Ministry of Culture and Innovation

References

[1] Ruiz, R., & Vázquez-Rodríguez, J. A. (2010). The hybrid flow shop scheduling problem. European journal of operational research, 205(1), 1-18. https://doi.org/10.1016/j.ejor.2009.09.024

[2] Abuhamdah, A. (2021). Adaptive elitist-ant system for solving combinatorial optimization problems. Applied Soft Computing, 105, 107293. https://doi.org/10.1016/j.asoc.2021.107293

[3] Colorni, A., Dorigo, M., Maniezzo, V., & Trubian, M. (1994). Ant system for job-shop scheduling. JORBEL-Belgian Journal of Operations Research, Statistics, and Computer Science, 34(1), 39-53.

[4] Mitchell, M., Holland, J., & Forrest, S. (1993). When will a genetic algorithm outperform hill climbing. Advances in neural information processing systems, 6.

[5] Escamilla-Serna, N. J., Seck-Tuoh-Mora, J. C., Medina-Marin, J., Barragan-Vite, I., & Corona-Armenta, J. R. (2022). A hybrid search using genetic algorithms and random-restart hill-climbing for flexible job shop scheduling instances with high flexibility. Applied Sciences, 12(16), 8050. https://doi.org/10.3390/app12168050

[6] Chu, F., Liu, M., Liu, X., Chu, C., & Jiang, J. (2018). Reentrant flow shop scheduling considering multiresource qualification matching. Scientific Programming, 2018(1), 2615096. https://doi.org/10.1155/2018/2615096

[7] Vaughan, D. E., Jacobson, S. H., Hall, S. N., & McLay, L. A. (2005). Simultaneous generalized hill-climbing algorithms for addressing sets of discrete optimization problems. INFORMS Journal on Computing, 17(4), 438-450. https://doi.org/10.1287/ijoc.1040.0064

[8] Zheng, X., Zhou, S., Xu, R., & Chen, H. (2020). Energy-efficient scheduling for multi-objective two-stage flow shop using a hybrid ant colony optimisation algorithm. International Journal of Production Research, 58(13), 4103-4120. https://doi.org/10.1080/00207543.2019.1642529

[9] Huang, K. L., & Liao, C. J. (2008). Ant colony optimization combined with taboo search for the job shop scheduling problem. Computers & operations research, 35(4), 1030-1046. https://doi.org/10.1016/j.cor.2006.07.003

[10] Van der Zwaan, S., & Marques, C. (1999, April). Ant colony optimisation for job shop scheduling. In Proceedings of the’99 Workshop on Genetic Algorithms and Artficial Life GAAL’99.

[11] Flórez, E., Gómez, W., & Bautista, L. (2013). An ant colony optimization algorithm for job shop scheduling problem. arXiv preprint arXiv:1309.5110.

[12] Qu, C., Fu, Y., Yi, Z., & Tan, J. (2018). Solutions to no-wait flow shop scheduling problem using the flower pollination algorithm based on the hormone modulation mechanism. Complexity, 2018 https://doi.org/10.1155/2018/1973604

[13] Wei, H., Li, S., Jiang, H., Hu, J., & Hu, J. (2018). Hybrid genetic simulated annealing algorithm for improved flow shop scheduling with makespan criterion. Applied Sciences, 8(12), 2621 https://doi.org/10.3390/app8122621

[14] E. Taillard, "Benchmarks for basic scheduling problems", EJOR 64(2):278-285, 1993.