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Harmony Search for Flow Shop Scheduling Problems: A Comprehensive Review

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Abstract. This article is a comprehensive review that presents the Harmony Search algorithm and how the Flow Shop Scheduling task can be solved with it. Harmony Search is a population-based metaheuristic algorithm. It is inspired by musical harmony. Flow Shop Scheduling is one of the best-known production scheduling tasks. During the task, n jobs must be performed by m machines, such that the jobs must be processed sequentially through each machine. The objective function is most often the makespan minimization. The article presents the Flow Shop task, the Harmony Search algorithm, and then the search results of the Google Scholar and Elsevier databases. After that, the most important articles published on the topic are presented and summarized in a table.

Keywords: Harmony Search, Flow Shop Scheduling, Review

1. Introduction

The purpose of this article is to demonstrate the power of the Harmony Search algorithm in the field of production scheduling problems. The most common such production scheduling problem was selected, which is the Flow Shop Scheduling Problem.

In the Flow Shop Scheduling [1] problem, a given number of jobs must be processed on multiple machines in the same order. Each job runs in the same machine order. A machine can only process one job at a time. A job can only be processed by one machine at a time. The machine cannot interrupt the processing of the job. The goal is the makespan minimization. Several variants of the problem have been developed, which are adapted to production environments:

- No-Wait Flow Shop (NWFS) [2]: A job cannot wait between two consecutive machines.
- Blocking Flow Shop (BFSP) [3]: If the next machine is busy, the job blocks on the previous machine (until then, the next job cannot be started on the previous machine).
- Flexible Flow Shop [4]: The levels have multiple parallel machines.
- Distributed Flow Shop (DFSP) [5]: There are several factories in the system and each factory has a Flow Shop structure. Jobs must first be assigned to a factory.
- Flow Shop with Sequence-Dependent Setup Times (SDST-FS) [6]: the system includes different setup times, where the setup time depends on both the previous job and the current job.

- Flow Shop with Release Dates [7]: not all jobs are available at the start time. There are jobs that are only available later.

Harmony Search [8] is a population-based metaheuristic that continuously improves the population of solutions. The algorithm was originally developed for continuous tasks, so a discretization process is required for discrete tasks. The inspiration for the algorithm comes from the process of musical improvisation. It is inspired by the fact that musicians draw from previous sounds, make small modifications to them, or try something completely new. The main components of the algorithm are:

- Harmony Memory (HM): Harmony Memory is a memory structure that stores: HMS (Harmony Memory Size) pieces of solutions and contains the best solutions so far.
- HMCR – Harmony Memory Considering Rate: The probability of selecting the value of a variable from memory.
- PAR – Pitch Adjustment Rate: The probability that the selected value will be modified.

The algorithm consists of the following steps:

1. Initialization: Parameters (HMS, HMCR, PAR, bw) setting and random initialization of Harmony Memory
2. Improvising a new harmony: Randomly selecting a value from Harmony Memory with probability HMCR, and generating a random value with probability 1-HMCR. Taking a small modification to the new harmony with probability PAR.
3. Memory updating: If the new solution is better than the worst element of HM, then it is replaced.
4. Checking the stopping condition: e.g. number of iterations, running time, convergence, etc.

The article then presents the application of the Harmony Search algorithm to the Flow Shop task. First, the article details the search results for the Google Scholar and Elsevier databases.

2. Search Methodology

2.1. Google Scholar search

This subsection presents search results from the Google Scholar database.

Figure 1. Google Scholar search:
"Harmony Search" and "review"

Figure 1 shows the Google Scholar search results, where the following keywords were investigated: "Harmony Search" and "review". The search is between 2010 and 2025. It can be seen that the number of articles published per year increased continuously until 2022. In 2010, 202 articles were published, in 2015 1180 articles, in 2020 3400 articles, while in 2025 4180 publications were published.

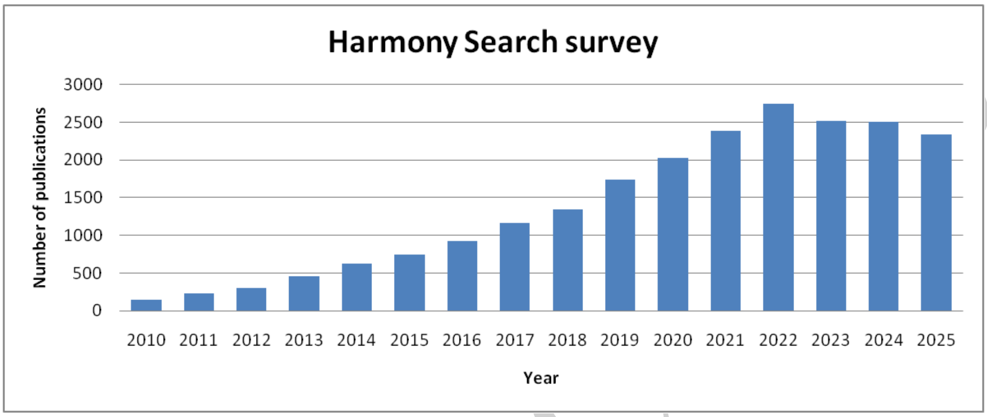

Figure 2. Google Scholar search:
"Harmony Search" and "survey"

Figure 2 shows the results for the keyword survey. Here too, we can see an increasing trend until 2022, followed by a small decrease in the number of annual publications. In 2010, 147 articles were published, in 2015 747, in 2020 2020 publications, and in 2025 2340 articles were published.

Figure 3. Google Scholar search:
"Flow Shop" and "Harmony Search"

The next search was for the keywords "Flow Shop" and "Harmony Search" (Figure 3.). According to the results, the number of articles has been increasing continuously (almost annually). In 2010, 33 articles were published, in 2015, 145 articles, in 2020, 228 articles and in 2025, 218 articles were published.

2.2. Elsevier search

This subsection presents the search results in the Elsevier database for various keywords. It presents the number of publications per year, the article type, and the subject areas.

Figure 4. Elsevier search: "Harmony Search" and "review"

Figure 4 illustrates that the number of articles increased continuously until 2022. In 2010, 14 articles were published, in 2015 167, in 2020 408, and in 2025 616 articles were written by the researchers.

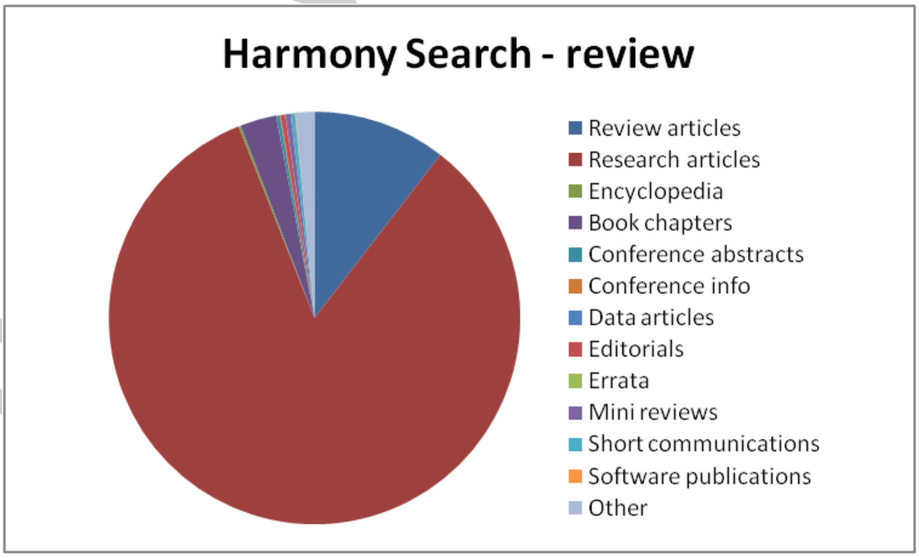

Figure 5. Elsevier search – article type: "Harmony Search" and "review"

Figure 5 shows the article type. It can be seen that research articles were the most common type (4124). There were also a number of review articles (516) and book chapters (140).

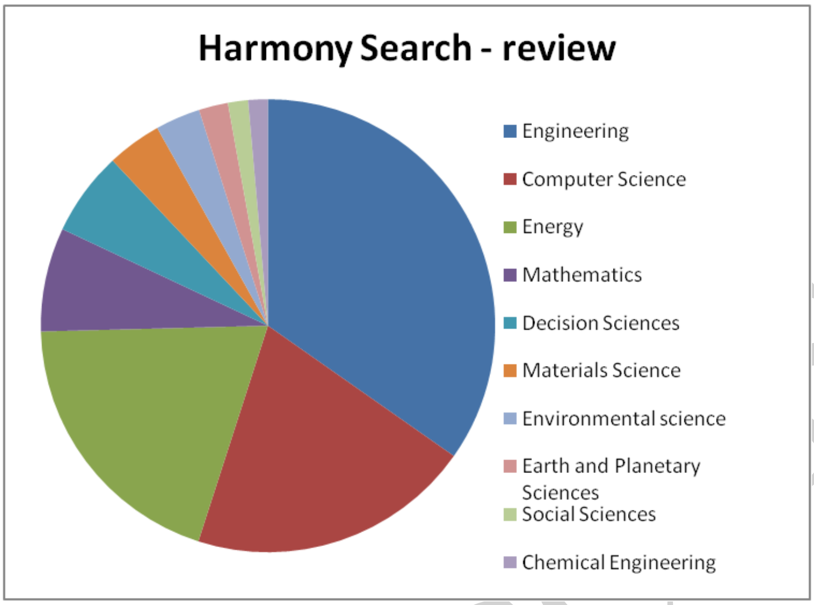

Figure 6. Elsevier search –subject areas: “Harmony Search” and “review”

Figure 6. illustrates the subject areas. It can be seen that most articles were in the engineering, computer science, and energy topics.

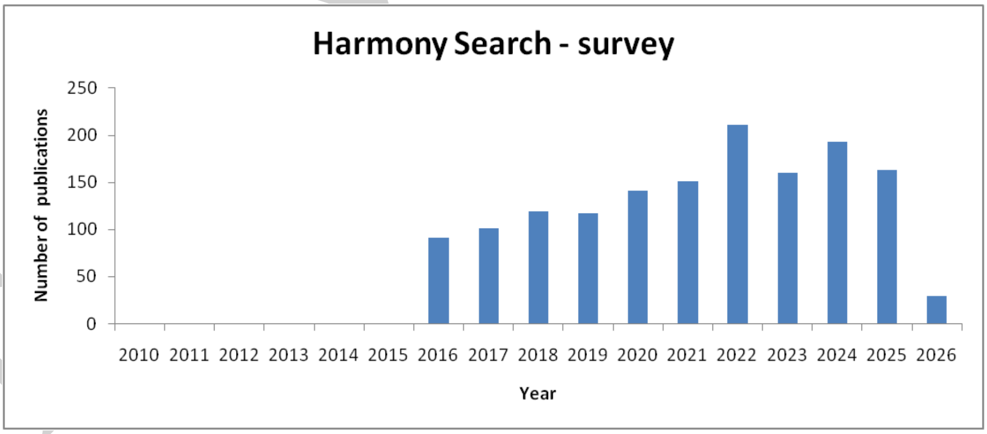

Figure 7. Elsevier search: “Harmony Search” and “survey”

Figure 7 shows the publications by year. We find articles for the keywords “Harmony Search” and “survey” starting in 2016. In 2020, 141 articles were published, and in 2025, 163 articles were written.

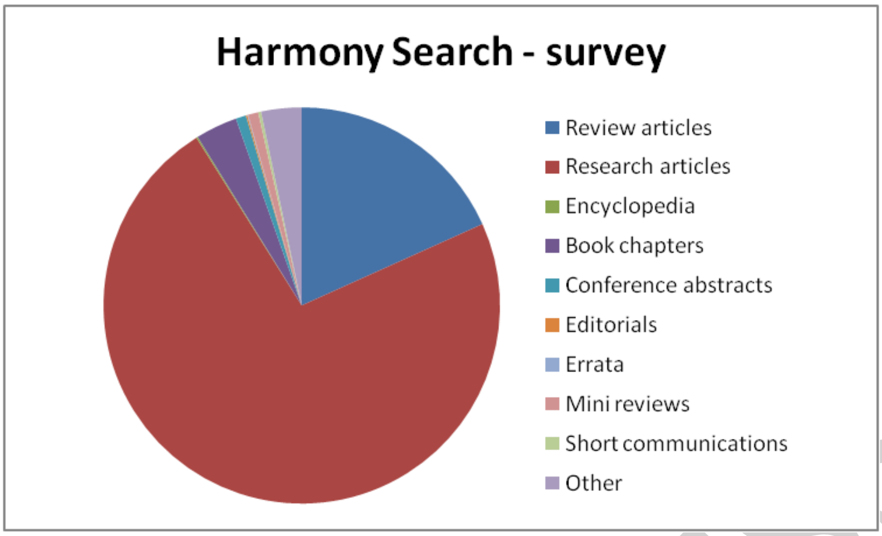

Figure 8. Elsevier search – article type: "Harmony Search" and "survey"

Figure 8 shows the article type. The most common type here is research article (1216). However, we also find articles in review article, book chapter and other types.

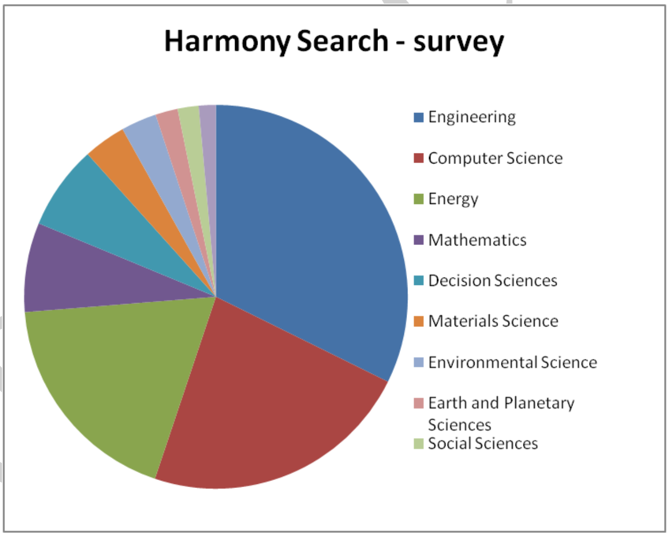

Figure 9. Elsevier search –subject areas: "Harmony Search" and "survey"

In terms of subject area (Figure 9), the following are the most important: engineering, computer science, energy, mathematics, decision sciences.

Figure 10. Elsevier search : "Harmony Search" and "Flow Shop"

Figure 10 shows the number of articles per year for the keywords "Harmony Search" and "Flow Shop". It can be seen that 4 articles were published in 2010, 21 articles in 2015, 25 articles in 2020 and 19 articles in 2025.

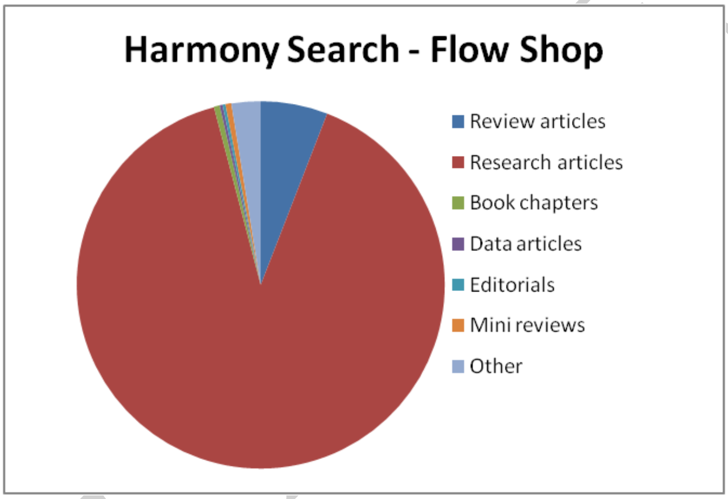

Figure 11. Elsevier search –article type: "Harmony Search" and "Flow Shop"

Figure 11 shows the article type. Here too, research articles were the most common, but we can also find articles on review and other topics.

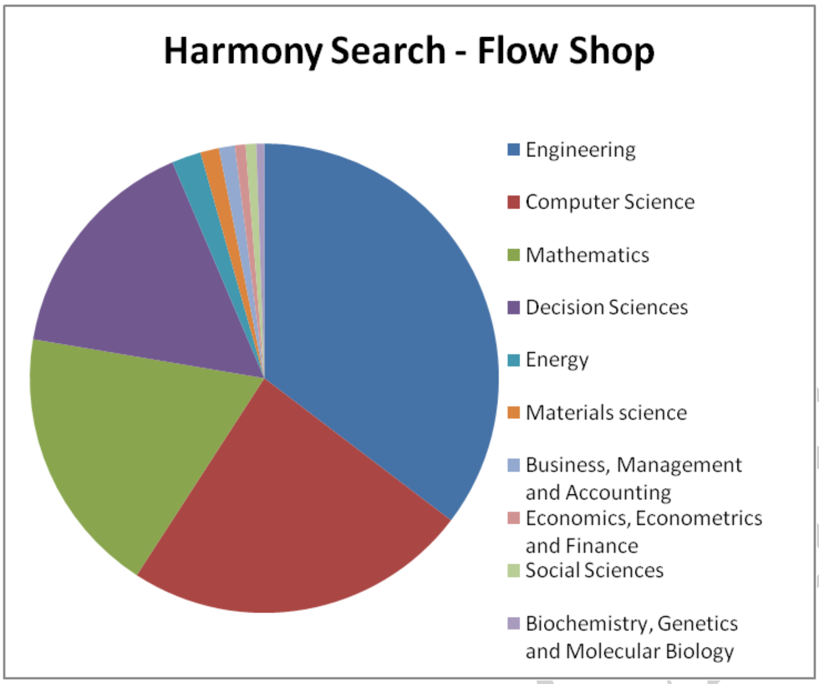

Figure 12. Elsevier search – subject areas: "Harmony Search" and "Flow Shop"

Figure 12 contains the subject areas diagram. It can be seen that engineering, computer science, mathematics, and decision sciences had the most publications.

3. Solving Flow Shop Scheduling Problems with Harmony Search: most important articles

This section presents the most important articles related to the Flow Shop Scheduling task and the Harmony Search algorithm. First, the descriptions of the articles, followed by a tabular summary containing the reference, authors, year of publication, problem and solution method.
In the publication of [9], the authors solve a multi-objective Flexible Job Shop Scheduling problem. The discrete Harmony Search algorithm is used to solve the problem. The problem has two objective functions: minimum completion time (makespan), and mean of tardiness and early completion. The coding consists of two parts during the task: Machine Assignment (MA) vector specifies which machine will perform each operation. The Operation Sequence (OS) vector determines the order of operations during processing. The algorithm uses several techniques for machine selection, such as the global minimum-processing time rule, two-step greedy rule. The authors also compared the test results with the following algorithms: AL + CGA, MOGA, MOPSO + LS, HSFLA, EPABC, PGDHS. The efficiency of the algorithms was tested for the following complexities, where n is the number of tasks and m is the number of machines: (n=4, m=5), (n=8, m=8), (n=10, m=7), (n=10, m=10), (n=15, m=10), and (n=10, m=6), (n=15, m=8), (n=15, m=4), (n=10, m=15), (n=20, m=5), (n=20, m=10), (n=20, m=15).
In the publication of [10], a Permutation Flow Shop Scheduling problem is solved,

which is of blocking type. The problem is solved using a hybrid Harmony Search algorithm. Here too, the goal is to minimize the total completion time (makespan). The authors present an efficient initial population initialization based on the Nawaz–Enscore–Ham (NEH) heuristic. The problem was solved using a permutation representation method, where the INSERT, SWAP and INVERSE operators were applied. The authors used the Taillard benchmark dataset to validate the efficiency of the algorithm. They examined data of all complexity, including small, medium and large sizes. Runs were also performed with the following algorithms to compare the efficiencies: mGHS, hmGHS, HGA, HPSO, IG, TS + M.

In the article of [11], the authors solve a Flow Shop Scheduling problem that also includes maintenance tasks. The machines are not available during the entire planned duration of the problem. Their availability is affected by random failures. They also perform corrective maintenance activities and planned maintenance operations in the plant. The goal is to optimally schedule work and maintenance so that processing times are minimized (makespan minimization). The authors developed an algorithm that combines Genetic Algorithm and Harmony Search.

In the publication of [12], the authors solve a Flow Shop problem where the goal is to minimize the total flow time. The algorithm also includes blocking type. The article presents three hybrid Harmony Search (HS) algorithms. The Harmony Search algorithm is also hybridized with local search. The test runs were performed on the Taillard benchmark set. Tests were performed on tasks of all sizes.

In the article of [13], the authors also solve the Flow Shop problem by using a modified Harmony Search algorithm to solve the problem. The authors present the test results of the DPSO, DLHS and HGA algorithms. Here, the runs are performed on the following complexity of test datasets, where n is the number of jobs and m is the number of machines: (m=3, n=7), (m=5, n=7), (m=3, n=10), (m=5, n=10), (m=3, n=20), (m=5, n=20), (m=10, n=20), (m=3, n=30), (m=5, n=30), (m=10, n=30), (m=3, n=40), (m=5, n=40), (m=10, n=40).

In [14], the authors solve a Flow Shop Scheduling problem involving a limited buffer. They use a chaotic version of Harmony Search. The authors use an efficient initialization scheme based on the Nawaz–Enscore–Ham (NEH) heuristic. The algorithm is validated on benchmark data, and the paper also presents and compares the efficiencies of several other algorithms, such as the IHS, GHS, CHS, and ACO algorithms.

In the publication of [15], the authors solve the production scheduling problem using a modified Harmony Search algorithm. They solve a Flow Shop Scheduling problem involving blocking. Harmony Search also uses neighbor heuristics. The Nearest Neighbor (NN) and Modified NEH construction methods (MNEH) were used for initialization. The paper uses operators such as swap neighboring heuristic, insert neighboring heuristic, insert reversed block neighboring heuristic, all-best neighboring heuristic. Test runs were created and compared for the following algorithms: Hybrid Whale Algorithm, Hybrid Genetic Algorithm, Tabu-mechanism Improved Iterated Greedy, Discrete Water Wave Optimization Algorithm, Discrete Self-Organizing Migrating Algorithm, Improved Iterated Greedy Algorithm. The Taillard benchmark dataset was used for the tests. Tests were made on the entire test dataset. In another experiment, the authors tested the efficiency with 16 other algorithms.

In [16], the authors developed an Island Harmony Search algorithm for the production scheduling problem. This problem is a Flow Shop Scheduling problem with uncertain processing times. The task is characterized by minimizing weighted delays while meeting deadlines. The uncertain processing times are given by a

large, discrete set of scenarios.
In [17], the Harmony Search algorithm is used to solve the No-Wait Flow Shop Scheduling problem. There are no-wait operations, and the setup time and processing time of some jobs are given separately. Not all jobs are available at the beginning of the schedule, so each job has a unique ready time. Machines are not always available due to preventive maintenance. The authors discuss this complex problem. Algorithms such as Population-Based Simulated Annealing (PBSA), the Adapted Imperialist Adversarial Algorithm (ICA), and a hybrid approach of PBSA and ICA (ICA+PBSA) are also tested.
In [18], the Hybrid Harmony Search algorithm was used to solve the Flow Shop Scheduling problem. Here, the authors used the NEH procedure for the initial solution set. The proposed algorithm is also compared with other algorithms, such as the Iterative Greedy Metaheuristic algorithm with a Limited Search Strategy, the improved Jaya algorithm. In this Flow Shop task, the tasks are first assigned to factories, and then the order of the tasks within the factories must be determined. The authors generated a new data set on which the test runs were performed.
In [19], the Flexible Job Shop Scheduling problem is solved. Here, the problem was solved with the discrete Harmony Search algorithm. The authors of the article used several improvisation operators in connection with Harmony Search, such as machine selection vector improvisation, order sequence vector improvisation. The authors used the Modified Intelligent Mutation (MIM) operator for mutation. The runs were performed on the following complexity test data, where n is the number of jobs and m is the number of machines: (m=6, n=10), (m=8, n=15), (m=4, n=15), (m=15, n=10), (m=5, n=20), (m=10, n=20), (m=15, n=20).
In [20], a version of the Job Scheduling problem is solved that is flexible and multi-objective. The authors solve it using the Discrete Harmony Search algorithm. It uses a weighted combination of two minimization criteria, which are: the maximum of the completion time (makespan) and the average value of the early/too late completion. The authors use the following machine assignment components: random rule, global minimum-processing time rule, two-step greedy rule, hybridization of operationminimum-processing time and local minimum-processing time rule.
In [21], the authors apply the Harmony Search algorithm with Levy Flight to a production scheduling problem, which is the Job Shop Scheduling problem. In this problem, each operation can be performed on several possible machines. The efficiency of the algorithm was validated on the 8×8, 10×10 and also 10 benchmark examples proposed by Brandimarte.

Table 1. Summary

Reference	Authors	Publication year	Problem	Solving method
[9]	Gao, K. Z., Suganthan, P. N., Pan, Q. K., Chua, T. J., Cai, T. X., & Chong, C. S.	2014	Multi-objective Flexible Job Shop Scheduling (FJSP); objective fuction: makespan + mean earliness & tardiness	Discrete Harmony Search (MA + OS encoding); comparison: AL+CGA, MOGA, MOPSO+LS, HSFLA, EPABC, PGDHS
[10]	Wang, L., Pan, Q. K., & Tasgetiren, M. F.	2011	Blocking Permutation Flow Shop Scheduling (makespan minimization)	Hybrid Harmony Search + NEH initialization; operators: INSERT, SWAP, INVERSE; comparison: mGHS, hmGHS, HGA, HPSO, IG,

				TS+M
[11]	Branda, A., Castellano, D., Guizzi, G., & Popolo, V	2021	Flow Shop Scheduling with maintenance (random failure, planned maintenance); makespan	Hybrid Genetic Algorithm + Harmony Search
[12]	Wang, L., Pan, Q. K., & Tasgetiren, M. F.	2010	Blocking Flow Shop; objective function: minimization of the Total Flow Time	Three hybrid Harmony Search + Local Search; Taillard benchmark
[13]	Pan, Q. K., Suganthan, P. N., Liang, J. J., & Tasgetiren, M. F.	2011	Three hybrid Harmony Search + Local Search; Taillard benchmark	Modified Harmony Search; comparison with the following algorithms: DPSO, DLHS, HGA
[14]	Pan, Q. K., Wang, L., & Gao, L.	2011	Flow Shop Scheduling with limited buffer	Chaotic Harmony Search + NEH initialization; comparison with the following algorithms: IHS, GHS, CHS, ACO
[15]	Doush, I. A., Al-Betar, M. A., Awadallah, M. A., Santos, E., Hammouri, A. I., Mafarjeh, M., & AlMeraj, Z.	2019	Blocking Flow Shop Scheduling	Modified Harmony Search + neighborhood heuristics (NN, MNEH, swap/insert/reversed block/all-best); comparison with several hybrid metaheuristics
[16]	Wang, B., Zhang, P., Wang, X., & Pan, Q.	2024	Flow Shop Scheduling with uncertain processing times; weighted tardiness	Island Harmony Search
[17]	Rahmanidoust, M., Zheng, J., & Rabiee, M.	2017	No-Wait Flow Shop Scheduling with ready time and maintenance	Harmony Search; comparison with the following algorithms: PBSA, ICA, ICA+PBSA
[18]	Shen, H., Cheng, Y., & Li, Y.	2025	Distributed Flow Shop Scheduling (order to factory + scheduling within factory)	Hybrid Harmony Search + NEH initialization; comparison with other algorithms: Iterated Greedy, Improved Jaya, new evolutionary algorithm
[19]	Gaham, M., Bouzouia, B., & Achour, N.	2018	Flexible Job Shop Scheduling	Discrete Harmony Search + improvisation operators + Modified Intelligent Mutation (MIM)
[20]	Gao, K. Z., Suganthan, P. N., Pan, Q. K., Chua, T. J., Cai, T. X., & Chong, C. S.	2016	Multi-objective Flexible Job Scheduling	Discrete Harmony Search; weighted makespan + mean earliness/tardiness
[21]	Li, J., & Zhou, Y.	2025	Job Shop Scheduling (flexible)	Harmony Search + Lévy Flight

4. Summary and future research directions

This article focuses on Harmony Search and Flow Shop Scheduling. A review was created, where the classic Flow Shop task and its most common variants were presented. In addition, the Harmony Search algorithm was introduced. Then, the results of searches in Google Scholar and Elsevier databases were presented. After that, the study reviewed the main articles published on the topic. A summary table also shows which Flow Shop task and which algorithm were used in the articles.

Based on the literature reviewed, Harmony Search (HS) and its various improved versions can be effectively applied to solve a wide range of complex scheduling problems. One of the defining directions of research is the combination of Harmony Search with local search and heuristic methods. Several studies use NEH-based initialization and various local modification operators, such as INSERT, SWAP and INVERSE operations. Nowadays, researchers also hybridize the Harmony Search algorithm with other metaheuristics, such as genetic algorithms, local search, NEH heuristics, neighborhood strategies.

A significant part of the problems contain complex constraints that pose a greater computational challenge, such as blocking conditions, limited buffers, no-wait requirements, machine maintenance, random failures, and uncertain processing times. In the case of problems, the goal is often not only to minimize the makespan, but also to investigate a complex objective function. This indicates that Harmony Search may be suitable for handling decision-making situations where the joint optimization of competing performance indicators is required.

Future research direction could be additional reviews in the field of metaheuristic algorithms and production scheduling, as well as solving specific production scheduling tasks and benchmark datasets with metaheuristics.

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