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# Genetic Algorithms as Optimalisation Procedures

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SUMMARY

*Drawing a parallel between biological and economic evolution provides an opportunity for the description of dynamic economic processes changing in time by using genetic algorithms. The first step in finding algorithms in biological and economic processes is to draw a parallel between the terms used in both disciplines and to determine the degree of elaboration of analogues. On the basis of these ideas it can be stated that most biological terms can be used both in economics and in the social field, which satisfies the essential condition for successful modeling.*
*Genetic algorithms are derived on the basis of Darwin-type biological evolution and the process starts from a possible state (population), in most cases chosen at random. New generations emerge from this starting generation on the basis of various procedures. These generating procedures go on until the best solution to the problem is found. Selection, recombination and mutation are the most important genetic procedures.*

## THE PLACE OF GENETIC ALGORITHMS IN THE SYSTEM OF OPTIMALISATION PROCEDURES

John Holland started dealing with computational modelling of genetic algorithms in 1975. These algorithms are derived on the basis of Darwin-type biological evolution and the process starts from a possible state (population), in most cases chosen at random. New generations emerge from this starting generation on the basis of various procedures. These generating procedures go on until the best solution to the problem is found. The characteristic features of the problem are coded with a determined string of words called chromosomes. Genetic algorithms produce a new population in the same way as generic reproduction. One or two entities are selected from the existing ones and with the help of generic processes new possible entities called descendants are generated.
Thus, a generic algorithm is an hospitalisation procedure which is essential to be placed in the system of procedures in order to establish its advantages and make it suitable for its application in practice. The optimalisation procedures can be classified into three groups. (Figure 1)
The gradient-based methods using differential calculus are the most widespread ones. The essense of the gradient-based direct method lies in the fact that the local extreme value can be found only if the most gradient steep is ascended or descended (local gradient). The indirect method is the best known optimalisation procedure where the local extreme value is found by zeroing the derivated function.
In the case of procedures based on detailed search a predetermined problem area is looked at under the rules of a particular algorithm, each point of it is analysed and evaluated taking into consideration the target function. With the application of the so-called stepping back technology the analysis of one and the same point several times can be avoided. As far as more complicated problems are concerned their problem areas are so large that the procedures based on detailed search cannot be applied because the final result cannot be achieved within reasonable time. In such cases supplementary algorithms are used which make the area smaller. But even the application of these algorithms do not allow to reduce the size of the area to such extend as to use the method based on detailed search efficiently. Neither can they be applied in the case of moderately complex problems.
The shortcomings of the above mentioned two procedure groups generated the development and application of methods based on random choice. The basic characteristic features of these methods lie in the fact that they do not examine the whole problem area. That is the reason why well before their application it is very essential t to define 'when we reach our target', namely the value of the target function that can 'already' be considered to be optimal. Obviously, this value usually means only local optimum. Thus, the application of the random choice method does not make obtaining the

global optimum easier. In spite of this fact in most cases the local optimum can also be used for solving practical problems. In the case of simulated cooling a method similar to physical cooling of materials is applied to search for local optimum point suitable for our purposes in the problem area. In the case of taboo search a taboo list is compiled in order to avoid stepping back and circulating round and round. As soon as the required step number and target value is achieved, the whole process is finished. The following method applying random choice is the genetic algorithm. The procedure - as introduced earlier – operates in the same way as the biological evolution. It conducts a multi-point search in the problem area. That is the reason why the process hardly 'freezes' and can provide several almost optimal solutions in the end.

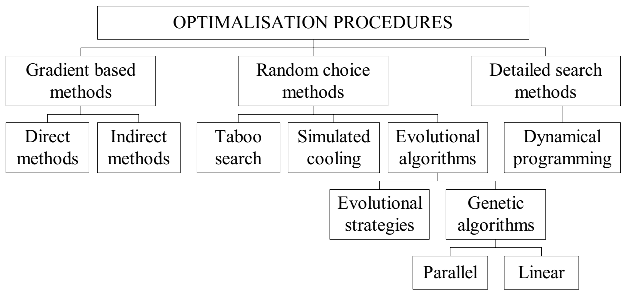


*Figure 1. Grouping of optimalisation procedures (Álmos et al., 2002)*

# PRACTICAL APPLICATION OF GENETIC ALGORITHMS

Generic algorithms are applied in several fields of science (technical and medical sciences). They are successfully applied in technical sciences in process design, network optimalisation, control engineering, automatisation and surface optimalisation. Besides the above mentioned ones they are also used in making up timetables, mapping and image processing. In the field of medical science they find their primary application in spatial segmentation of pictures taken of human bodies when various techniques are used.

## *Economic Applications*

Since the beginning of 1990 there have been several publications about application and applicability, testing and research of genetic algorithms in the description of economic processes. Arifovic (Arifovic, 1994) and Dawid (Dawid, 1997) applied genetic algorithms in Cobweb-type market model in order to examine the production and profit formation. Brenner (Brenner, 1998) was interested whether evolutional algorithms could be adapted to describing learning processes. Birchenhall (Birchenhall, 1995; Birchenhall et al., 1997) analysed the Cobb-Douglas type function of production. Lawrenz (Lawrenz, 1999) wanted to find out whether the securities market guided by genetic algorithms had as actual and rational results as the ones similar to a real situation. Arifovic (Arifovic, 1998) modelled the formation of exchange rates on the basis of two interrelated countries. Reichmann (Reichmann, 1999) with the help of genetic algorithms analysed the stability of economic attitude.

## *Genetic Algorithms in the Cobweb Model*

Arifovic (Arifovic, 1994) provided analysis of several variants of genetic algorithms from the view point whether in a simple Cobweb-model genetic algorithms lead to the same results as economic processes observed in real life. He claimed that genetic algorithms simulated learning ability of the companies aiming to capture the market. According to the model market demands are external possibilities for companies:

$$p(t) = A - B \cdot \sum_{i=1}^{n} P_i(t) ,$$

where p(t) shows the price in t period, $P_i(t)$ means the production of the i[th] company.

According to one of the ideas the learning algorithm is the so-called one population algorithm where each and every individual ($P_i(t)$, i=1,…, n) of the population (P(t)) of the genetic algorithm represents a company, that is its decision about the volume of production. The entity (company) namely the produced amount can be expressed by binary code (genotype). For instance, let us take a company ($P_1(1)$), whose volume of production is 1001010110110100 encoded, which means 38324 units of production in the period of t=1. It is obvious that each company has costs of production: $C[P_i(t)]$. Making it simpler, but essentially reflecting the real situation, let us suppose that there is a direct proportion between production costs and volume of production:

$$C[P_i(t)] = a \cdot P_i(t), \text{ where: } 0<a<1.$$

The profit – in the terminology of genetic algorithm it is the entity's fitness value – on which the decision about the volume of production depends can be defined as follows:

$$Q[P_i(t)] = P_i(t) \cdot p_t - C[P_i(t)].$$

This value plays an essential role in selection. If we assume that in the initial stage the $p_1=1$ and a=0,7, the fitness value of an entity is as follows:

$$Q[P_1(1)]=38324 \cdot 1-0{,}7 \cdot 38324=11515{,}2.$$

The selection between various quantitative decisions is made on the basis of the accustomed method of the genetic algorithm. The probability of the particular entity to be selected corresponds to the relative fitness value. If the solution to the above mentioned problem is developed on and it is assumed that the total fitness value of the population (total production) equals to 945321, the relative fitness value of the entity is $P_1$, that is the survival probability in biological term is:

$$Q_r[P_i(1)] = \frac{11515{,}2}{945321} \approx 0{,}01$$

Arifovic developed the Holland-type basic algorithm further, introduced the so-called selection operator. The operator operates as follows: each company takes new decisions about the volume of production during a certain period of time [$P_i(t)$]. Both recombination and mutation play essential roles in decision making. However, before this amount is really produced and launched on the market, the company provides comparison analysis and takes into consideration the amount produced in the previous period $P_i(t-1)$, namely its profitability. The profit gained in the previous period $Q[P_i(t-1)]$ is compared with the one expected from the sales of the new amount, namely with the potential fitness. The company defines this figure on the basis of the available data, market prices and production costs of the previous period. Thus, the potential fitness value of the $P_i(t)$ quantity can be calculated as follows:

$$Q^p[P_i(t)] = P_i(t)\cdot p_{t-1} - C[P_i(t-1)].$$

Let us see how a company makes a new decision $P_1(2)$ on the basis of this model. We selected two previous production strategies $P_1(1)$ and $P_8(1)$. The $P_1(2)$ was established from two previous quantities with the help of recombination:

$P_1(1)$: 1001010110110100
$P_8(1)$: 0100110000111010
$P_1(2)$: 1001010110111010

After this the new entity $P_1(2)$ suffers a mutation on the fifth place:

$P_1(2)$: 1001110110111010

This genotype means 40378 units of production and its potential fitness value (profit) is:

$$Q^p[P1(2)] = 40378\cdot 1 - 0{,}7\cdot 40378 = 12113{,}4.$$

The company applying the selection operator actually produces the amount which has a higher fitness value (profit), so the $P_i(t)$ production volume is as follows:

$$P_i(t) := \operatorname{argmax}\{Q[P_i(t-1), Q^p[P_i(t)]\}.$$

In our example the company produces 40378 units in the second period, because the fitness value (12113,4) of this amount is higher than the actual fitness value of the previous period (11515,2).

The second possible variation is the so-called multipopulation algorithm. It assumes several 'parallel' populations at one time and every single population represents feasible decisions of a company. The existing and the most viable volume of production is the one that is actually chosen by the company, namely launched on the market. The company finds out the market price only when it enters the market. The multipopulation algorithm happens by analogy with the one population with the difference that decision possibilities of companies turn out to be more differentiated. It is assumed that there is no exchange of information between particular populations (companies) of the algorithm, for instance market experience is not shared.

Arifovic provides analysis of the effectiveness of the selection operator. He simulates one and multi population procedures with the operator and without it. In the case of modelling without the operator neither algorithm converges toward the equilibrium. Thus, it can be concluded that genetic algorithms are not applicable in real learning and optimum search processes and do not have relevant outcome. On the other hand, algorithm expanded with a selection operator does converge toward the equilibrium. The significant conclusion to the simulation is that the price even in the unstable Cobweb-model does not withdraw from the equilibrium. Both in the stable and unstable models the price and the amount converge toward the equilibrium. The convergence in its temporal function and at about equilibrium values is characterised by smaller fluctuation amplitudes than in the unstable case. These results coincide with the results gained during real observations, but on the other hand, they are inconsistent with the theory.

## *Modelling of Technical Changes*

According to Birchenhall (Birchenhall, 1995) technical progress is a social level learning process. In his model he looks at learning in a very broad meaning of this word. In his interpretation learning means spread of knowledge in the society on the one hand and acquisition of new knowledge, on the other. Regarding technical progress this definition involves not only 'real' innovation on the basis of which new 'ideas' start emerging, but spread of the existing technical knowledge and its completely new combinations as well. Birchenhall shows the ways how technological innovations and learning processes spread among mixed players of various economic spheres as a result of interaction.

His initial assumption is that the economic players do not have clear ideas about limits of technical facilities. He considers the change in the technical level to be a process which defines this limit and broadens it with the help of new and very promising ideas. He takes the so-called concepts of modular technologies as a basis. They are technologies consisting of components with defined functions.

According to the model economy consists of two basic sectors: engineers and financiers. Engineers and designers create technological variants from which financiers select and use the appropriate one. In the sectors (populations) behaviour and decision making mechanisms of entities are generated by genetic algorithms. There is a competition among engineers for technologies and only the ones for which there is a great demand can survive. This is the essence of the selective process. New technologies are developed either by placing the existing technological elements in a new context (recombination) or by establishment of new technological modifications (mutation). 'Users' create decision models in order to evaluate potential possibilities of technologies. Their activities involve market observation of available technologies and their evaluation on the basis of expected results. Decision making models are in competition with one another (selection) and that is the reason why they are reviewed and modified, their components are combined in a completely new way (recombination) or some components are newly modified (mutation). Financiers chose from technologies offered by engineers.

Different players imitate technologies having tremendous success, modify them all the time and assure resources for their application. The success of technologies is reflected in their profitability. The quality of the evaluation and decision models depends on their ability to explain the relative efficiency or failure of a strategy. Apart from the above mentioned genetic operators (recombination, mutation and selection) Birchenhall also applies the selection operator which in this model means comparison of profit-oriented technologies not introduced on the market yet.

After this let us see how the model is built up: in a farm a y domestic animal can be produced with the help of various technologies (X stands for technological set). $y=f(x)$, $x \in X$, $X=\{x^h \mid h=1,\dots,N\}$. This technologies consists of different units, the so-called modules:

$$x^h = (x_1^h, \dots, x_n^h).$$

The function of production is homogeneous and linear and expressed by components as:

$$f(x) = \prod_{i=1}^{n} x_i^{a_i} \text{ , where } \sum_{i=1}^{n} a_i = 1$$

In the genetic algorithm of the 'designer' sector (in other words technological algorithm) the population means set of technologies. Each entity corresponds to xh technology consisting of one module. The production factors $a_i$, $i=1,\dots,n$ are known to the engineers. Innovations, namely technological inventions originate during recombination and mutation between technologies. The x value of the technology used in the process of production is defined by the profit attained with the help of it:

$$\Pi(x) = R[f(x)] - C(x),$$

on the basis of previous ones:

$$\Pi(x) = \prod_{i=1}^{n} x_i^{a_i} - \sum_{i=1}^{n} x_i \text{ .}$$

Although according to the model the users (financiers) know the structure of the production function, they do not know the real values of production parameters ($a_i$). Consequently, they have no clear ideas about the real form of the profit function. Models are created in order to define the unknown factors. The users' models differ from each other only in supposed values ai. The supposed production parameters can be shown with the help of a vector in the form of $\hat{a}=(\hat{a}_1,\dots,\hat{a}_n)$ and the profit gained with the help of these technologies can also be defined:

$$\hat{\Pi}(x^h) = \hat{R}[\hat{f}(x)] - \hat{C}(x) \text{ .}$$

The users' sphere can also be expressed with the help of a genetic algorithm. Technological models to which profit values can be established create population:

$$\hat{\Pi}^k(x), k = 1,\dots,M \text{ .}$$

Profit values of the model are compared with the real profit of technologies appearing on the active market. The smaller the divergence between the 'model profit' $\hat{\Pi}(x^h)$ and the real profit $\Pi(x^h)$ is, the more applicable the model is. This divergence related to all existing technologies can be expressed as follows:

$$E(\hat{\Pi}, X) = \sum_{h=1}^{N} \left| \hat{\Pi}(x^h) - \Pi(x^h) \right| \text{ .}$$

The algorithm of the users certainly contains genetic operators, recombination, mutation and selection, which leads to replacement, change and selection of the best model. In this case the selection operator is also at disposal and creates connection between the model and technological algorithm, producers and users of technology. The x' technology changed by recombination or mutation will replace the technology applied at that particular time only if there is at least one user's model (k) which puts higher profit down to this technological variation, namely.

$$\hat{\Pi}^k(x') > \Pi(x)$$

On the basis of information about the introduced technology the users obtain ex post they can define the accuracy of the model ex ante.

Birchenhall repeated the algorithm of the technological and model sectors in completely different circumstances. The simulation proved his expectations namely that the technology is going towards equilibrium: the technical knowledge if spread evenly in the economy and the players in economy acquire new knowledge exponentially. As far as the selection operator is concerned he came to the conclusion that if the selection of the technology is not made on the basis of forecasting models of the users, the development of the technological algorithm does not depend on the model algorithm. Producers do not take into account their own knowledge based on results of the users' technological algorithms or their knowledge about market competitiveness of particular technologies. The comparison of outcomes of the algorithms run in quite different circumstances show that economies modelled by a selective operator develop quicker and in a broader circle than the ones without an operator.

# Conclusion

On the basis of the introduced examples it can be stated that with the help of genetic algorithms economic models make every effort to dinamise the statistic neoclassic models and this activity is proved in both models by a simple, one-factor fitness function. The weakness of the models lies in the fact that they use simple and linear correlation and target functions which do not coincide with reality and consequently considerably reduce their strength.

With the help of selection operator model processes (similar to economic processes) become manageable because the negative mutation and recombination in the population do not obtain validity at all, which speeds the achievement of the optimum solution up.

The multipopulation genetic algorithm model gives a more differentiated description of the behaviour of the economic players than that of the single population model. If a two-level analysis is conducted (company, branch), a parallel application of both seems to meet the objectives, because both company and market processes can be connected despite the time divergences in the processes.

On the basis of the above mentioned ideas it can be concluded that in the case of economic optimalisation problems genetic algorithms are advised to be used only when the problem area and target function are as complicated and complex as the ones in the real life. Otherwise any of the above mentioned optimalisation procedures lead to success.

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