Transforming the Net Present Value for a Comparable One

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SUMMARY

This study examines the nature of net present value. It defines the economic content of the net present value and mathematically proves that definition is correct. This economic content inducts that the net present values are not comparable. The study systematically eliminates the distortion affects. The net present value transforms into a special kind of rate, namely, the modified difference between the factual and the required rate of return. The ranking list according to this transformed net present value corresponds to the list according to the internal rate of return. This is a new cognition and a very important correspondence.
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INTRODUCTION, PURPOSE AND METHODS OF RESEARCH

The literature of decision preparation methods is very large and polychromatic. Even creating a comprehensive picture only about the literature of capital budgeting decisions is seems to be impassable. The discussion about the best method of capital budgeting decisions has been long and intensive. Many writings already were born in the 50s of the last century (Alchian, 1955; Solomon, 1956; Bierman – Smidt, 1957; and so on). Since then the discussion is running.

In the forefront of discussion are the net present value (NPV) and the internal rate of return (IRR) as rival methods. One of the peculiarities of the discussion is that increasingly appear more and more complicated refinements of the basic methods. One of the typical variant of this is the modified internal rate of return (MIRR). One of the academic supporters of MIRR is Kierulff. (Kireulff, 2008.)

This method was appearing in the 18th century, was newly rediscovered in the 50s of the last century, and nowadays is coming to the front as well. The MIRR is calculated as follows

$$MIRR = \sqrt[n]{\frac{\text{Future value of positive cash flows (with reinvestment rate)}}{\text{Present value of negative cash flows (with finance rate)}}} - 1,$$

where n is the number of total life-span of project. It would be difficult to define exactly the economic content of MIRR.

The net present value and the internal rate of return are two well known categories of capital budgeting decisions. During the decision preparation processes of investment projects both of them can provide useful information. The way of approaching and interpretation problems of the two methods and their calculation results as this appears in the literature are not cleared up deeply enough. Therefore a lot of inadequate explanations and contradictions occur in connection with them. For example the literature does not define exactly what does the sum of the net present value means. Despite of this in the literature exist a strong tendency – with improvements appearing time and again – emphasizes the advantages of net present value method and its better quality from the decision-making process point of view. This method is suggested for examination of an investment project's acceptability and to create a ranking list as well. In contrast, the practical experts usually prefer the internal rate of return method for both purposes in an important part of developed countries. For example Arnold - Hope (1990: 262-263) cite two surveys done in the 70s and 80s of the last century demonstrating that the largest British companies (that are otherwise able to pay the best experts, or to train specialists) definitely rank the internal rate of return higher than the net present value in practice. After this review they mention that lots of American surveys prove that the practice prefers the internal rate of return in the USA as well.

The knowledge of the content of the information used for decision making can provide practical aspects of choosing between the two methods. Reviewing the question of how the methodological process and the results of this match the logic of the management practice and the thinking routine of the decision makers is reasonable as well.

Business economics, among others, has the function of providing methodological aid, methodologically-founded ideas for economists in practice. Probably based on this function, the business economics literature often mentions the importance of the professional clarity and practical implications of the suggested methodology (e.g. Garrison, 1988: 712; Arnold - Hope, 1990: 260; H. Schmalen, 2002: 602-605). That is, the majority of the company experts can only apply the methodology correctly if they can fit methods to their way of thinking, if they can somehow connect them to the logic of the economic process.

This study intends to clarify some basic issues of the net present value. The main questions of the research are: applicability area of the method; economic content of the index number received based on the calculation; comparability of the net present values of different investment projects; purifying this index number from distorting effects. The internal rate of return is regularly used in the study to make the analysis and findings unambiguous.

The main methods of the research are the logical analysis and the use of understandable, relatively not difficult mathematical models. The proving ways of findings are logical and mathematical processes as well.

THE FUNDAMENTAL QUESTIONS OF NET PRESENT VALUE AND INTERNAL RATE OF RETURN METHODS

The methodologies of both procedures are widely known in the profession; therefore, this study does not explain the methodologies in detail. This section mainly serves as starting point, and to draw attention to some general issues.

The Required Rate of Return

Three decades ago the required rate of return was interpreted in the business economics literature as the interest rate, or the company's average rate of capital profitability. For example Clifton and Fyffe (1981) also mention these two requirements in their book: "…in the method of the net present value the required rate of return is the rate of return which is analogue with interest rate", as written on page 164. Later this content was slightly modified: "The required rate can be the interest rate or the effective rate of the company's own invested capital" (p. 329). In 90-th years of the last century in business economics became obvious that determination of the required rate of return is the same as the opportunity cost interpreted for capital and defined by microeconomics. Applying this approach became general in the business economics literature. Partly parallel with this process, the conception appeared in the financial studies that the required rate of return is not equal to both main elements of capital. The cost of capital applied to the equity and that applied to the debt cannot be the same. By this conception concerning the equity, besides the price of using the capital, the commodity market risk-premium requirements also must be returned; concerning the debt, however, that is enough if the interest is repaid. The required rate of return concerning the equity is typically defined as if this were independent from the level of indebtedness.

The most characteristic methodological solution for this is the weighted average cost of capital. Here the required yield rate from equity and the interest rate are averaged according to the weight rate of equity and debt. This solution seems to be the growing trend; however, the conception is not scientifically clarified. In the literature lots of critiques count with weighted average cost of capital as well. (E.g. Luehrman, 2007.) These critiques are very important, but always are not about the matter of the principle of a required yield rate.

Net Present Value Method

In theory, the net present value method is calculated so that the discounted sum of all cash outflows is subtracted from the discounted sum of all cash inflows that are associated with an investment project. The calculation can also be made with the time series of the difference of cash inflows and outflows. The annual differences are discounted and summarized. In business practice the logic of the calculation can be transformed: subtraction of the discounted sum of the investment expenses from the discounted sum of the difference of revenues and out-of-pocket costs. The content of the difference of revenues and out-of-pocket costs is very similar to the earnings before interest, taxes, depreciation, and amortization. According to this method the investment project is acceptable if the net present value is not less than zero.

Internal Rate of Return Method

The internal rate of return was earlier known in business economics as the time-adjusted rate of return (Garrison, 1988: 657). Calculation of the internal rate of return means searching for the rate of return which makes the cash inflow line and the outflow line equal to each other. In essence, that is the rate at which the net present value would be zero. In case of investment projects that can have only one internal rate of return, the internal rate of return shows the factual time-adjusted profitability rate of the investment. The acceptability criteria of the project are decided by how large factual profitability rate is generated compared to the required rate of return. The difference shows how large surplus rates (or lack of rates) are generated compared to the required rate of return. Generally, this difference needs not be defined numerically; this becomes visible when writing the two rates next to each other. Where the two rates are equal, that still means that economic efficiency and required profitability are exactly achieved.

Applicability

In the case of investment projects or activities where the yield effect of the decision can be calculated numerically in a relatively obvious way, the main question is whether the generated yields are enough to meet the required rate of return. The relatively correct solution of the choice between variants excluding each other includes comparison of return options as well.

A wide range of investment projects have no direct yield effect or this cannot be measured. Every investment item concerning an office building and equipment of administration can be classified as such. The yields of the individual elements of the technological chain of the production process, or the yield of a given machine or equipment in itself cannot be defined either. For example, in the case of a power plant the cash flows arising from a single turbine cannot be calculated.

At this question the economic survey sets as its goal to minimize the time-adjusted average annual repayment requirements connected to the given function performance. This type of decisions can be called as function-oriented decisions. The significant appearance in business economics of function-oriented capital budgeting decision methods has a history longer than half a century. In the function-oriented capital budgeting decisions the time-adjusted average amount of annual repayment requirement gives methodologically correct grounds for comparison.

Methodologically, measurable yield effect projects can also be divided into two groups. The business economics literature differentiates between investment projects with orthodox and unorthodox cash flow patterns (e.g. Arnold – Hope, 1990: 262-263). The name can be different; for example: typical and non-typical, conventional and non-conventional, orthodox and unorthodox cash flows. The financial literature usually does not handle this differentiation. (Using this differentiation is not a general practice in business economics literature, either.)

A main characteristic of the orthodox cash flow patterns is that the time series of the difference between cash inflows and cash outflows starts with one or more negative sign amounts, and from a point in time where this difference turns into positive first, this positive sign does not change. So no other year will be in the duration of the project when the amount of cash outflow will exceed cash inflow. Thus the cash flow series starts out with a negative sign member or members, and sign change occurs only once.

In the case of orthodox cash flow pattern projects, the project itself creates and produces all the yield elements appearing in the examination. So the examination of economic efficiency can follow the logical question arising before decision making, that yields of investments in connection with the project would be enough to fulfill the return requirements. Investment projects with orthodox cash flow patterns can have only one internal rate of return. This statement results from a theorem of Rene Descartes (French scholar and philosopher 1596-1650). From this as follows the time series of the difference of cash inflows and outflows may have as many internal rate of return, as the number of sign changes. The basic relationship can be considered as generally known our days.

Having more than one internal rate of return is the characteristic of unorthodox cash flow pattern projects only. Because of the financial literature usually does not handle the distinction of orthodox and unorthodox cash flow patterns often deny expedience of internal rate of return (e.g. Brealy - Myers, 1992: 76-82), and often says that the internal rate of return is without meaning (Hill, 2008: 36). These statements are valid only the case of unorthodox cash flow patterns.

Investments belonging to the group of unorthodox cash flow pattern projects are those in which sign changes occur at least twice in the cash flow series. (For example the outflow is larger than the inflow in case of periodical renovation, a partial rebuilding of traditional iron furnaces, or in the period of remediation works after the closing of open-pit mines.) A main characteristic of the majority of unorthodox cash flow pattern projects is that a part or the whole payback, once disengaged from the project, must later be reinvested into the same project. This reinvesting matter for the economic efficiency of the project how large yields can be achieved with amounts of money temporarily utilized in other areas or projects. (For a detailed explanation and demonstration with an example, see Illés, 2007.) Classical methods automatically assume that amounts temporarily invested result in yield effects according to required rate of return or the internal rate of return. When they are used, the yield effect inside the project becomes inextricably mixed up with the yield effect of amounts temporarily invested being assumed automatically by the method. As the result of all this, neither the net present value, nor the internal rate of return of unorthodox cash flow pattern projects provide adequate information for decision making. This relationship can be considered generally known in connection with the internal rate of return. However, in contrast with the suggestions in the literature even the net present value cannot be considered to be clear information in case of unorthodox cash flow pattern projects.

Thesis 1 Net present value and internal rate of return can only provide well interpreted, clear information in the case of investment projects with orthodox cash flow patterns.

THE ECONOMIC EFFICIENCY BY FOLLOWING UP THE REPAYMENT PROCESS, AND THE ECONOMIC CONTENT OF NET PRESENT VALUE

The economic efficiency of investments can be examined correctly by several methods, not only by the published ones. Such no familiar method can be the method based on following up the repayment process. The essence of this method is calculating for each given year until the end of the project the difference between the sum of cost of capital not repaid and the yield. At the end of the process the output is the sum of factual extra yield or lack of that compared to the required rate of return. The last step could lead to the net present value if this sum is discounted back to the zero point of duration (Illés, 1997).

The method itself involves significantly more steps than the net present value method and requires more background calculations. From this regard, this calculation process cannot compete with the net present value method. However, the extended information background gives the opportunity to survey and to follow the repayment process throughout time. Following up the repayment process can serve as a contemplation support for a company and can be stimulating, since the strategic visions and project performance can be followed up continuously throughout the life of the project.

This method is based on the fact that through discounting back the extra yield (or lack of that) generated by the end of the duration of the project leads to the net present value. According to this calculation the economic content of the net present value can be clearly defined. The net present value is the discounted sum of the surplus yield (or lack of that) generated above the yield requirement according to the required rate of return (Illés, 1990: 103-105). The following examination shows the deduction through a simple example, and then this context proves generally valid relationship applying to orthodox cash flow pattern projects.

Demonstrating the Follow-up of the Repayment Process Through a Simple Example

The cash flow series are calculated as the difference between the annual cash inflows and cash outflows of an investment project (in order of years; generated at the end of each year) as follows: - units 820; + units 420; + units 440 and + units 194. The required rate of return is 12%.

NPV = - 820 + 0.89286 × 420 + 0.79719 × 440 + 0.71178 × 194 =43.8 units.

The examined project meets the repayment requirement; besides surplus yield is generated with a present value of 43.8 units. The development of the repayment requirements through time and the repayment process itself is shown below (amounts counted in units).

The steps of calculation process:

At the end of Year 1: - 820× 1.12+420 = - 498.4

At the end of Year 2: - 498.4×1.12+440 = -118.2

At the end of Year 3: -118.2× 1.12 +194 = 61.6

NPV= 61.6× 0.71178= 43.8

Explanation of the calculation:

From all the repayment requirements occurring 420 units were repaid at the end of the first year. At that time remains 498.4 units to return. This 498.4 units and the required yield generated because this amount continuing to be locked up, that is 558.2 must be returned. At the end of the second year 440 units were repaid from the amount of repayment requirement. 118.2 units were not repaid. At the end of the third year a yield of 194 units are generated. This exceeds the 132.4 units required repayment, and leaves 61.6 units. This is the surplus compared to the required repayment at the end of the third year.

The surplus yield (61.6 units) generated at the end of the third year is discounted back to time zero and equals to 43.8 units, which is exactly the same amount as the amount of the net present value calculated above.

The relationship introduced based on the example can be proved to be generally valid for all orthodox cash flow pattern projects.

General Demonstration

Starting point:

The net present value calculation variant applied to orthodox cash flow pattern projects.

$$NPV = - E_0 + \sum_{t=1}^{n} H_t \frac{1}{(1+i)^t}, \qquad (1)$$

where

E_0 = Initial investment. The investment sum occurring in the zero point of time, and investment amounts occurring earlier added up with required rate of return.

t = Serial number of years ($t > 0$).

H_t = Difference between cash inflows and cash outflows in year t, where $H_t > 0$ for orthodox cash flow pattern projects.

n = Duration of the project, where the time of investment realization does not constitute part of the duration.

i = Required rate of return.

From the demonstration point of view, the content relationship that further investment-like expenses for maintaining the working ability of a given fixed asset or to restore the asset can occur during the working period of the investment project bears no relevance in this relationship. The demonstration consists of two stages and several steps at each stage.

Stage one:

The first stage of the demonstration describes the repayment process of the cost of capital. The cost of capital is the sum of the face value repayment requirement of the capital and its required yield according to the required rate of return. The repayment process describes the numerical definition of the cost of capital not yet repaid at individual points in time and the comparison to the concerning annual yield.

The amount of the cost of capital not yet repaid

at the end of Year 1: $-E_0(1+i)+H_1$

at the end of Year 2: $\left[-E_0(1+i)+H_1\right](1+i)+H_2$

at the end of Year 3:

$$\left\{\left[-E_0(1+i)+H_1\right](1+i)+H_2\right\}(1+i)+H_3$$

and so on.

Assuming that the status of the repayment at the end of Year 3 already shows the pattern of the development of the process through time, simplification is introduced to the above inscription in parentheses.

Eliminating the curly brackets:

$$\left[-E_0(1+i)+H_1\right](1+i)^2+H_2(1+i)+H_3$$

Eliminating the square brackets:

$$-E_0(1+i)^3+H_1(1+i)^2+H_2(1+i)+H_3$$

The numerical definition of the repayment status inscribed for the end of Year 3 can be applied further for the full duration of the investment. Formula (2) defines numerically the amount of the surplus repayment or lack of that generated at the end of the duration of the investment project.

$$-E_0(1+i)^n+H_1(1+i)^{n-1}+H_2(1+i)^{n-2}+\ldots+H_{n-1}(1+i)+H_n \qquad (2)$$

Stage two of the demonstration:

In this stage the end result of the repayment process is transformed into present value form. The present value of the amount of the surplus repayment or lack of that is calculated for the end of the duration in three steps.

Step one: sign up the discounting formula.

$$[-E_0(1+i)^n+H_1(1+i)^{n-1}+H_2(1+i)^{n-2}+\ldots+H_{n-1}(1+i)+H_n]\frac{1}{(1+i)^n} \qquad (3)$$

Step two: perform the discounting process.

$$-E_0+H_1\frac{1}{(1+i)}+H_2\frac{1}{(1+i)^2}+\ldots H_{n-1}\frac{1}{(1+i)^{n-1}}+H_n\frac{1}{(1+i)^n} \qquad (4)$$

Step three: simplification of the (4) formula.

$$- E_0 + \sum_{t=1}^{n} H_t \frac{1}{(1+i)^t} \quad (5)$$

At step three the net present value calculation formula inscribed as the starting point (1) is arrived. The proof is complete.

The deduction proved that in the case of orthodox cash flow pattern projects the net present value is the discounted amount of the surplus yield (or lack of that) generated above the yield requirement according to the discount rate. The existence of the surplus yield and its amount depends on the required rate of return, too.

In case of generating no lack of yield at the end of the period, the project is economically efficient and acceptable. The advantage of the method is that the process of the calculation logically follows the repayment process. The information content of the index number can be quite transparent from a practical point of view, in contrast with the net present value which is sometimes mystifying. Through discounting the face value of the surplus yield (or lack of that), the net present value can also be defined numerically.

Thesis 2 In the case of investment projects with orthodox cash flow patterns the net present value shows the sum of the surplus yield above the required one (or lack of that), discounted for present value. (This content is proved mathematically.)

Thesis 3 In the case of investment projects with orthodox cash flow patterns the sum of the surplus yield (or lack of that) calculated for the closing date of the project can also be suggested for practical experts as a correct economic index number. The advantage of the method is that the calculation process logically follows the repayment process. The information content of this index number is easily conceivable from an economic point of view; by discounting this index number the net present value is reached.

COMPARABILITY OF NET PRESENT VALUES

The net present value has a general content contradiction. This contradiction results from the fact that the method only handles the all-time extent of the capital investment correctly in the matter of the required rate of return. The surpluses (or lacks) of yield generated above the requirements are simply discounted and summarized. From the viewpoint of the method the sum of the capital and its presence in the project concerning the surpluses and the lifetime of the project are irrelevant. (The deduction above clearly proves this.)

Following from all these, the net present values of different investment projects are not suitable for comparing the economic efficiency of the projects. The net present value contains distortions regarding three relationships.

Distortion Effects

1. The initial investment requirements of the project can be different. Otherwise assuming unchanged conditions, the initial investment of the lesser amount is the more advantageous.
2. The duration of the investment project can be different. Otherwise assuming unchanged conditions, shorter duration is more advantageous. The re-investment of capital can happen earlier and the new yields are thus generated earlier; in addition in the case of the other conditions being the same the project bears smaller risk behind the repayment process of shorter period.
3. The rapidity of capital payback can be different. Otherwise assuming unchanged conditions, faster payback is more advantageous. In the case of slower payback, for example those that are concentrated at the end of the lifetime of the project, the reinvestment can commence later, affecting a significant part of the invested capital.

The distortion affects or either of them are performed a lots of publications (for example Keane, 1975, Van Horne – Vachowicz, 2008). Although the demonstration of the above three distortion effects makes clear that investment projects are generally not comparable based on their net present values, the literature is far from being homogeneous in this question. Some of the sources suggest the comparison based on net present value without any restraints. For example „Mutually exclusive projects: Accept the project with the highest positive NPV." (Brigham – Houston, 2009:340.) Furthermore „Projects … can also be ranked according to their NPV." (Hill, 2008:36.) In fact, the Fisher's intersection was borne on the basis of comparability of net present values as well. This intersection is the rate which brings the NPVs of two investments into equality (Fisher, 1930). Fisher's intersection is often referred nowadays as well (Baker - Powell, 2005, Hill, 2008, Van Horne – Vachowicz, 2008. and so on).

Possibilities for Eliminating the Distortion Effects

There are literary sources for suggestion of certain corrections to eliminate of the distortion effects. These suggestions, however, only correct the first or the second out of the three distortion effects demonstrated above. As I see, there is not the pursuit for a complex correction.

The elimination of the distortion affects is solvable in many ways. In this study the start-up basis of the applied cleaning method is the one-problem-oriented proposals of literature. (Supposedly this way of solution will give a hand to survey and apprehend the relationships.)

Some sources suggest dividing the net present value by the initial investment [$\frac{NPV}{E_0}$] or the profitability index. [$1 + \frac{NPV}{E_0}$] For example Brealey & Myers (1992:115.).

This index number or the profitability index only eliminates the distortion effect of differences in initial investments. The distortion effects of the differences in duration and rapidity of payback still remain.

The suggestion of the time-adjusted average of the net present value often appears – mainly in the financial literature – in the last two decade. (E.g. Helfert, 1991: 250-251, Baker - Powell, 2005: 262, or Lee, A. C. - Lee, J. C. - Lee, C. F. 2009: 473-475.) Defining the average is calculated in the way that the net present value is divided by the annuity factor, which in business economics means multiplying by the loan repayment factor. (Business economics always used the formula of loan repayment factor. The financial literature uses the annuity factor, which is the reciprocal of the previous formula.) The formula of this time-adjusted average is: $q_n\ NPV$ (where q_n is the loan repayment factor, within the required rate of return and lifetime of the project). This solution only eliminates the distortion effect of the differences in duration; the distortion effects in the differences between capital requirements and rapidity of payback remain.

Another step could be – although I did not find any suggestions for this in the literature – merging of the two methods demonstrated above, that is, the numerical definition of the time-adjusted average of the net present value divided by initial investment, that is $q_n \frac{NPV}{E_0}$.

After eliminating the distortion effects of the initial investment and the duration at the same time, the distortion effect would be only in the rapidity of capital payback remaining in the index number. (Demonstration of this with an example can be found at the end of this study.)

This way, however, the transformed net present value is getting closer to the main information used during application of the internal rate of return method, that is, how much the factual profitability rate differs compared to the required one. The methodological elaboration of calculating the coefficient which can measure payback rapidity seems to be very complicated. I believe that the calculation of this coefficient is not necessary, but very important to know its essence. Fundamental cases:

- Payback by years is uniform. Than the coefficient is 1.
- Payback is quickly. The bigger cash flows arise at the beginning of the life-span. In this case the coefficient is bigger, than 1.
- Payback is slow. The bigger cash flows arise at the end of the life-span. In this case the coefficient is lesser, than 1.

When the third problem, the distortion effect in the difference between the rapidity of payback, is successfully eliminated, the transformed index number arrives at a corrected difference between the internal rate of return and the required rate of return.

The Cleansed Formula

The transformed net present value is a special rate-difference. This content is followed from exhibited calculating procedure. The rate-character has appeared when the net present value was divided by initial investment. The matter of rate-difference is following from that, the net present value is a surplus yield (or lack of this). Multiplying this surplus yield rate with the loan repayment factor it transforms the time-adjusted average of net present value rate. The last step is the correction with the coefficient of payback rapidity. This rate will be the modified difference between the internal rate of return and the required rate of return.

This way cannot lead to the accurate difference of two rates. This is coming from the special cleaning method in which are mixing the elements of static and dynamic procedures for capital budgeting. The formula as follows

$$q_n \frac{NPV}{E_0} \lambda = (r - i)\varepsilon, \qquad (6)$$

where, besides the above,

λ = Coefficient of payback rapidity,

r = Internal rate of return,

ε = Modifying factor to the difference of internal rate of return and required rate of return.

Thesis 4 From the point of view of the comparability of decision variants, the net present value contains distortions in three relationships. These are the initial investment, the duration and the rapidity of capital payback. By systematically eliminating these distortions the net present value transforms into a special kind of rate, namely, the modified difference between the factual and the required rate of return.

Reduction of the Formula in the Case of Existing Annuity Terms

According to the relationship demonstrated above, the systematic elimination of distortion effects in the net present value leads to the corrected difference between the factual and the required rate of return. This is clear and understandable when the terms of annuity method exists.

Two conditions should be met to apply the annuity method:

1. The investment should be processed in a very short time (this must be an investment point).
2. The difference between revenues and out-of-pocket costs should be the same every year, so the generated cash flow size must be a constant amount per year.

In this case the net present value method can become simpler (as this well known):

$$NPV = -E + \frac{h}{q_n}$$

where

E = Sum of the investment where the investment process is very short. That is, $E_0 = E$.

h_t = Difference of revenues and out-of-pocket costs in year t.

h = Constant difference of revenues and out-of-pocket costs. That is, $h_t = h$, if $t > 0$.

With these two conditions the internal rate of return method is simpler as well.

$$0 = -E + \frac{h}{q_{techn}} \qquad \text{and} \qquad q_{techn} = \frac{h}{E}$$

where qtechn is a technical loan repayment factor, using the factual profitability rate (internal rate of return) "r" instead of required rate of return "i".

Starting formula (6):

$$q_n \frac{NPV}{E_0} \lambda = (r - i)\varepsilon$$

If the yearly payback is constant (ht = h; and E0 = E as was discussed before), then the coefficient of payback rapidity is 1 (that is λ =1). According to these

$$q_n \frac{NPV}{E} = (r - i)\varepsilon, \qquad | \; E_0 = E \text{ and } h_t = h \qquad (7)$$

After rearranging equation (7):

$$NPV = \frac{(r-i)\varepsilon E}{q_n} \qquad | \; E_0 = E \text{ and } h_t = h \qquad (8)$$

That is,

$$-E + \frac{h}{q_n} = \frac{(r-i)\varepsilon E}{q_n} \qquad | \; E_0 = E \text{ and } h_t = h \qquad (9)$$

and

$$\frac{h}{E} - q_n = (r - i)\varepsilon \qquad | \; E_0 = E \text{ and } h_t = h \qquad (10)$$

As noted above $\frac{h}{E} = q_{techn}$, so

$$q_{tech} - q_n = (r - i)\varepsilon \qquad | \; E_0 = E \text{ and } h_t = h \qquad (11)$$

or

$$\frac{r(1+r)^n}{(1+r)^n - 1} - \frac{i(1+i)^n}{(1+i)^n - 1} = (r-i)\varepsilon \qquad | \; E_0 = E \text{ and } h_t = h$$

In this special case the transformed net present value is the difference between the technical and the true loan repayment factor as shown in formula (11). This difference by absolute value is smaller than the difference of the factual and required rates of return because of the influence of specific construction of the net present value divided by initial investment.

In the case of existing annuity terms the modifying factor ε can count as well.

$$\varepsilon = \frac{q_{tech} - q_n}{r - i} \qquad | \; E_0 = E \text{ and } h_t = h \qquad (12)$$

For example if the lifetime of the project is 10 years, the internal rate of return is 20% and the required rate of return is 15%, then in the case of existing annuity terms the transformed net present value from formula (11) is: (0.2 – 0.15) ε = 0.23852 – 0.19925 = 0.03927. The 3.9 percentage point is smaller than the percentage point of 5, as the difference of the factual and required rates of return is. The numerical value of ε in this case is 0.7854.

Thesis 5 In the case of existing annuity terms the net present value transformed to comparable is the difference between the technical and the true loan repayment factor, where the technical repayment factor is the constant difference of revenues and out-of-pocket costs divided by investment.

Ranking of Investment Projects According to Economic Effectiveness

In the case of investment projects with orthodox cash flow patterns the net present value and the internal rate of return methods lead to the same result when selecting the acceptable variants, in spite of the fact that their information content differs. However, the ranking list according to the two methods can differ.

The ranking list according to comparable (transformed) net present value corresponds to the list according to the internal rate of return. Consequently the highest profitability project according to the internal rate of return is also on the first place in the ranking list according to the net present value cleared of main distortion effects. Ranking by internal rate of return is in conformity of operational principle of the economy, by which the capital is wandering to the possibilities of highest profitability (by the given risk).

The everlasting debated question is which one is the better from two economically efficient investment variants that exclude each other. The investment of course yields more profit in the project variant of larger profitability, but that is not sure the one with the larger profitability is to be chosen.

The initial investment, the duration and the rapidity of payback should also be taken into consideration, namely how much investment, how long time and with what rapidity the relative large profitability takes. That can happen, especially when the less investment, shorter life-span or quicker payback refers to modest yield sum opportunities, that the somewhat less profitable, but bigger investment, longer duration, slowly payback variant (compared to the largest profitability variant) becomes more advantageous. (This can be seen at first sight; for example, that – with otherwise unchanged conditions – a 50 million unites or 20-year duration project of 24% profitability is more advantageous than one 5 million unites or lasting for 2 years having 25% profitability.)

The relatively higher sum of net present value or relatively high rate of surplus yield by the net present value divided by initial investment can refer to the latter advantages as well.

So the solution is: setting up a three-index number for ranking where the internal rate of return is the primary and the sum of the net present value with the net present value divided by initial investment appear as secondary ranking indicators. If all of the three ranking indicators show the same ranking list, then this will be at the same time the practical ranking list as well. If, however, the three indicators lead to different ranking lists, then further analysis is needed.

To continue the setting up of the ranking list, that is practical to determine the critical profitability level (critical internal rate of return) of the difference of investment sum, or reinvestment amounts. This critical profitability level should be reached at the difference of investment sum or reinvested amounts of the less sum, shorter duration or quicker payback project to make the internal rates of return of the two projects the same. Only with this critical profitability rate would both options be considered equally favorable. If larger profitability chances than the critical profitability rate can be foreseen, then that is favorable to choose the project with higher internal rate of return; otherwise that is favorable to choose lower capital profitability rate project (Illés, 1997: 131-135).

EXAMPLE OF A DIFFERENT PAYBACK RAPIDITY AND RANKING

The example below demonstrates the relationships discussed above using real numbers. For the sake of this purpose the example disregards the fact that the occurrence of the problem with such clarity is not probable in practice.

A company has an investment option of 350 units. The practical experts worked out two investment project variants. The market risk of the two variants is the same, so a uniform 12% required rate of return was assigned to each of them. Table 1 summarizes the main data of the individual variants.

Table 1
Main Data of the Two Project Variants in the Example

(in units)

Year	Project variant 1				Project variant 2			
	Amount of investment	Revenues	Out-of-pocket costs	Yields	Amount of investment	Revenues	Out-of-pocket costs	Yields
0	350	0	0	0	350	0	0	0
1		408	403	5		867	467	400
2		412	407	5		661	656	5
3		747	247	500		523	518.49	4.51

Decision preparation information:

1. The net present value of the two variants is the same (where var. = variant).
   $NPV_{(var.1)}$ = -350 + 0.89286 ×5 + 0.79719×5 + 0.71178×500 = 14.3
   $NPV_{(var.2)}$ = -350 +0.89286 ×400 +0.79719×5 + 0.71178×4.51 = 14.3
2. Since the net present value and the invested amount of the two projects are the same, the net present value divided by initial investment and the profitability index (PI) will be the same as well.
   $$\frac{NPV}{E} = \frac{14.3}{350} = 0.041; \qquad PI = \frac{364.3}{350} = 1.041$$
3. The internal rate of return of the two variants shows a significant difference:
   Internal rate of return $_{(var.1)}$ ~ 13.5%.
   Internal rate of return $_{(var.2)}$ ~ 16.5%.
   The factual profitability shows a significant difference, which is approximately 3 percentage points. The reason: the rapidity of payback is significantly higher in the case of the second variant; consequently the average locked-up capital is smaller. Most of the invested amount's nominal value is repaid at the end of year one.
4. For the critical profitability rate of the reinvestment amounts freed in the second project (with the intention to estimate roughly) that is practical to examine with what profitability the 400 units freed at the end of first year should be invested to ensure a net yield of 500 units at the end of year three.
   $400\,(1+r)^2 = 500$; $(1+r)^2 = 1.25$;
   $1+r = 1.118$; $r = 0.118$, that is 11.8%
   The 11.8% does not reach the required rate of return, so by a rough estimate the second variant is the better one.
   For more exact estimates that is practical to consider each of the annual differences in yield:
   $395(1+r)^2 = 495.488$; $(1+r)^2 = 1.2544$;
   $1+r = 1.12$; $r = 0.12$, that is 12%
   According to the more accurate calculations, choosing the second variant is more practical in the case of a reinvestment that exceeds the required rate of return by any small amount. (However considering the smaller risk behind the shorter repayment period, then definitely the second variant is better.)
   Should the required rate of return be raised to 15% due to an unexpectedly occurring higher commodity market risk, then the first variant would drop out of the group of acceptable variants. Its internal rate of return would not reach the required rate of return and its net present value also would turn negative.
   NPV(var.1) = -350 + 0.86957× 5 + 0.75614× 5 + 0.65752×500 = -13.
   NPV(var.2) =-350 +0.86957×400 + 0.75614× 5 + 0.65752×4.512 = 4.6
   In the case of a 15% required rate of return, the discounted amount of the lack of yield is 13.1 units for the first variant, while the discounted amount of the surplus yield is 4.6 units for the second variant.
5. With the 12 % required rate of return the dynamic average of the net present value divided by initial investment will not show a ranking list. Considering that the investment amount, net present value and duration of the two projects are the same, the resulting dynamic average will also be the same.
   $$q\frac{NPV}{E} = 0.41635 \times 0.04097 = 0.017$$

The time adjusted average of net present value divided by initial investment is 1.7 % in the case of both projects. That is because of the fact that this step cannot take into consideration the rapidity of payback. By handling rapidity of payback the annual rate of the net present value would be different. The payback rapidity indexes are: less than 1 of the first variant and higher than 1 of the second variant. So the net present values cleared of main distortion effects are 1.5 ε_1 percentage points and 4.5 ε_2 percentage points, respectively.

The example shows that the numerical definition of the time adjusted average of the net present value divided by initial investment leads to information that can be interpreted in practice as well (the researchers did not combine the two steps so far). In addition, by systematically eliminating the effects of the net present value distorting comparison possibilities, the corrected index number get closer and closer to the difference between the internal rate of return and the required rate of return. These will not be completely equal because of the applied examination terms.

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