

Fisher's Rate and Aggregate Capital Needs in Investment Decisions

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SUMMARY

Fisher's rate means the interest rate where the net present values of two mutually exclusive projects become equal. The paper examines the background and the circumstances of conformation of Fisher's rate in connection with the aggregate capital needs. Aggregate capital needs is a new conception and gives a new viewpoint to investment project decisions. The paper defines the special content of aggregate capital needs, and compiles an index number for it. The analysis widens knowledge regarding the content of net present value, and highlights the importance of taking the aggregate capital needs into consideration. Fisher's rate only means useful information in practice if the ranking is made based on the net present value. However, this principle of ranking is in contradiction with the concept of long-term profit maximization. The transformed net present value, which is free of distorting effects (and assuming equal required rates of return,) gives the same ranking list as the internal rate of return. Therefore, Fisher's rate has no importance in business decisions.

Keywords: aggregate capital needs; investment project decisions; net present value; internal rate of return; ranking; Fisher's rate
Journal of Economic Literature (JEL) code: M21

INTRODUCTION

Fisher's rate (or intersection) is nowadays an often quoted and illustrated category in financial books. This rate shows the interest rate that would provide the same net present value for two mutually exclusive projects. The relating intersection is that point where the two net present value curves drawn according to the series of interest rate intersect each other. The illustration covers a range of rates from zero interest rate to a rate slightly higher than the internal rate of return of the two projects. The value of the intersection on the x axis gives the rate; on the y axis it shows the same net present value of the two projects. In the range of interest rates where the interest rate is smaller than the Fisher's rate, the ranking based on the net present value differs from the ranking based on the internal rate of return. After the intersection the ranking of net present values changes, and the rankings based on the net present value and on the internal rate of return become equal. The literature mainly deals with the demonstration and ranking-changing role of this rate; (to my best knowledge) there has been no substantive mapping of the background of these relationships yet.

The financial literature recommends net present value as a tool of ranking. Since the middle of the 20th century, financial recommendations say that the ranking of investment alternatives must support the maximization of

shareholder wealth, which means the realization of projects with the highest net present value. The authors usually refer to the work of Fisher. Illés (2012b) gives a brief literary overview on this topic.

If net present value (NPV) is used as a ranking indicator, the intersection can mean important decision-making information, as this is the point where the net present value based ranking of the two projects changes order. However, net present value is not suitable for ranking. After removing distorting effects from the net present value method (and assuming equal required rate of return) this ranking list leads to the same ranking as the internal rate of return (IRR). Illés (2012a) proves this relationship. Based on this article doubt is cast upon several statements of the literature, including the role of Fisher's rate in the process of investment project decisions.

The main objectives of this paper are:

1. To discover the causality relationships between net present value and required rate of return from a business economics perspective.
2. To define and analyze the concept of aggregate capital needs.
3. To compile an index number of aggregate capital needs.
4. To discover and explain the mechanism and internal coherences of the formation of Fisher's rate and Fisher's intersection.

5. To discover the causality-based pre-conditions of appearance of this rate and intersection.
6. To highlight that this rate should not be treated as substantive information in investment project decisions.

The paper analyzes the net present value and its internal relationships by using an economic detour approach. The analysis applies in principle a business economics view and system of conditions that makes it fundamentally different from any financial analyses. Illés (2012b) works out the most significant differences between the financial and the business economics approach.

One of the basic categories of examinations is the yield, which means the difference between the total revenues and total expenditures of a year. (In the literature yield is used in different contexts and so can mean different things. In this paper the yield always means the difference of the total revenues and total expenditures of a year.)

In this paper investment decisions concern investment projects with orthodox cash flow patterns. (The well-known criteria of orthodox cash flow patterns are: a series of the difference of annual revenues and expenditures starts with negative amount or amounts and the sign of these differences changes only once. That is, from a point in time where this difference first turns into positive, this positive sign does not change.)

In order to be clear, the paper only calculates with the usage of equity capital and defines profit as pre-tax profit and at project level. (At company level a loss-reducing investment does not give profit, therefore at this level it is not taxable.)

THE ORIGINAL NAME OF THE RATE

The name Fisher's rate refers to the work 'The Theory of Interest' of Irving Fisher published in 1930. This study was issued more than eight decades ago. Its significance is indisputable. Besides its financial hypotheses, another significant benefit of this work is that it is one of the establishers of the methodology of dynamic profitability calculations. The terminology has changed with time. For example, the expression 'net present value' is not mentioned in Fisher's book.

Fisher actually examines the interest rate according to which the income streams of two mutually exclusive projects will be equal. "That is, this equalizing rate is such that the present values of the two options would be equal" (Fisher 1930:155). This equality has an implicit assumption that the two projects have equal initial investments.

The author names the interest rate where the present values of two projects are equal 'the rate of return over cost'. This expression is used altogether 77 times in the book. The denomination does not refer to the real content. Scientific articles and studies of the last century refer to

the category according to Fisher's denomination (for instance: Alchian 1955; Renshaw 1957; Dudley 1972; Keane 1975; Meyer 1979; Hirst & Ma 1983). The economic content of the rate would be slightly better described by the expression "equalizing rate", also used by the author; however, this expression is only mentioned three times in his book.

There are several misunderstandings in the contemporary literature due to the novelty of Fisher's topic and to the rather specific denomination of the category. One of the biggest and most often referred to misunderstandings can be found in the work of Keynes (1936). Several authors quote and analyze this oversight (for instance Alchian 1955; Carlson et al. 1974; Keane 1975). The substance of this is that (in today's terminology) Keynes defines in his work the internal rate of return that he has drawn up as equal to "the rate of return over cost" developed by Fisher. Although considering Fisher's denomination this misunderstanding is not surprising, it is still remarkable because besides the significant differences in content – with some simplification – two projects are necessary to calculate Fisher's rate, while for the internal rate of return only one project is needed (Alchian 1955).

It should be noted, however, that in some cases a different interpretation of the rate of return over cost is also possible. For instance: "In general, the rate of return over cost has to be derived by more complicated methods. As already indicated, the rate of return over cost is always that rate which, employed in computing the present worth of all the costs and the present worth of all the returns, will make these two equal. Or, as a mathematician would prefer to put it, the rate which, employed in computing the present worth of the whole series of differences between the two income streams (some differences being positive and others negative) will make the total zero." (Fisher 1930:168-169.) The second sentence of the quote is a clear explanation of the type of misunderstanding seen in Keynes.

Dudley (1972), as well as Hirst & Ma (1983), finds it important to highlight that according to Fisher's examples the initial investment of the two projects (implicitly) is the same. In fact, this fisherian solution can be the reason that in the related examples of financial literature, the initial investment of the two examined projects is always the same. The equality of the initial investment in the examples is basically uninteresting; the information about the aggregate capital needs would be relevant. Fisher gave preference to the project, from the mutually exclusive projects, with the highest present value, because he thought that this could make the highest contribution to the growth of shareholders' wealth. This view of Fisher that prefers (net) present value even in case of ranking is referred to in the recommendations of the mainstream of today's financial studies.

Furthermore, according to Fisher's approach it is worth mentioning the explanation of the rate at the

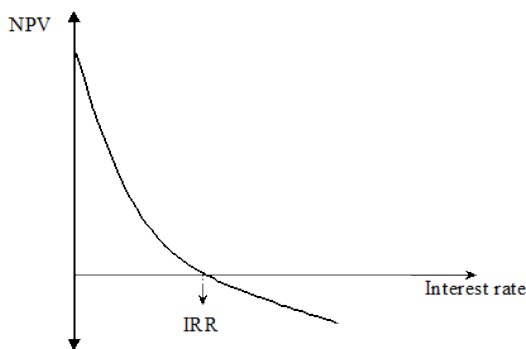
intersection as the relevant re-investment rate (Dudley1972; etc., critical analyses: Keane 1975; orthodox micro-economic approach: Meyer 1979).

Today in most cases only the name of the rate and of the intersection refers to the scientist. Besides these general references there is usually no concrete literature reference (for instance Van Horne & Wachowicz 2008; Baker & Powell 2009). In business economics publications it is mostly typical that the categories related to the intersection of the net present value curves do not use his name, not even in their denomination (e.g., Adelberg et al. 1986; Arnold & Hope 1990). Among the financial publications there are also some where neither the denomination of the intersection nor the name of the related rate refers back to Fisher (for instance Brealey & Myers 1988, or Firer & Gilbert 2004).

THE BACKGROUND OF THE NET PRESENT VALUE CURVE

The plotting of the curve of the net present value is a regularly appearing topic within the related literature. (This curve is illustrated by Figure 1.) The authors consider this figure well-known and widely used in this form; therefore do not use any professional reference. The figures and related explanations can be introduced from different perspectives: once a general theoretical relationship (Arnold & Hope 1990:254), another time as a solution of an exercise or an introduction of a problem through an example (Brealey & Myers 1988:79; Van Horne & Wachowicz 2008:329).

The literature usually illustrates that the higher the interest rate is, the smaller the net present value. As a result of increasing interest rate, this decrease first reaches the zero net present value, then further increasing results in higher and higher negative net present values. The interest rate that is at the intersection of axis x and that results in zero net present value is, as we all know, the internal rate of return. I am aware of no literature sources on the pre-conditions and detailed content background of the illustrated relationship.



Source: Widely-used illustrations of the relationship

Figure 1 The sum of the net present value in the function of interest rate

In order to specify the problem it must be laid down that the curve in Figure 1 is valid for most but not all projects in question. The relationship is only valid for profitable projects with orthodox cash flow pattern. The illustrated relationship can also be applied for protracted investments. In this case, at the beginning of the duration the initial investment contains the compound interest as well. Hereinafter, in order to simplify modeling, that sort of model will be examined where the payment of initial investment occurs at the same time as operation is started. This is date zero. The first revenues will occur one year later, by this time the annual revenues exceed the annual expenditures.

The survey starts from the content background of economic correspondence. In this way the causality relationships may come to be evident. Related topics:

- > to discover the process of the yield requirements formation,
- > to analyze the changing content structure of the yield in function of interest rate,
- > to show that when the yield requirements are fulfilled then all of the further yields become surplus profit, the discounted sum of which is the net present value.

The net present value method is applied to orthodox cash flow pattern projects, where the initial investment occurring at the zero point of time is the following:

$$NPV = \sum_{t=1}^n (B_t - K_t) \frac{1}{(1+i)^t} - E_0 \quad | B_t - K_t > 0 \quad (1)$$

where

- B_t = revenues in year t ,
- K_t = expenditures in year t ,
- E_0 = initial investment,
- i = required rate of return,
- t = serial number of years ($t > 0$),
- n = duration of the project.

The illustrated net present value curve starts from zero percent interest rate. When the required rate of return is zero, then the net present value is calculated as follows:

$$NPV = \sum_{t=1}^n B_t - \sum_{t=1}^n K_t - E_0 = M \quad | i = 0 \quad (2)$$

M = Total accounting profit that is the difference of the nominal value of total revenues and total expenditures including initial investment arising during the whole duration of the project.

As Equation (2) shows, at zero percent interest rate the net present value is the difference between the nominal value of total revenues and total expenditures including initial investment. This difference can also be considered as an accounting profit summed up in nominal value for the total duration of the project. This gives the conclusion that if no profit is gained at nominal value, the starting point of net present value curve cannot be in a

positive range. Because of this the figure is not valid for projects that do not generate accounting profit

In this calculation, which considers the nominal value and the whole duration of the project, the problem caused by accounting not defining the profit as a difference between revenues and expenditures but as annual differences between revenues and total costs disappears. (Nevertheless, the profit of a project only means profit at the company level when the company itself is not in an accounting loss.)

The profit calculated at nominal value for the whole duration means the coverage of the profit requirements according to the required rate of return. In fact, the content mechanism of net present value method subtracts the amount of profit requirements – that can be calculated for the not yet returned investment using compound interest calculation – from this nominal profit sum. If the profit requirements are smaller than the sum of all accounting profits, the mechanism of the calculation discounts and summarizes the surpluses for the years concerned. This content mechanism is difficult to recognize because of the calculation is made on the basis of yields.

In case of an interest rate of 1 percent, in the first year this method calculates 1 percent interest-like profit requirements for the initial investment. The yield in the first year first of all covers the profit requirements and the remainder part of the yield covers a part of the initial investment. In the following years, the 1 percent profit requirements will be calculated for the sum of the not yet returned capital. (This way compound interest will be realized.) When all of initial investment and profit requirement is returned then all of the further yields become surplus profit. The higher the interest rate used in the calculation, the smaller surplus will remain from the total accounting profit, and then, when this does not provide coverage for the requirements any more, a yield lack arises. Discounting the surplus profit (or the lack of yield) for the date zero gives the net present value.

In the case of investment projects with orthodox cash flow patterns the net present value shows the sum of the surplus yield above the required one (or lack of that), discounted for present value. (The definition is in Illés 1990 and its mathematical proof can be found in Illés 2012a.) Consequently, increasing the rate of interest continuously decreases the surplus sum of accounting profit, then, after equality, it gradually increases the yield lack compared to the requirements.

The capital recovery process is not illustrated in Figure 1. In order to identify the relationships, it is practical to clarify that the annual difference of revenues and expenditures, that is, the annual yields of examined investment project, first of all are expected to cover the profit requirements according to the required rate of return and the possible part of not yet returned capital. The yearly profit requirements are calculated by multiplying of the not-returned capital and the interest rate.

Considering that the return of capital and profit requirements must be covered by the annual yields, and furthermore that the return of the nominal value of the initial investment and the profit requirements are not differentiated, the essence of the method does not become clear. The essence is that the profit requirements calculated with compound interest should be returned from the sum of the total accounting profit. This latter part of the relationship is illustrated by the amount of net present value at zero percent interest rate. (The details will come later.)

Assuming otherwise constant conditions, the higher the interest rate is, the higher the amount that needs to be returned. The interest rate of which the required profit sum is just covered by the sum of all accounting profit is the factual profit rate of the time-varying tied-up capital. At this interest rate there is no surplus profit or lack of yield (and this interest rate is named as internal rate of return). A profit requirement that is higher than the factual profit rate cannot be fulfilled according to the project's database, thus the calculation will show lack of yield. In such cases, the value of the yield lack at the end of the period after discounting will be the net present value with negative sign.

Of course, net present value can be defined as a difference between the present value of future yields and the initial investment. From the aspect of management process, this seems to be only a theoretical approach; it does not give sufficient information about the content background.

CALCULATION OF THE PROFIT REQUIREMENTS ACCORDING TO THE NOT RETURNED PART OF CAPITAL

The profit requirements calculated for the duration of the project are the following:

$$\text{In the first year: } M_{s1} = E_0 i \quad (3)$$

$$\text{In the second year: } M_{s2} = E_1 i ; \text{ where } E_1 = |H_1 - E_0 i - E_0| \quad (4)$$

$$\text{In the third year: } M_{s3} = E_2 i ; \text{ where } E_2 = |H_2 - E_1 i - E_1| \quad (5)$$

For the $t > 1$ year, where the payback period (in years) is not smaller than the duration of the project:

$$M_{st} = E_{t-1} i ; \text{ where } E_{t-1} = |H_{t-1} - E_{t-2} i - E_{t-2}| ; | 1 < t \leq z \quad (6)$$

H_t = the yields (that is the difference of revenues and expenditures) in year t , where the value of H_t is always

positive for years $0 < t \leq n$ by the terms of orthodox cash flow pattern projects, and the initial investment occurring at the zero point of time,

E_t = the not returned part of capital at the end of year t ,

M_{st} = the profit requirement in year t according to the not returned part of capital and required rate of return,

z = the number of years of the dynamic payback period.

In order to show the content relationships, in Equation (6) the following mathematical merger of the components is not included: $H_{t-1} - E_{t-2} i - E_{t-2} = H_{t-1} - E_{t-2}(I+i)$

Between the first and the $z-1$ -th years the yield has two content components. These are profit requirement and certain capital return (provided that the yield is greater than the profit requirement). If the yearly profit requirement is larger than the current year's yield, then the difference is added to the capital still to be returned. In the z -th year the yield has a third content component as well, provided that the return cannot be met at the end of the year. This third component is a surplus profit.

The total profit requirements that arise during the lifespan of the project (as an accounting approach) can be defined as follows:

$$\sum_{t=1}^n M_{st} = \sum_{t=1}^n E_{t-1} i \quad | \quad n \leq z; \tag{7}$$

where

$$E_{t-1} = |H_{t-1} - E_{t-2} i - E_{t-2}|; \quad | \quad t > 1$$

If the payback period is not shorter than the project duration in years, then the present value of surplus profit is the net present value itself:

$$NPV = \left[M - \sum_{t=1}^n M_{st} \right] \frac{1}{(I+i)^n} \quad | \quad n \leq z \tag{8}$$

If the payback period is shorter than the project duration in years, there will be some years without capital and profit requirements. In this case the yields of the additional years are surplus profits as well. (The number of these years is $n-z$.) According to the continuance of the conformity with net present value method, these surplus profits can be compounded to the end of the duration. The amount of nascent quasi interest will disappear when discounting. (See Table 2 and the following calculation.)

EXAMPLE FOR CALCULATING INTEREST RATE DEPENDENT PROFIT REQUIREMENTS

The above statements are illustrated by a simple example. The basic data of a project – let us call it Project

S – are the following: at date zero, 240 units of expenditure arise (as an initial investment), and then for 4 years a 100-unit yield (revenues minus expenditures) is gained annually.

According to the database, during the total lifespan of Project S an accounting profit of the nominal value of 160 units is gained. ($400-240=160$) This amount can be divided between the return based on profit requirements and surplus profit, or in case of higher profit requirements, the difference means a lack of yields.

The relevant data are summarized in Table 1. The detailed and explaining calculations can be found in Tables 2 to 4. For the better identification of relationships, the results of these calculations are presented as rounded numbers.

Table 1
The total profit requirements and surplus profits according to different interest rates of Project S

Measurement unit: unit

Interest rate as a required rate of return	0 %	8 %	20 %	24 %	30 %
Nominal value of the total profit requirements	0	38	121	160	219
Surplus profit /lack of yield at the end of Year 4	160	124*	39	0	- 59
NPV (present value of the surplus profit /lack of yield at the end of Year 4)	160	91	19	0	-21

*This amount contains 2 units of technical (quasi) surplus profit. See details in Table 2.

In Table 1, the sum of profit requirements and the nominal value of surplus profit always add up to 160 units as the sum of all of accounting profits. (In case of yield lack, the difference of profit requirements and yield lack gives the accounting profit of 160 units.) The 24 percent is at the same time the internal rate of return as well. In this case there is no surplus profit or yield lack. The sum of the profit requirements is 160 units. (This is the whole accounting profit of the project.)

As is visible in Table 2, the annual amounts of profit requirements are accounted by multiplying the amounts of the not yet returned capital and the required rate of return. The annual yield of 100 units on the one hand covers the profit requirements of the current year, then, on the other hand its remaining part decreases the not yet returned capital until the total amount of capital is returned. In the payback year, the yield that remains over the profit requirements and not yet returned capital is the surplus profit. The yields in the following years mean clear surplus profit.

Table 2
Details of the economic calculation in the case of 8 percent required rate of return

Measurement unit: unit

Year	Capital to be returned at the beginning of the year	Amount of profit requirements (8 %)	The structure of 100 units annual yield		Capital still to be returned (-)/ surplus profit at the end of the year (+)
			Profit requirements	For capital return	
1.	240	19	19	81	- 240 + 81 = - 159
2.	159	13	13	87	- 159 + 87 = - 72
3.	72	6	6	94	- 72 + 94 = + 22
4.	surplus profit 22	2*	Surplus profit in the current year 100 + 2*		+22 + 102 = +124

*In order to ensure the consistency of the net present value method, the surplus profits gained before the end of the duration need to be compounded to the end of it. ($22 \times 0.08 = 2$) This is only a technical solution and only concerns those cases where the payback period in years is shorter than the duration of the project. The calculated quasi-surplus profit disappears when compounded surplus profit will discounted to date zero.

Below is a control calculation of net present value where the required rate of return is 8 per cent: $-240 + \frac{100}{0.30192} = -240 + 331 = 91$, and $124 \times 0.73503 = 91$

Table 3
Details of the economic calculation in the case of 20 percent required rate of return

Measurement unit: unit

Year	Capital to be returned at the beginning of the year	Amount of profit requirements (20 %)	The structure of 100 units annual yield		Capital still to be returned (-)/ surplus profit at the end of the year (+)
			Profit requirements	For capital return	
1.	240	48	48	52	- 240 + 52 = -188
2.	188	38	38	62	- 188 + 62 = -126
3.	126	25	25	75	- 126 + 75 = - 51
4.	51	10	10	90	- 51 + 90 = + 39

Table 4
Details of the economic calculation in the case of 24 percent required rate of return

Measurement unit: unit

Year	Capital to be returned at the beginning of the year	Amount of profit requirements (24 %)	The structure of 100 units annual yield		Capital still to be returned (-)/ surplus profit at the end of the year (+)
			Profit requirements	For capital return	
1.	240	58	58	42	- 240 + 42 = -198
2.	198	48	48	52	- 198 + 52 = -146
3.	146	35	35	65	- 146 + 65 = - 81
4.	81	19	19	81	- 81 + 81 = 0

THE AGGREGATE CAPITAL NEEDS

The method of net present value only considers the amount and period of capital investment up to the level of required rate of return in a correct way. Any surplus profit over this level is simply being discounted and summed up. The amount remaining as a surplus profit is not affected by the capital-focused corporate efforts that are what was the average tied-up capital and how long it was needed to gain a certain surplus profit. In practice, from an economic viewpoint, this means the main deficiency of the information content of this category and this is the main reason that net present values cannot be compared on the merits.

A net present value with positive sign has the following message for the decision-maker: the profit requirements according to the required rate of return will be fulfilled and in present value a surplus profit of a

certain amount will also be gained. If the question is whether the profit requirements are fulfilled, this answer is satisfactory. But it is not easy to tell what advantage this amount exactly means. A quite ordinary example: for an individual who is fixing 300,000 euro in a bank for one year, it is more meaningful to do so at an interest rate of 4.5% than to be told that he will get 3% interest on his deposit plus 4,500 euro more. This special sort of deposit is more difficult to see through with a long-term commitment (Illés 2012b).

The literature that deals with net present value is not concerned about the aggregate capital needs. However, it is absolutely evident that if a higher capital sum is needed for a longer time, then the aggregate capital needs will be higher. Knowledge of the aggregate capital needs can help in the comparison of net present values. In the clarification process it can be considered as a step forward when the aggregate capital needs may help us to see that the net present values (with their original

content), and the net present values divided by initial investment or multiplied by the loan repayment factor cannot be compared. These recommendations can be found in various literary sources. (On the latter in more detail see Illés 2012a.)

The project's aggregate capital needs means the amount of capital needed for the operation of the project during its full duration. For investments with an orthodox cash flow pattern, the capital needs are basically determined by three factors: the amount of initial investment, the duration of the project and the rapidity of capital payback. The latter depend on the required rate of return as well. As illustrated in the example of Project S, the higher the required rate of return is, the larger the part of the annual yield that is used to cover the profit requirements and the less remains to cover the not yet returned capital. By increasing the interest rate, the return of nominal value of the capital will be delayed, and the aggregate capital needs of the project increase.

The exact calculation of aggregate capital needs for investment projects is not a simple task. This issue has not arisen; no correct method of calculation has been discovered yet. As mentioned above, there are three reasons for the differences in aggregate capital needs of the project variants: different initial investment, different duration of the project and the different rapidity of capital payback. According to this, main manifestations of the higher aggregate capital needs – higher initial investment, longer time of return, lower rapidity of return – and different combinations of these. In the case of one-way differences, it might be obvious which variant has the higher aggregate capital needs. However, in reality, these three factors can appear in different combinations and at the same time can have effects in different directions. For example, one variant has a higher initial investment that will return in a shorter time but at with slower capital payback, etc.

The series of annual tied-up capital are built up in the Equations (3)-(6). According to these, the not yet returned capital concerning the payback period of the project can be calculated as the following:

In the first year of operation, the full amount of initial investment tied up in the projects is: E_0

In the second year, the amount of the tied-up capital is decreased by the capital returned as the result of the first year's operation: $E_1 = |H_1 - E_0 i - E_0|$

In the third year, the tied-up capital is decreased by the capital returned during the second year's operation: $E_2 = |H_2 - E_1 i - E_1|$

During $1 < t \leq z$ years the amount of tied-up capital is:

$$E_{t-1} = |H_{t-1} - E_{t-2} i - E_{t-2}|; \quad | 1 < t \leq z \quad (9)$$

The index number of the aggregate capital needs should be worked out on the basis of yearly tied-up capital and the duration of the project. Tied-up capital is a state indicator, and therefore the tied-up capital of

different years cannot be summed up from this aspect. However, in consideration the income-producing potential needs to take into account simultaneously the tied-up capital and tied-up time. The solution is to sum up the yearly tied-up capital, that is, the not-returned parts of the capital for each year. To do this correctly as a measurement unit may be one unit tied-up capital for one year. According to the calculation method used above a three-year tie up of one capital unit is equal to three capital units tied up for one year. This solution is considered to be correct because the tied-up capital is computed with a database where the profit requirements are subtracted from the yields. (In the general case a three-year tie up of one unit of capital would be more burdensome than a one-year tie up of three units. This is because of using the compound interest method.)

Based on all these, the index number of aggregate capital needs (E_{ACN}) is as follows:

$$E_{ACN} = \sum_{t=1}^n E_{t-1} \quad (10)$$

This index number reflects the effect of initial investment, the duration and the rapidity of capital payback in the aggregate capital needs rather well, but concerning the content it means a capital amount which is tied up for one year.

In case of equal lifetimes the index number of annual average capital needs ($E_{average}$) can also provide useful information.

$$E_{average} = \frac{\sum_{t=1}^n E_{t-1}}{n} \quad (11)$$

The average tied-up capital shows how much tied-up capital is used in average during the lifetime of the project. This average index number is comparable only if the projects' duration is the same. (The rapidity of capital payback is taken into account as well.)

The category of the aggregate capital needs has a significant opinion-forming role in the sphere of using the net present value method. For the evaluation of the relative amount of surplus profit this index number is also necessary. When using the internal rate of return, the aggregate capital needs are important information as well; because of this the index number shows the real comparable capital amount which results in this profitability.

As was mentioned above, the required rate of return can also have a great effect on the aggregate capital needs. The higher the required rates of return, the smaller the part of the annual yields for capital return. Consequently, when the required rate of return is higher the return of the capital takes longer and the index number of the aggregate capital needs will be higher. This is illustrated through the calculations related to

Project S (see Tables 2-4). In regard to the fact that when calculating the annual tied-up capital, the profit requirements were already considered, the examination concerns the nominal value of capital to be returned. The results of the calculations related to the capital needs are summarized in Table 5.

Table 5
The effect of interest rate on the capital needs for Project S

Measurement unit: unit

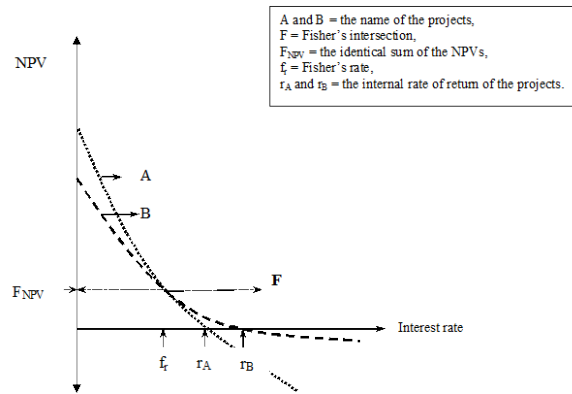
Year	Yearly capital needs depending on the interest rate				
	0 %	8 %	20 %	24 %	30 %
1	240	240	240	240	240
2	140	159	188	198	212
3	40	72	126	146	176
4	-	-	51	81	99
Aggregate capital needs					
Aggregated	420	471	605	665	727
Annual average capital needs calculated for the duration of the project					
Average	105	118	151	166	182

The analysis of Table 5 refers to Project S, where the duration is 4 years. This is the relevant information for calculating the annual average capital needs. In the case when the average calculation refers to the real tie-up period of the capital instead of the whole duration of the project, the index number does not show comparable information. (As Table 5 shows applied interest rates of 0 and 8 percent, the tied-up capital only lasts 3 years, while at 10 and 24 percent this would be 4 years. In case of an interest rate of 30 percent the profit requirements are not covered.)

THE RATE AND THE INTERSECTION

As was mentioned above, the finance literature usually calls the interest rate Fisher's rate when the net present values of the two investment projects are equal. The intersection of the net present value curves of the two projects is called the Fisher's intersection. The intersection and the rate are illustrated by Figure 2. The names of the projects are: Project A and Project B. The rate that belongs to the intersection on the x axis is the Fisher's rate, while the intersection reflected on the z axis gives the same net present value of the two projects.

In Figure 2, the net present value of Project A considering zero percent interest rate (i.e. the accounting profit of the whole duration of the project) is significantly higher than that of Project B. The aggregate capital needs of Project A must be significantly higher so that the gradually increasing profit requirements according to the rate of interest will use up higher amounts of the accounting profit than in the case of Project B. Under certain capital and profit conditions, the intersection comes into being as a result of the differently decreasing surplus profits. At the interest rate that belongs to the intersection, the net present values of the two projects are equal.



Source: Edition by the author, based on commonly used figures in the related literature (For instance, Keane 1975:22; Adelberg et al. 1986:708; Brealey & Myers 1988:83; Arnold & Hope 1990:257; Firer & Gilbert 2004:42; Van Horne & Wachowicz 2008:332; Crundwell 2008:189; and Baker & Powell 2009:259)

Figure 2 The intersection of NPV curves and the related Fisher's rate

At interest rates over the intersection, the project variant with lower capital needs will have the higher net present value. Where the surplus profit disappears, there is the internal rate of return. The surplus profit of the variant with lower capital needs lasts up to higher interest rates, which shows higher profitability of capital. The profit requirements of the variant with higher aggregate capital needs 'uses up' the total surplus profit faster, thus the internal rate of return will be lower. (The capital invested in this variant will have a lower average profitability.) This is the main reason that above the intersection the rankings based on the net present value and the internal rate of return are the same.

But it cannot be a question whether in terms of equal net present values the two projects can be considered economically equal. The aggregate capital needs are significantly higher for Project A. By investing the difference into another project, further net present values could be gained. We cannot leave out of consideration whether the same surplus profit can be gained with lower aggregate capital needs. This relationship can only be disregarded in case of unlimited capital resources and under other special conditions (Illés 2012b).

THE CAUSALITY BASED PRE-CONDITIONS FOR EMERGENCE OF INTERSECTION

Keane (1975) highlights that the existence of Fisher's rate is not necessary, furthermore when illustrating two net present value curves, even more intersections can emerge. The latter means that the net present values of two projects can be equal at several interest rates.

Keane (1975:23) cites the findings of Mao (1969) about the conditions that are necessary for existence of the Fisher's rate:

“There will be no Fisher intersection in the interval (0, r_m) where r_m = the smaller of the two rates of return, if

- (1) a) A's NPV > B's NPV at zero discount rate, and
b) A's NPV decreases at a greater rate than B's, in response to a given increase in k
c) A's IRR > B's IRR.
- (2) a) A's NPV > B's NPV at zero discount rate, and
b) A's NPV decreases at a lesser rate than B's in response to a given increase in k .

There will be a unique intersection between the two NPV functions where:

- a) A's NPV > B's NPV at zero discount rate, and
- b) A's IRR < B's IRR
- c) A's NPV decreases at a greater rate than B's in response to a given increase in k .”

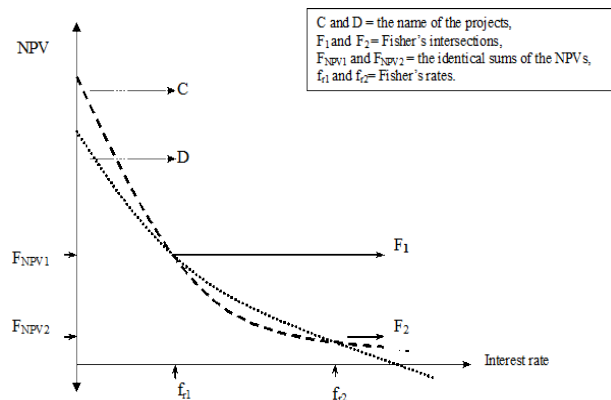
In their study Hirst & Ma (1983) look for the explanation of the emergence of Fisher's intersection. They built upon the fact that Fisher (implicitly) used equal initial investments in his calculations; therefore this can be excluded as a possible reason. Further root causes can be the duration of a project and the rapidity of capital payback. In order to consider the common effect of the two factors, the authors introduced the category of weighted duration, which can be used as an explanation for the intersection. They say that the net present value of the variant with higher weighted duration is more sensitive to changes in interest rate. (According to the terminology of the current study the reasons for the longer weighted duration are the longer duration and/or the slower rapidity of capital payback.) For the calculation of weighted duration Hirst & Ma (based on the works of other authors) use the present values of annual yields as weights. (In the numerator of the fraction there is a sum that is calculated by multiplying the ordinal numbers of operating years with the annual discounted yields. The denominator contains the sum of all discounted yields. The authors also analyze the effect of applied interest rate on the weighted duration.) Hirst & Ma (1983) show some interesting examples in the field of intersections in context of weighted duration. It must be emphasized that this weighted duration can only be accepted as a root cause if the initial investment is the same in both projects.

According to the related thesis of Descartes (1596–1650), the maximum number of internal rate of return of an investment project is determined by the number of sign changes in the yield series. The project may have as many internal rates of return as the number of yield series sign changes. A lower number of internal rates of return than this are possible, but no more. In our days this basic relationship can be considered as generally known. Having more than one internal rate of return is the characteristic of unorthodox cash flow pattern projects only. The number of internal rates of return means how many times the net present value function intersects the x axis.

The Fisher's rate – from this aspect – is the interest rate that makes the series of yield differences of the two

projects zero. The internal rate of return of the series of annual yield differences will be equal to the same interest rate where the net present value of the two projects becomes equal. So at the Fisher's intersection the present value of the yield differences of the two projects becomes zero. Consequently the Fisher's rate is the internal rate of return of yield differences as well.

If the criterion of an orthodox cash flow pattern is fulfilled for both variants and the sign change of the yield differences occurs only once, there can be no more than one point of intersection. However, several occurrences of intersections of two projects with orthodox cash flow patterns are possible as well when the sign of yield-differences changes on several occasions. This is illustrated in Figure 3.



Source: This sort of illustration can be found in the study of Hirst & Ma (1983:169) according to another conception. Illustration in this context is the author's work.

Figure 3 The possible occurrence of Fisher's rates and intersections of two projects with orthodox cash flow patterns where the yield differences changed sign twice

The several sign changes in the yield difference series do not necessarily mean several intersections. If intersections of the net present value curves of arbitrary type investment projects (e.g. with unorthodox cash flow patterns) are examined, then general quantifying of these intersections may become very complicated.

The classical Fisher's rate defines only one intersection. The majority of recent (English) financial books do not deal with the possibility of more than one intersection, either. A causality-based comprehensive background of this is not analyzed in the literature.

The simultaneous fulfillment of the following six conditions is necessary to have one and only one intersection:

1. The criterion of orthodox cash flow pattern must be fulfilled in case of both variants.
2. The sign change of the difference of the two series of yields occurs only once.
3. Both projects must have an accounting profit.
4. The sum of all accounting profits and the aggregate capital needs must be different for the two projects.

5. The variant with the higher sum of all accounting profits must have higher aggregate capital needs.
6. The variant with lower capital needs must have a higher internal rate of return.

The illustration of net present values of the two project variants starts with a discounting by zero percent interest rate (Figure 2). If the accounting profit sums differ, the starting points on the y axis will also be different, thus one requirement of the intersection is already fulfilled.

The variant with higher sum of all accounting profits must have higher aggregate capital needs, so that by increasing the interest rate, the net present value of this variant would decrease faster. The different rapidity of decrease is also a requirement of the intersection.

In addition, to have equality besides a positive net present value, it is also necessary that the variant with lower capital needs must have a higher internal rate of return. (The first and second conditions are necessary so that only one internal rate of return of yield differences may come into being.)

THE IMPORTANCE OF THE INTERNAL RATE OF RETURN OF YIELD DIFFERENCES

In his quoted work, Fisher deals a great deal with the question of internal rate of return of the series of annual

yield differences of the two projects. He attaches very great importance to it.

As mentioned in detail above, the internal rate of return of the yield differences means exactly the same interest rate where the net present values of the two projects are equal. This is obvious, as this interest rate has made the differences of yield series disappear.

According to my current knowledge, this is only a technical feature, it has no substantive significance. The literature on modern corporate finances uses the internal rate of return of the yield differences as a rather simple way of calculating the Fisher's rate. This method of Fisher's rate calculation is much simpler than calculating the net present values of the two projects starting from zero percent interest rate to higher percentages until the two net present values become equal (or this interest rate will be determined in approaching from two sides).

ILLUSTRATION AND EXPLANATION OF THE INTERSECTION THROUGH AN EXAMPLE

The purpose of the example below is to illustrate and explain the Fisher's rate and the Fisher's intersection through numerical relationships. The most important data of the two mutually exclusive project variants are summarized in Table 6.

Table 6
Revenues, expenditures and yields of the two project variants
(assuming equal initial investment, equal duration, but different aggregate capital needs)

Date	Project A ₁			Project B ₁			Measurement unit: unit
	Expenditures	Revenues	Yields	Expenditures	Revenues	Yields	The difference of the two series of yields A ₁ – B ₁
0	350	0	-350	350	0	-350	0
1	552	557	5	771	1171	400	- 395.000
2	691	696	5	1017	1022	5	0
3	352	852	500	861.488	866	4.512	495.488

According to the database the main economic characteristics of the projects are the following. The initial investment and the duration of the two projects are equal but the rapidity of capital payback is significantly different. In case of Project B₁, the nominal value of the 350-unit initial investment is exceeded by the yield of 400 units already at the end of the first year. In the following two years – despite the fact that the sales revenues only decreased by 8-12 percent – only 4.5-5 units of yield are gained. In the same time, in the case of Project A₁, the yields of the first two years only support the return of capital needs and profit requirements with a rather small amount of yield, 5 units; 98% (500 units) of the yield is only gained at the end of the third year. At an

interest rate of zero percent the total accounting profit of variant A₁ is 160 units, while of variant B₁ it is only 59.5 units.

Considering the fact that the two series of yields are orthodox cash flow patterns, the internal rate of return shows the average profitability of the projects. The aggregate tied-up capital shows how much capital operates with this profitability rate.

The internal rate of return for Project A₁: ~13.5 %, aggregate tied-up capital: 1182 units.

The internal rate of return for Project B₁: ~16.5 %, aggregate tied-up capital: 362 units.

Variant B₁ with its significantly lower capital sum results in the internal rate of return of 16.5 percent.

Variant A₁ with higher aggregate tied-up capital provides a capital profitability of 13.5 percent. For making a good choice between the two projects, it is practical to examine the profitability possibilities of re-investing the yield difference of 395 units for two years, and then compare

this with the critical profitability rate of the re-investment. (Details are shown in Illés 2012a.) Table 7 contains the data necessary for the illustration of Fisher's rate and Fisher's intersection.

Table 7
The net present values of two project variants with different interest rates

Project	Measurement unit: unit							
	0 %	4 %	6 %	8 %	10 %	12 %	15 %	20 %
A ₁	160.0	103.9	79.0	55.8	34.3	14.3	-13.1	-53.0
B ₁	59.5	43.3	35.6	28.2	21.2	14.3	4.6	-11.6

The net present values of the project variants can differ due to the different interest rates as well. When increasing the interest rate, due to the higher aggregate capital needs, the net present values of Project A₁ are decreasing more rapidly than those of Project B₁. The equality of net present values occurs at the interest rate of 12 %. The significant points calculated from the data of the above example are illustrated in Figure 4.

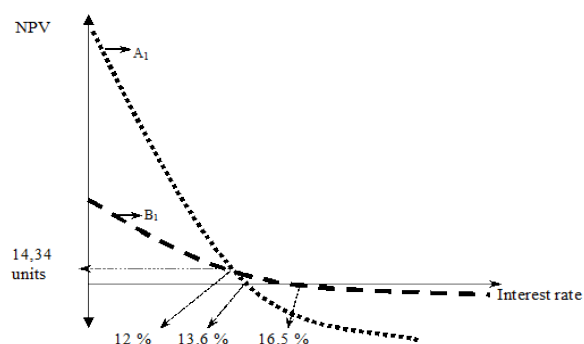


Figure 4 The Fisher's rate and the Fisher's intersection calculated from the data of the above example

Calculating the internal rate of return (r) from the differences of the yield series of the two projects can help to estimate the Fisher's rate:

$$\text{Basic relationship: } -395 \frac{1}{1+r} + 495,488 \frac{1}{(1+r)^2} = 0$$

$$\text{After rearrangement: } \frac{495,488}{395} = (1+r)^2$$

After executing the calculation: $(1+r)^2 = 1.2544$; where $r = 0.12$

The internal rate of return calculated from the data series of yield differences is 12 %, which is equal to the Fisher's rate. As we can see, this rate of 12 % is irrelevant in the economic evaluation of projects.

SUMMARY

According to the approaches and recommendations of finance, the net present value has a primary importance in supporting investment decisions. Therefore the existence of Fisher's rate and the rate itself are significant business

information in finance. The literature calls attention to the fact that conformation of the intersection is not necessary. This study works out the causal relationships that are necessary for existence of the intersection, in order to gain a better understanding of internal relationships of the net present value method.

The net present values of different projects cannot be compared in general; therefore the net present value is not suitable as a ranking indicator. Factors that cannot be disregarded are how much initial investment, for how long and how much tied-up capital is needed to gain a certain net present value. After eliminating the three main distorting factors, the net present value will be a sort of rate that gives the difference between the internal and the required rate of return (corrected by a calculation error factor). Thus this corrected net present value rate is approximately equal to the surplus profitability that exceeds the required rate of return (Illés 2012a).

If the risks of the projects are equal or risk management of the projects has been done in advance (e.g. decreasing the series of revenues or increasing the series of expenditures and so on), the ranking based on the net present value rate that is free of distorting effects and the ranking based on the internal rate of return will be the same. The surplus profitability over the required rate of return will be the highest for the project where the internal rate of return is the highest. Thus the same projects will gain the first, second, etc. place in both rankings. No change of ranking is possible with Fisher's rate.

In theory, the company can reach the best growth if the projects with the highest internal rate of return are carried out. (Provided, that is, that the required rate of return is equal.) Based on the rates of critical profitability and the real profit opportunities of the difference of initial investment and other capital need differences, the expedience of choice can prefer solutions that differ slightly from the ranking based on the internal rate of return. (This topic is showing in detail by work of Illés 2012a.) But even in these cases the decision criterion is still not the net present value. The relationship that in case of limited capital-investment opportunities at a certain risk level, the highest profitability can lead to the highest profit is still valid.

From this aspect, ranking based on the simple net present value is not relevant as well. The interest rate that would provide the same net present value for two projects is also irrelevant. In most cases when the net present values are equal, the aggregate capital needs are different.

The paper also defines the special content of aggregate capital needs, and elaborates an index number for this content. The project's aggregate capital needs means the amount of capital needed for the operation of the project during its full duration. It considers the income-producing potential necessary to take into account simultaneously the tied-up capital and tied-up time. The measurement unit is one unit tied-up capital for one year. The solution is to sum up the yearly tied-up capital that is the not-returned parts of the capital for each year. This solution considered to be correct because of the tied-up capital is computed with a database where the

profit requirements are subtracted from the yields. The category of the aggregate capital needs has a significant opinion-forming role.

A highly important relationship is the following: profit requirements calculated with compound interest should be returned from the sum of the total accounting profit. The larger the aggregate capital needs of the project, the greater the profit requirement that arises.

Based on these conclusions, considering a real economic system of conditions and the purpose of long-term profit maximization (maximization of shareholder value), Fisher's rate and the Fisher's intersection can only have theoretical significance. Their analyses can give a better understanding of the content background of the net present value, but are irrelevant for making investment decisions.

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