The Real Reinvestment Rate Assumption as a Hidden Pitfall

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SUMMARY

The paper explores a few hidden problems of the reinvestment rate assumption. The automatism of net present value method creates and applies a very special reinvestment rate assumption. This assumption does not disturb the evaluation of investment projects with orthodox cash flow patterns. However, in the case of unorthodox cash flow patterns, automatism constructs a serious mistake in the calculations. In this case, the net present value provides wrong information about the economic efficiency. However, according to the general academic opinion, the net present value method is suitable for evaluation in the case of unorthodox cash flow patterns as well, as there can be only one net present value as opposed to the opportunity of several internal rates of return. The paper sets out to prove that this way of evaluation is wrong, and works out a solution based on the real economic basis.

Keywords: reinvestment rate assumption, net present value, internal rate of return, orthodox and unorthodox cash flow pattern, ranking, aggregate capital needs.

Journal of Economic Literature (JEL) codes: M21, G31, G32

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INTRODUCTION

The debate about the reinvestment rate assumption began in the 1950s (Solomon 1956; Renshaw 1957), and is still underway. This debate is essentially about whether the net present value method and the internal rate of return method may contain a kind of assumption concerning profitability of reinvestment of the annual yields. The essence of the disputed assumption is as follows: the two methods assume different rates of return concerning the reinvestment of annual yields (as long as the project lasts). According to this, the net present value method assumes the required rate of return, while the internal rate of return method takes the internal rate of return as the reinvestment rate.

The contested conception emerged as a kind of treatment of the ranking conflict which often occurs between the net present value (NPV) and the internal rate of return (IRR). The supporters of the described above reinvestment rate assumption concept ensure automatic priority to the NPV method by emphasizing that reinvestment according to the high IRR is hard enough. A typical example: "Projects can be ranked from highest to lowest IRR, with the highest being considered superior. The reinvestment rate assumption constitutes a drawback of this approach, as it assumes that every time a cash inflow occurs it can be reinvested to earn the IRR for the remainder of the project's life. Sometimes this is an unrealistic assumption, especially for high-IRR projects." (Laux 2011:30)

The content band covered by the debate may be narrowed to some extent by the fact that only orthodox cash flow patterns are involved in the topic of ranking conflict (there is only one sign change in the cash flow line). In the case of unorthodox cash flow patterns, the IRR method is inadequate for project evaluation. Therefore in such cases the ranking conflict cannot even occur between the two methods.

The debate is slightly one-sided. One of the dominant groups does not argue and does not react to the opposing views, just repeats the validity of the reinvestment rate assumption as a well-known relationship. These views can be considered roughly uniform (and they are typical in the finance literature). The representatives of the significantly smaller group of those who partially or fully reject the reinvestment rate assumption try to prove that this assumption is wrong. They use different logical arguments as well as mathematical or exemplary evidence. Their methodical solutions are also varied (for example Dudley 1972;

Carlson et al. 1974; Keane 1979; Lohmann 1988; Johnston et al. 2002; Crean 2005; Rich and Rose 2014).
A great number of studies on this topic have been published during these six decades. In these publications a number of unclear conditions, categories and phrases can be found. For instance, the reinvestment rate assumption itself is interpreted as either an explicit, implicit, or some kind of general assumption. The reinvestment amount is not always obvious, either. These amounts can mostly be interpretable as yields coming from a project in different years of its duration, more rarely as differences computed from yields of two examined projects. Sometimes the examined problem is not actually the reinvestment rate assumption, but the critical reinvestment rate (for example Alchian 1955; Dudley 1972; Meyer 1979). In the latter cases, references date back to Fisher (1930). Meyer's paper (1979) examines this question according to the system tools and categories of microeconomics.

Several authors point out their disapproval with the one-sided teaching of faulty doctrines. Among them, Johnston et al. (2002) call attention to the fact that a number of finance textbooks completely ignore scientific findings that disprove the reinvestment rate assumption of the two methods. Due to this, they urge reforms. In the introduction of their paper, Walker et al. (2011) give a detailed description of teaching completely controversial materials. Their research joins the study of Keef and Roush (2001), which draws the attention to the fact that finance textbooks use the reinvestment rate assumption higher proportion than textbooks in management accounting. The findings of their own research done a decade later show a similar direction. They make an important statement emphasizing the lack of consistency amongst disciplines: "Finance books fall at one end of the continuum with 64 percent using the assumption while the engineering economics books fall at the other end with just 20 percent using the assumption" (Walker et al. 2011:11-12).

Considering the given aim and content, the debate about this reinvestment rate assumption is unnecessary. The bottom line is that the problem of ranking conflict mistakenly occurred because the NPV method is inherently unsuitable for ranking. The differences in initial investments, durations and rapidity of capital returns may distort the comparability of net present values. (Today this problem is well known.) In the case of orthodox cash flow patterns a systematic and correct elimination of distorting factors leads to a special NPV rate, which is the difference between IRR and the required rate of return. This rate difference as an authentic NPV rate gives the same ranking list as the internal rate of return (assuming equal required rates of return). Therefore, in the case of an equal required rate of return, the ranking according to the correctly computed net present value rate and the ranking according to the internal rate of return cannot differ from each other. As two different and correct rankings cannot emerge, the ranking conflict between these two methods cannot occur either. (This topic is shown in detail in Illés 2012a and 2014.) Note that the literature of business economics applies different methods for taking the risk into consideration, and thus the correction of required rate of return due to the project risk is not the one and only solution.

The topic of the reinvestment rate assumption is still an essential one, despite the fact that this question is not relevant to the original problem. This topic arises in other structures and contexts that are definitely different from those presented above. The present paper proves that the NPV method automatically creates a special kind of reinvestment rate assumption in all cases. This one is the real reinvestment rate assumption. In the case of evaluation of orthodox cash flow patterns, this assumption does not have any disturbing effects. However, in the case of unorthodox cash flow patterns, this automatism involves an error, which precludes the possibility of a correct application of the NPV method for project evaluation in this field. In the case of unorthodox cash flow patterns the automatism of the real reinvestment rate assumption come into being for the IRR method as well. The possibility of multiple internal rates of return markedly shows that this method is unsuitable for evaluation in this field. A brief description of the distorting mechanism may contribute to a better understanding of details.
The main objectives of this paper are:

1. to reveal the essence and effect mechanism of the real reinvestment rate assumption;
2. to present the automatic realization of this assumption on the basis of mathematical models and to explain the process with help of examples;
3. to present the misleading effects of the real rate assumption resulting from the automatism of the method in the case of unorthodox cash flow patterns, and
4. to look for possibilities for eliminating misleading effects.

RESEARCH METHODOLOGY

This paper examines the circumstances and the effect mechanism of the real reinvestment rate assumption under the conditions and methodological solutions listed below.
1. *The traditional concept of the NPV and the IRR methods:* The paper interprets and analyzes the content background of NPV and IRR methods in the classic sense. Among others, the paper uses the term 'capital' as a homogeneous sum in terms of ownership. Profit is interpreted as a pre-tax profit. The interpretation and analysis of methods are related exclusively to investment projects. The analysis of financial market projects does not fall within the scope of the research. The paper does not cover the analysis of further branches and

combinations of traditional methods. It does not consider inflation effects, either.

2. *Business economics approach and system of aspects:* In the literature, there are two different trends of the comprehension and analysis of the NPV and IRR methods. The business economics interprets and manages the database as well as the results of calculations according to the conditions in reality. Finance is inherently built on standard microeconomic foundations. This trend has a relatively high level of abstraction, and applies categories partially different from those used in real life. Furthermore, in finance the conditions of a number of respects differ from reality. Therefore, the paper is based on business economics foundations. Illés (2012b) reveals the main discrepancies occurring in the relevant topic between finance and business economics.

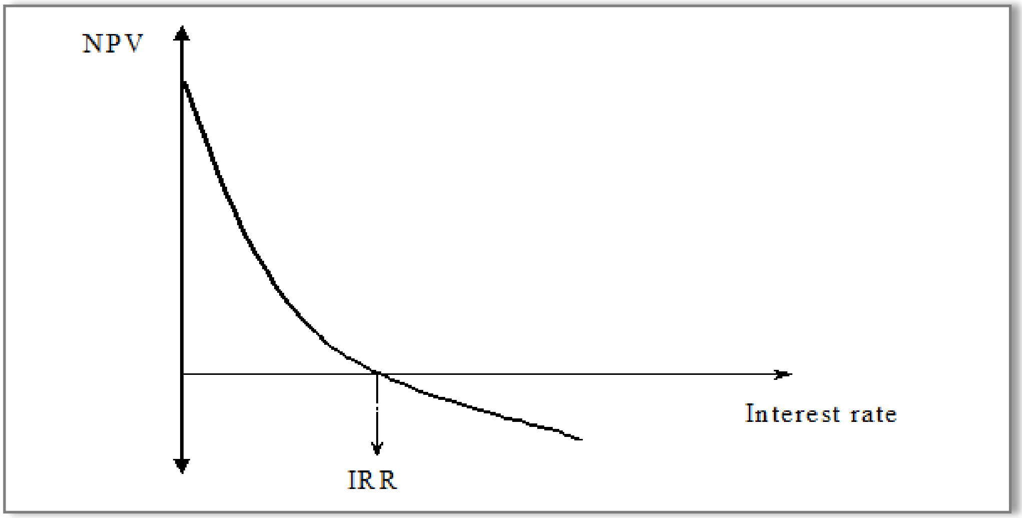
3. *The calculation logic follows the real process of management and after the close of this, returns to the discounting method:* The paper assumes that the management relations can be clarified moving forward in time according to the management process. (The planning and thinking of corporate executives works the same way) This is the only way in which the emergence and realization of return requirements as well as the process of the surplus profit formation can be seen through (Illés 2012/a, 2014). Therefore, the study uses a detour to substitute for the classical methods. In order to show the content tally with the classical methodology of project evaluation, after the systematic exploration of content relations, the analysis returns to the classical method. Discounting back to the start time makes the examination of the management process impossible.

4. Yield analysis according to the return structure: Exploration and analysis of the return process can be solved by following the formation of internal structure of the yield. The yield is the difference between the annual revenues and annual expenditures. A positive amount of yield is surplus revenue in terms of the project's financial needs. Therefore the yield exits from the project at the end of the given year. The conditions of further utilization of this usually do not affect the evaluation of the analyzed project. In the case of orthodox cash flow patterns, the content of the yield with a positive sign can consist of capital return and/or profit. In the NPV method, the profit part of the yield may consist of further two parts: profit according to the required rate of return and surplus profit. Until meeting the return requirements, the yield consists of profit according to the required rate of return and capital return. After the fulfillment of return requirements the content of the emerging yield is surplus profit. In the IRR method, the yield all along consists of capital return and profit according to the interest rate, there is no surplus profit. In this case, collation with the return requirements takes place after calculations.

ORTHODOX CASH FLOW PATTERNS, NET PRESENT VALUE

The importance of profit sum calculated at nominal value

The profit sum calculated at nominal value does not appear in the database of the NPV method. The significance of the nominal profit sum can be presented by the NPV curve, the general shape of which is well-known. This shape frequently appears in publications dealing with net present value. The curve shows what sum of net present value comes, with what interest rate (*Figure 1.*).



Source: Widely-used illustration

Figure 1 The sum of the net present value as a function of interest rate

Explanations related to the curve generally interpret only the surface. They point out that the higher the interest rate is, the lower the net present value becomes. The reduction first reaches the zero NPV, after that, because of the increase in interest rate the NPV becomes more and more negative. The interest rate, which results in the zero NPV, is the IRR itself. (This is well known.) Deeper explanations about the curve are not known.

However, there are quite significant connections in the content background of Figure 1. First of all the fact should be emphasized that this curve may be used only for profitable projects with orthodox cash flow patterns, for two reasons. First, the basic condition of a monotonic decrease is orthodox cash flow patterns. Second, the curve starts from a positive value range, and for that, the project should be profitable. With zero interest rate, the NPV quantifies the nominal value of the profit occurring during the whole duration of the project. (After the substitution of zero interest rate to the general formula, the NPV turns out to be the difference of the amount of all annual revenues and the amount of all annual expenditures calculated at nominal value.) *The content of the net present value related to the zero interest rate also makes it clear that the nominal profit occurring during the whole duration of the project can only serve to cover the profit requirements*. The maximum amount of profit requirement that can be covered is equal to the nominal profit generated during the project duration (for further details see Illés 2014).

The source of the net present value is the remaining surplus profit, which is the difference between the profit calculated at nominal value and profit requirement according to the required rate of return. The present value of this difference depends on the date of emergence and the required rate of return. In the case of investment projects with orthodox cash flow patterns the net present value shows the sum of the surplus profit above the required profit (or lack of it), discounted for the present date (Illés (2012a) proves this mathematically).

Content of the real reinvestment rate assumption

According to the logic of time going forward, firstly the capital and profit requirements should be recovered. These items gradually quit the project and calculations, according to their return. (The method does not charge farther return requirements for these items.) The yields generated after the fulfillment of return requirements are the surplus profit. *These sums also leave the project; however, they remain in the calculations. The NPV method focuses on the enumeration of sums interpreted in this paper as surplus profit.*

In the course of calculations according to the logic of moving forward, surplus profits must be increased by the interest rate by the end of the period to the possibility of summation. The interest income occurring this way is not real, rather technical item, which support the possibility of summation of surplus profits emerging at different times. As a consequence, there will be a surplus profit higher than the nominal value at the end of the given period. *In the course of discounting back to the zero point of time, false interest income disappears from the calculations. However, the yield rate assumption does not disappear.*

The required rate of return has a role in discounting the surplus profits, despite the fact that the surplus profit also quits in the year of emerging, and furthermore the surplus profit cannot be regarded as the organic part of the project. The discounting mechanism related to the surplus profit automatically assumes that the profitability of this surplus will be equal to the required rate of return according to the project. This way the reinvestment rate assumption will prevail, but only regarding the surplus profit appearing above the profit as to the required rate of return. It is important to emphasize that *the assumption concerns only the surplus profit and it concerns neither the total yield, nor the yield part for capital return, nor the yield part for return on profit requirement.* (The content band of the real assumption is significantly narrower than that in the literature.)

In this case the reinvestment rate assumption exclusively enforced for the surplus profit does not cause any inconvenience. (The intended proper content can be reached by the average reinvestment rate on the market.)

Presentation of content relationship based on model editing

At the beginning of modeling it is required to determine the formula according to which – as long as the invested capital and its required profit return – the annual yield is appropriated for the return requirement. Except for the last year of the pay-off period, the emerging yield consists of two content elements: capital return and the profit according to the required rate of return. Formula (1) describes the calculating process of this. According to the calculation, firstly the profit requirement is extracted from the sum of the given year's yield. The remainder sum is the current year's capital return. This sum decreases the next year's tied-up capital. Further details are given in Illés 2014. [Formula (1) could be mathematically simplified, but then the formation of the structure itself cannot be seen.]

$$(E_{t-1}\, i + E_{t-1}\,) - H_t = E_t \qquad \mid\; H_t < E_{t-1}(1+i);\; 0 < t < z \tag{1}$$

where:

H_t = the yields (that is, the difference of revenues and expenditures) in year t, where the value of Ht is always positive for years $0 < t \leq n$ by the terms of orthodox cash flow pattern and the initial investment occurring at the zero point of time,

E_t = the not-returned part of capital at the end of year t,

i = required rate of return,

t = serial number of years,

z = number of years of the pay-off period (including the last commenced year).

As the second step is required to make up a formula, showing the economic content of yield generated in the final year of payoff period. Economic content of this yield consists of three elements: profit return according to required rate of return, capital return and surplus profit. On this basis formula (2) describes the calculation of surplus profit concerning the final year of the pay-off period.

$$H_z - (E_{z-1}\, i + E_{z-1}) = \Delta H_z \qquad \Big|\ H_z > E_{z-1}(1+i) > 0 \tag{2}$$

where ΔH_z = sum of the surplus profit in the last commenced year of pay-off period.

The yield occurring in the years after the return totally consists of surplus profit. The description of its quantification begins by making formula for the calculation of the annual amounts.

The surplus profit at the end of the first year after the pay-off period:

$$H_{z+1} + \Delta H_z\,(1+i)$$

The summed surplus profit at the end of the second year after the pay-off period:

$$H_{z+2} + [H_{z+1} + \Delta H_z\,(1+i)]\,(1+i)$$

The summed surplus profit at the end of the third year after the pay-off period:

$$H_{z+3} + \{\, H_{z+2} + [H_{z+1} + \Delta H_z\,(1+i)]\,(1+i)\}(1+i)$$

Considering the third year's formula, the sum of all of the annual surplus profits charged with the interest rate can be calculated by the end of the duration as follows (3):

$$FV\Delta M = \sum_{j=1}^{s} H_{z+j}(1+i)^{s-j} + \Delta H_Z (1+i)^s \tag{3}$$

where:

$FV\Delta M$ = the sum of the surplus profit charged with interest rate at the end of the duration,

j = the ordinal number of the years of the operating period after the pay-off,

s = the number of years of the operating period after the pay-off $(s = n - z)$.

Formula (3) contains some false interest rate income. The false interest income falls out during discounting. Surplus profits will be discounted from the year of their occurrence. The present value of the discounted and summed surplus profits is the net present value itself. As for the applied resolution, it realizes according to formula (4).

$$NPV = [\sum_{j=1}^{s} H_{z+j}\,(1+i)^{s-j} + \Delta H_z\,(1+i)^s\,]\,\frac{1}{(1+i)^n} \tag{4}$$

where n = duration of the project $(z + s)$.

After simplification [using $(n = s+z)$] there will be a clear formula, according to which the net present value can be reached by discounting and assuming the surplus profits according to the date of occurrence:

$$NPV = \sum_{j=1}^{s} \frac{H_{z+j}}{(1+i)^{z+j}} + \frac{\Delta H_z}{(1+i)^z} \tag{5}$$

(A non-structure-follower proof of this content is included in Illés (2012a).)

Starting from the classical version of the NPV inscription it cannot be seen that only surplus profits remain among the really discounted items. The above formulas prove that the automatic reinvestment rate assumption can occur only concerning them.

Illustration and explanation with a simple example

Example: The cash flow pattern of Project A in order of commencement years:

units -300, +200, +150, +50, +20

Table 1 shows the formation of the return process according to the yield structure.

Table 1

The return process of Project A at 10 percent required rate of return

Measurement unit: unit

Year	At the beginning of the year	The yield structure at the end of the year			Balance
		Profit requirement/ false interest income	Capital return	Surplus profit	
1	-300	30	170	-	-300 + 170 = -130
2	-130	13	130	7	-130 +137 = +7
3	+7	0.7*	-	50	+7+0.7+50 = +57.7
4	+57.7	5.8*	-	20	57.7 + 5.8 +20 = +83.5
Net present value: 83.5 × 0.68301 = 57.0					

*false interest income

According to the conventional calculation method the net present value of Project A is as follows:

$NPV_{A,10\%}$= -300 +200 × 0.90909 +150 × 0.82645 + 50 × × 0.75131 + 20×0.68301=57.0

In the false interest income column of *Table 1*, units 0.7 and 5.8 marked by an asterisk (*) do not represent the real yield. The two items are functioning as technical factors ensuring the additivity of surplus profits emerging at different times. It is obvious that both of them disappear during discounting.

According to formula (5) the net present value can be defined as the sum of discounted surplus profits. Amounts of surplus profits emerging in certain years can be seen in column 5, *Table 1*. The calculation is as follows:

$NPV_{A,10\%}$= 7×0.82645 + 50×0.75131 + 20×0.68301 = 57.0

ORTHODOX CASH FLOW PATTERNS, INTERNAL RATE OF RETURN

As it well known (and can be seen in *Figure 1*), the IRR is an interest rate in terms of which the NPV is zero. According to this as timing and as much profit is generated which exactly results in the profitability, according to the internal rate of return. (This sameness principle gives the essence of the IRR method.) Thus, technically there is no surplus profit (no lack, either). This also means that the calculation mechanism of the IRR method does not create any reinvestment rate assumption in the case of orthodox cash flow patterns.

Illustration and Explanation with an Example

Example: the cash flow row of Project B in order to the serial number of years is as follows:

units -240, +100, +100, +100

The internal rate of return is 12%. *Table 2* shows the formation of the structure of the yields.

Table 2

The content structure of the yields of Project B at 12 percent interest rate

Measurement unit: unit

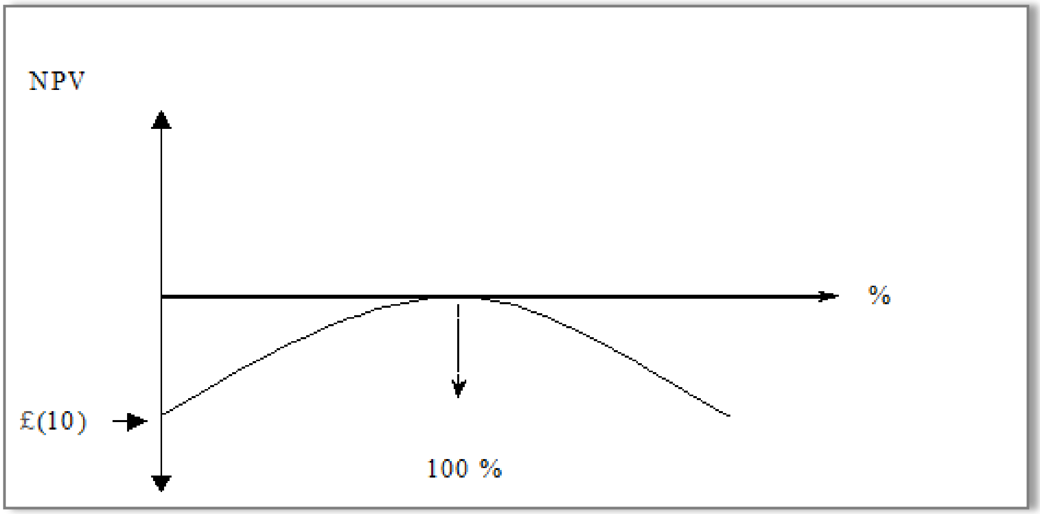
Year	Capital to be returned at the beginning of the year	The structure of 100 units annual yield		Capital still to be returned at the end of the year
		For profit requirements	For capital return	
1	240	29	71	- 240 + 71 = - 169
2	169	20	80	- 169 + 80 = - 89
3	89	11	89	- 89 + 89 = 0

Computational materials in *Table 2* present as well that in the case of the IRR method there is no surplus or lack of profit compared to the amount created according to the interest rate (in case of orthodox cash flow pattern). The calculation mechanism does not show any reinvestment rate assumption.

INVESTMENT PROJECTS WITH UNORTHODOX CASH FLOW PATTERNS

A wide variety of unorthodox cash flow patterns can be imagined. Usually the number of sign changes has the greatest importance. Furthermore, the relative size of the initial investment, the occurrence of yearly yields and their sum, and the time of changing signs also create different specialties.

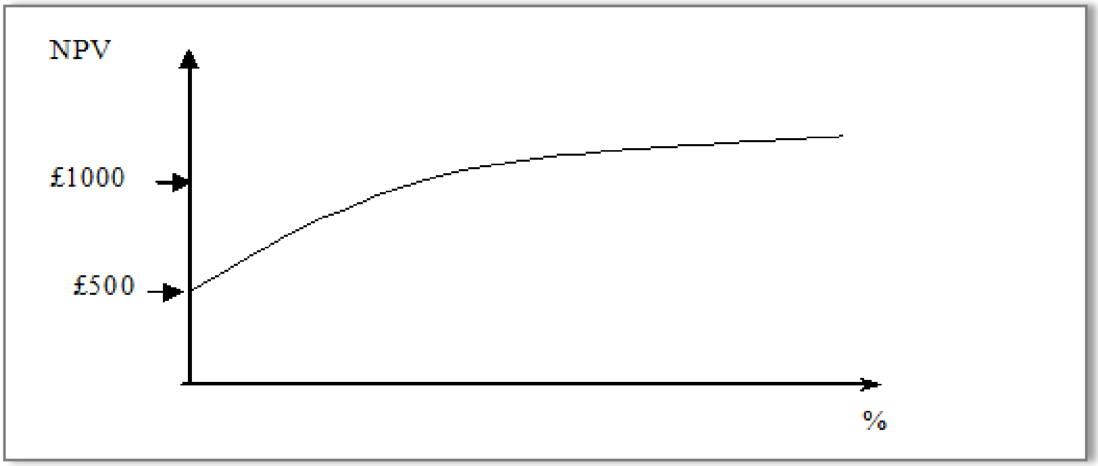
Keane quotes Mao's (1969) example with a figure in which – despite the change in two signs – only one IRR can be realized, and there is not any interest rate that would result in a positive net present value (Keane 1975: 16-17). The annual yields of the mentioned example are as follows: -£10, +£40, -£40. (The project has a loss according to its nominal value. That is why the net present value curve in *Figure 2* starts at -£10.)



Source: cited in Keane (1975:17)

Figure 2 The curve of Mao's net present value example

In addition, Keane (1975:18) uses an example demonstrating a project possibility that has an always positive NPV curve, and where there is no internal rate of return. In his example the cash flow line is as follows: +£1000, -£3000, +£2500. The pattern is rather peculiar. At the moment of the beginning (that is, the zero point) an income surplus of £1000 quits the project, then a year later an expenditure surplus of £3000 occurs, which is then followed by an income surplus of £2500. The profit calculated at nominal value is £500. (That is why the curve in *Figure 3* starts at £500.)



Source: Keane (1975:18)

Figure 3 The curve of Keane's special net present value example

Another example with a similar structure can be found in the book written by Arnold and Hope (1990:258): +£1000, -£2000, +£2000. The result is $r = \sqrt{-1}$ calculated by the authors as an internal rate of return.

In Van Horne and Wachowicz (2008:342-343) there is an example where – despite the double sign change – there is only one internal rate of return. The cash flow line is as follows: +$1000; -$1400; +$100. In addition, there is another example with three sign changes and three internal rates of return. (Cash flows are: -$1000; +$6000; -$11000; +$6000. The internal rates of return: 0, 100 and 200 percent.) A wide choice of extreme examples can be selected from publications focusing on the topic of multiple internal rates of return (e.g. Schafrick 2003).

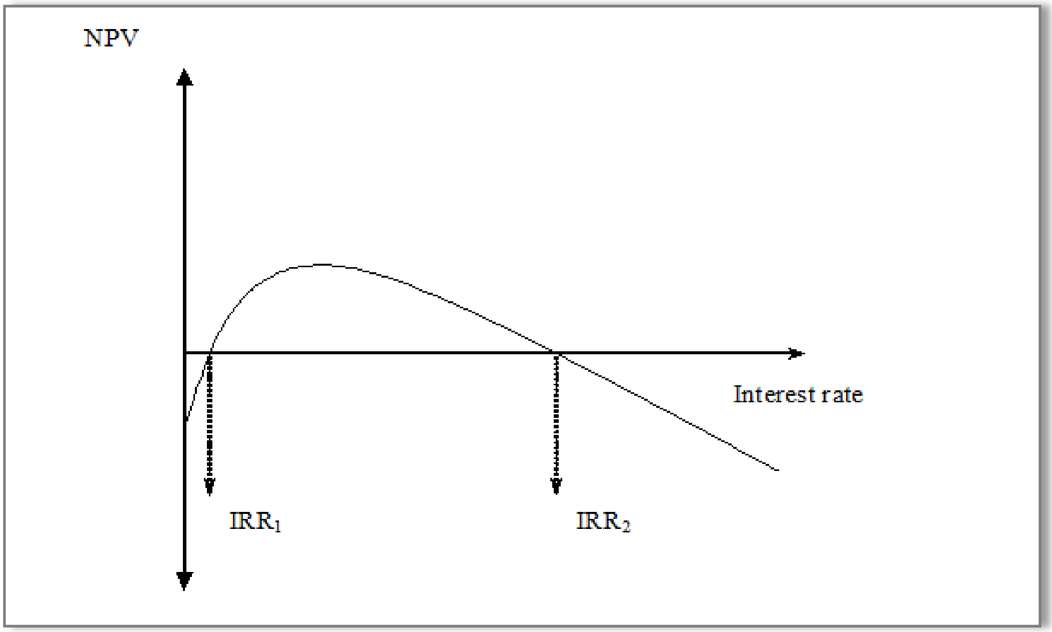
In the majority of the accessible publications there are examples concerning unorthodox cash flow patterns, where initial investment is relatively low and a very high income surplus occurs at the end of the first year as compared to the initial investment, then the second year also finishes with a similarly high expenditure surplus. In these examples, the cash flow raw has two internal rates of return. Some examples are given in Table 3.

Table 3
Published examples of unorthodox cash flow patterns with double internal rate of return

Source	Unit	Year			IRR, percent
		0	1	2	
Solomon (1956: 128)	$	- 1,600	+ 10,000	- 10,000	25 and 400
Brealey & Myers (1988: 80)	$	- 4,000	+ 25,000	- 25,000	25 and 400
Arnold & Hope (1990: 258)	£	- 2,000	+ 5,100	- 3,150	5 and 50
Plath & Kennedy (1994: 82)	-	- 16	+ 100	- 100	25 and 400
Firer & Gilbert (2004: 43)	-	- 1,600	+ 10,000	- 10,000	25 and 400
Van Horne & Wachowicz (2008: 341)	$	- 1,600	+ 10,000	- 10,000	25 and 400
Bierman & Smidt (2012:93)	$	-100	+310	-220	10 and 100

Below, the content of the main relations is revealed for this type of cash flow pattern. (The relatively low initial investment and this sum with its required profit can return in the first year of the project period, but a high sum of expenditure surplus occurs at the end of the period.) Certainly, the hidden context can be revealed as to other example types however, such complexity would spoil the transparency of the models. With the knowledge basis of the essence of these models, logically, the main problems of other unorthodox cash flow pattern models can become transparent.

In the case of unorthodox cash flow patterns the NPV calculated with zero interest rate also quantifies the nominal profit sum emerging along the total duration of the project. Each of the examples listed in Table 3 is calculated at loss-making nominal value. All of the net present value curves start from the negative range of values, and cross the x axis twice (*Figure 4*). All of the possible net present values are positive in the section between the two internal rates of return. Each interest rate outside this section leads to a negative net present value.



Sources: Brealey & Myers (1988:81); Firer & Gilbert (2004:43);Van Horne & Wachowicz (2008:342) and so on.

Figure 4 Net present value curve of the examples shown in Table 3

If any of the projects in Table 3 wishes to achieve profitability of at least zero net present value (according to the required rate of return), *the loss should be settled from some kind of sum, in addition, the project should generate the profit according to the interest rate, but this is impossible in the terms of the database*.

The riddle of net present value of projects with unorthodox cash flow patterns

One important question is how the positive net present value occurs in the interest rate band between the two internal rates of return in the case of the studied example types. These projects are loss-making. How can the loss and the required profit return, moreover how can some surplus profit emerge, creating a positive net present value? Where do these sums come from? (The yields of the quitting sums are related to other projects' result and are not applicable here.)

The problem can obviously be rooted in the cash flow with the negative sign emerging in the final year of the project. In the examined example types the initial investment and its profit requirements return in the first year. However, the surplus yield is not the surplus profit, but a temporary surplus profit, the amount of which is not even enough to cover the final year's negative cash flow. According to the calculation mechanism the temporary surplus profit can also have interest income. In reality this is only false income. Despite this fact, the automatism of the method utilizes them (partially or totally) in financing the negative cash flow.

The sum of the temporary surplus profit quitting the project can logically be included as a return element of the later expenditure, but the interest income related to it cannot. (The interest income will mean a chance for another project, providing the yields of reinvested temporary surplus profit.)

The false interest income does not disappear at rediscounting. In the case of orthodox cash flow patterns, its disappearance is implemented during the discounting process of the surplus profit. In this connection the key factor of the disappearance of the false interest income is the discounting of surplus profit. However, in the case of the studied unorthodox cash flow patterns there is no surplus profit, only temporary surplus profit, which is totally used for partially covering the sum of the negative cash flow at the end of the period. For this reason the temporary surplus profits cannot be discounted and their false interest incomes cannot disappear.

According to the mechanism of the method the false interest income of temporary surplus profit is able to cover the real losses caused by the project and meet the profit requirements according to the interest rate, and even surplus profit can emerge.

The most important point of the problem is as follows: the calculation mechanism of the net present value method handles the false interest income of temporary surplus profit as real money. The method uses the false income to cover the loss and profit requirements. In the x axis section between internal rates of return there is an unused part of the false interest income. Discounting the unused part of the false interest income leads to a positive sum of net present value. The net present value emerging this way is false as well. The false yield cannot become real yield, not even by discounting. (In reality, all of the net present values of the studied unorthodox cash flow patterns are false.)

In the case of studied unorthodox cash flow patterns the reinvestment rate assumption is the real problem during the formation of the false interest income as well as the discounting of the false surpluses.

Using the model analysis

The model analysis also refers to the cash flow pattern type in Table 3. Conditions for this model:

a) the cash flow pattern starts with a negative sum at the zero point of time,
b) the cash flow pattern changes its sign twice, the first change is by the end of the first year, and the second one takes place during the final year of project period,
c) the total profit sum according to nominal value is negative, that is the project makes losses.

Keeping in mind the above-mentioned features, the false surplus profit can be estimated as follows [where (6) is the corrected version of formula (3)]:

$$FV\Delta M_f = \sum_{j_f=1}^{s_f-1} H_{z_f+j_f}\left(1+i\right)^{s_f-j_f} + \Delta H_{z_f}\left(1+i\right)^{s_f} - H_n \qquad \Bigg| \; \sum_{t=1}^{n} B_t < \sum_{t=0}^{n} K_t \,; \quad i>0 \tag{6}$$

$FV\Delta M_f$ = false surplus profit at the end of the project period.
z_f = the number of return years of the initial investment and its profit requirements (including the last commenced year).
ΔH_{z_f} = temporary surplus profit appearing in the year of the return of the initial investment and its profit requirements.
$H_{z_f+j_f}$ = the yearly occurring temporary surplus profit after the z_f year.
j_f = the serial number of years of duration after the return of initial investment and its profit requirements ($j_f = t - z_f$).
s_f = the number of years of the period following the return of initial investment and its profit requirements ($s_f = n - z_f$).
B_t = revenues in year t ($t=1...n$).
K_t = expenditures in year t ($t= 0…n$).
t = serial number of years (in the s_f time section: $t = z_f + j_f$).
n = duration of the project ($n = z_f + s_f$).

As a corrected version of formula (4), the calculation of the false net present value is as follows:

$$NPV_f = [\sum_{j_f=1}^{s_f-1} H_{z_f+j_f}\,(1+i)^{s_f-j_f} + \Delta H_{z_f}\,(1+i)^{s_f} - H_n\,]\,\frac{1}{(1+i)^n} \qquad \Bigg| \sum_{t=1}^{n} B_t < \sum_{t=0}^{n} K_t\ ;\ i>0 \tag{7}$$

NPV_f = false net present value.

After reduction:

$$NPV_f = \sum_{j_f=1}^{s_f-1} \frac{H_{z_f+j_f}}{(1+i)^{z_f+j_f}} + \frac{\Delta H_{z_f}}{(1+i)^{z_f}} - \frac{H_n}{(1+i)^n} \qquad \Bigg| \sum_{t=1}^{n} B_t < \sum_{t=0}^{n} K_t\ ;\ i>0 \tag{8}$$

Formula (6) demonstrates that the net present value method handles the temporary surplus profit and its non-existent interest income as homogeneous payback elements. Formula (8) shows that in this case the net present value is not the sum of the discounted surplus profits. (A project with losses cannot produce surplus profit.)

Illustrative examples and explanations

The structure of the data of the example to be shown is similar to that of the examples in *Table 3*. The annual yields of Project C are as follows: units -100, +625, -625. The two internal rates of return are 25 and 400%. Thus, at each interest rate between 15 and 400 percent, the net present value has a positive sum and in the case of interest rates which fall out of this band, a negative net present value appears. The yield structure at interest rates of 15 and 27% is presented in *Table 4*.

Table 4

The content structure of the yields of Project C at 15 and at 27 percent interest rate

Measurement unit: unit

Percent	The structure of 625 units at the end of 1st year			Surplus profit/deficit calculated for the end of 2nd year			NPV
	For capital returns	For interest	Temporary surplus profit	Carried over	False interest income	At end of 2nd year	
15%	100	15	510	510	76.5*	586.5-625= -38.5	-29.1
27%	100	27	498	498	134.5*	632.5-625= 7.5	4.6

*The false interest income: 510×0.15 =76.5 and 498×0.27=134.5

According to the conventional calculation method the two net present values are calculated as follows:

$NPV_{C15\%}$ = - 100 + 625×0.86957 - 625×0.75614 = -100 + + 543.5 -472.6 = -29.1

$NPV_{C27\%}$ = -100 + 625×0.78740 - 625×0.62000= -100 + +492.1-387.5 = 4.6

According to formula (7), discounting of the temporary surplus profit and the final year's negative cash flow also leads to the same net present value:

$NPV_{C15\%}$ = 510×0.86957- 625×0.75614= -29.1

$NPV_{C27\%}$ = 498×0.78740 - 625×0.62000= 4.6

The data of *Table 4* show that the main content problem is the utilization of false interest income. This mechanism can be realized at both positive and negative net present value. In the reality, at the end of the first year the whole yield of 625 units quits the project. As a part of this the temporary surplus profits also quit. (If they will reinvest, their yield will be among the results of another project.) In the line of the 15% interest rate the temporary surplus profit is 510 units. Its false interest income is 76.5 units. The sum of these two items is not enough to cover the expenditure of 625 units at the end of the year. Contrary to this, the false interest income of 134.5 units at 27% together with the 498-unit temporary surplus profit is enough to meet the return requirement according to the method, and even surplus profit remains. Due to the false surplus profit's discounting the false net present value is 4.6 units.

Unorthodox cash flow patterns and internal rate of return

In the case of unorthodox cash flow patterns the main point of the problem with the IRR method is the same as that explored above. The only difference is that in this case there is no surplus false interest income, nor lack of it. Here the false interest income is in two parts. The first part is as much as needed to eliminate the loss and the second part is equal to the sum of the internal rates of return. (The net present value calculated with an internal rate is always zero.)

The two false internal rates of return formulated by false interest income support the view that these rates do not have a sensible economic content, and they do not give any useful information. *Table 5* presents the yield structure behind two internal interest rates of Project C.

Table 5

The content structure of the yields for Project C at 25 and 400 percent interest rate

Measurement unit: unit

Percent	The structure of 625 units at the end of the 1st year			The false total return at the end of the 2nd year		
	For capital returns	For interest	Temporary surplus	Carried over	False interest income	The covering of 625 units expenditure
25%	100	25	500	500	125*	625-625 = 0
400%	100	400	125	125	500*	625-625 = 0

*The calculation of false interest income: 500×0.25 = 125 and 125×4 = 500

In the given structure the false interest incomes of 125 and 500 units are not generated by the project. At the rate of 25% the false interest income covers the 100-unit loss and the 25 units that are needed for interest at that rate. At 400%, the 500 units cover the 100-unit loss, and the 400 units needed for interest (125 = 100 +25 and 500 = 100 + 400).

THE HIDDEN PITFALL AND THE POSSIBILITIES OF AVOIDANCE

It is well known that investment projects with unorthodox cash flow patterns have only one net present value. To this end, the literature suggests applying the net present value method during the evaluation of projects. This paper uses two citations to show the nature of suggestions. Arnold & Hope (1990:259) say, "In view of the technical difficulties associated with IRR, it is always preferable to use NPV to evaluate projects with unorthodox cash flows." Bierman & Smidt express an even more explicit view when they say, "In this case a simple calculation of the net present value of the investment at the appropriate rate of discount would have provided the correct answer and would have bypassed the problem of multiple internal rates of return" (Bierman & Smidt 2012:95).

It is beyond doubt that in contrast to the unmanageable information content of multiple internal rates of return, the net value calculations provide only one type of final result for unorthodox cash flow patterns. However, this sort of net present value uses non-existent interest income and for this reason lacks a meaningful economic content. It is playing with numbers that determines whether the net present value will be negative or positive.

If a company based on a positive net present value decides to invest in the implementation of a loss-making project, it creates a loss resource, which may be extremely disadvantageous. This is the main point of the hidden pitfall of the real reinvestment rate assumption.

Focused and targeted analyses are required to determine whether there are any counterbalancing forces that may make the losses generated by a specific project worth undertaking. In order to evaluate a project, a method is to be used that properly fits the context of management and results in providing information with appropriate content. In this case there are two main methodological solutions. They are as follows:

a) Examination of a project combination based on the internal rate of return and aggregate capital needs.

b) In some cases the yield line can be divided into sections by orthodox cash flow sections, for instance according to reconstruction or overhaul system of fixed assets (in such cases the evaluation can be solved as usual).

Examination of project combinations based on the internal rate of return and aggregate capital needs

If the company has a project possibility with orthodox cash flow patterns which is combined with the examined unorthodox one, and the combined project's cash flow patterns still remain orthodox, it is sensible to examine the two projects together as a project combination. It is very important to note that only the IRR method can be effective for comparison. The NPV and its traditionally derived indexes are not suitable for comparing projects.

First, the IRR of the project with orthodox cash flow patterns and its aggregate capital needs should be quantified, and then those of the project combination. The term “aggregate capital needs” is a new business economics category. It means the capital sum that is used for the whole duration of the project. Its measurement unit is one unit of tied-up capital for one year. (Details of calculations are available in Illés 2014). As decision-making information, it needs to made clear how the internal rate of return and the aggregate capital needs of the project and those of the project combination relate to each other. The economic impact of the unorthodox cash flow patterns is favorable if the indexes of the project combination are better than those of the project with orthodox cash flow patterns that functions as part of the project combination. Further analysis also may be necessary.

An example and explanations

Using Project C described above, we can combine it with Project D to form a C & D project combination, as shown in the following example. The cash flow row of Project D is as follows: units -500, +200, +625. The total profit sum on the nominal value is 325 units; the internal rate of return is 33.6%. The results of basic calculation are given in *Table 6*.

Table 6

Internal rate of return and the aggregate capital needs of Project D and the project- combination

Measurement unit: unit

Project	Cash flow row	Total profit in nominal value	Aggregate capital needs	IRR %
C	-100, +625, -625	-100	not interpreted	25; 400
D	-500, +200, +625	325	500 + (500 – 32*) = 968	33.6
C & D combination	-600, +825, ±0	225	600	37.5

* 200 – 500 × 0.336 = -32

According to the data in *Table 6* the internal rate of return of the project combination is 3.9 percentage points higher than that of Project D. This is a favorable impact. However, the higher profitability relates to a significantly lower capital sum. (Higher profitability with a lower profit sum can only occur if the related aggregate capital needs are also lower.) The aggregate capital needs of the project combination became lower because the very high yield of Project C in the first year led to a substantial capital return.

In the given case the decision makers should decide which is more favorable for the company: 33.6 percent profitability for the capital of 968 units, or 37.5 percent for the capital of 600 units. The identification and analysis of the critical profitability rate of difference of the aggregate capital needs can improve the chance of making a good decision.

The 33.6 percent profitability of the D project can be considered excellent. The critical profitability of the aggregate capital needs difference is 27%. That is, the 368 units to be invested in another project and its 27% profitability should be achieved, which together with the project combination would reach the results ensured by the D project itself. (The control calculation is as follows:

$$\frac{0.27 \times 368 + 0.375 \times 600}{968} = 0.335$$

The theoretical background of this calculation is given in Illés 2015.) It is not very likely that the difference of 368 units of the aggregate capital needs can be invested at the rather high profitability of 27%. The critical rate of profitability of the aggregate capital needs difference can be lower if combined with a project with lower profitability (assuming that among the projects to be implemented there are also such projects).

The method of project combination analysis has a series of simplification possibilities (an example of a simplified version is in Illés 2007).

SUMMARY

The nearly six-decade debate about the reinvestment rate assumption is in connection with the ranking conflict between the net present value and the internal rate of return. Nevertheless, a correct ranking conflict will not appear. The reason is that the NPV method is inherently unsuitable for ranking. In the case of orthodox cash flow patterns the elimination of distorting factors of comparability leads to a net present value rate, which is the difference between the internal rate of return and the required rate of return. For an equal required rate of return, the two rankings will be identical. In the case of unorthodox cash flow patterns the internal rate of return cannot generate ranking, so the ranking conflict cannot appear at this point, either. Without a real ranking conflict the long-term debate is rootless, therefore the reinvestment rate assumption coming from it does not have any sense.

However, considering the real content impact of the methods, the question of whether some kind of reinvestment rate assumption is realized by calculation automatism is important. On the basis of the yield structure formation analysis, this paper proves logically and mathematically that a special reinvestment rate assumption exists. In the case of NPV this assumption concerns only the surplus profit, and applies the required rate of return as reinvestment rate assumption. In the case of orthodox cash flow patterns, this assumption does not disturb the evaluation of investment projects.

For orthodox cash flow patterns the IRR method does not applies any reinvestment rate assumption. The IRR covers the whole profit sum and there is no surplus profit. Accordingly, the automatic reinvestment rate assumption cannot be realized.

However, in the case of unorthodox cash flow patterns the content relations strongly differ from the orthodox one. The study constructs a model for examinations. The model is based on the wide range of types in unorthodox cash flow patterns that frequently occur in the literature. Its characteristics: the initial investment is relatively low; a very high income surplus occurs at the end of the first year, then the final year finishes with a very high expenditure surplus. Furthermore, this examined type of cash flow pattern is loss-making and has two internal rates of return.

In this model the NPV and the IRR methods are of similar character as far as the yield assumption is concerned. Automatic yield assumption plays an important role in both methods. The NPV method realizes the yield assumption according to the required rate of return, while the IRR method does so according to the internal rate of return. In the examined type of projects there are no surplus profits because of the loss, only temporary surplus profits. With the automatism of the two methods the temporary surplus profits generate interest income, but these are false (non-existing sums). Both methods handle the false interest income as real money, and they use these false sums for covering the losses and meeting profit requirements, too. In the case of IRR method the false interest income apparently covers the losses and the profit is needed according to the interest rate. In the case of positive NPV the false interest income is able to cover the real losses, meet the profit requirements, and the unused false interest income can even emerge as a surplus profit. Discounting of this false surplus profit leads to the false NPV. The false interest income of temporary surplus profit results in net present values and internal rates of return with false content.

If a company decides to invest in the implementation of a loss-making project with a positive NPV this may be extremely disadvantageous. If there is no profit, the profitability expectancy cannot be realized. On implementing the loss-making project the losses of the project become real. In this case the NPV is misleading. This is the main point of the hidden pitfall of the real reinvestment rate assumption.

One reasonable step is to examine the question if there is a situation when it is worth taking up a project with losses. The analysis can go into two main directions. One of them is the examination of project as a part of a project combination with an orthodox cash flow pattern. The other one is dividing the yield line into sections by orthodox cash flow sections, if possible.

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